Chapter 01 Electrostatics

CONTENT

➢ The branch of physics which deals with phenomena of charge (study of forces, fileds and potentials) at rest is called electrostatics.

1. ELECTRIC CHARGE :

It is the internal property of certain elementary particle like electron or proton.

➢ Charge produces and experiences electric and magnetic effects.
➢ S.I. unit of charge is coulomb(C).
➢ C.G.S. unit of charge = esu = stat coulomb and franklin
1 coulomb = 3×10^9 stat coulomb (stat C)
➢ M.K.S. unit of charge = e.m.u. = Absolute coulomb (abC)

1 coulomb = $\frac{1}{10}$ absolute coulomb $= \frac{1}{10}$.

1 electromagnetic unit (emu) = 1 ab coulomb = 10C
1C = 3×10^9 stat C ab C

➢ Dimensional formula of charge = $[M^0L^0T^1A^1]$

Charge is of two types :
(i) Positive charge (ii) Negative charge

➢ 1C positive charge is equal to –1C negative charge. Here negative sign represents that the nature of positive charge is opposite to the nature of negative charge.

1.1 Properties of Charge

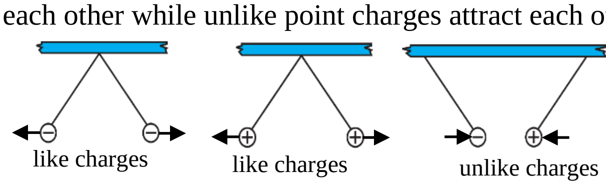
➢ Charge is a scalar quantity.
➢ Charge always resides on mass & associated with mass.

Particle	Relative mass	Relative charge	Charge (C)	Mass (kg)
Electrons	0.0005	–1	-1.6×10^{-19}	9.1×10^{-31}
Protons	1	+1	$+1.6 \times 10^{-19}$	1.67×10^{-27}
Neutrons	1	Neutral	0	1.67×10^{-27}

➢ **Charge is conserved:** In an isolated system, total charge remains constant whatever change takes place in that system.

SPOT LIGHT
Quantum of charge : It is the minimum amount of charge that can exist freely in nature.

➢ **Quantization of charge:** Charge is quantized and quantum of charge is ±e i.e., charge on a body always exists in integral multiples of a fundamental unit of electric charge. So charge on a body is Q = ± ne, where n is an integer and e is the electric charge and carried by protons and electrons.
➢ Like point charges repel each other while unlike point charges attract each other.



➢ Charge cannot exist without mass though mass can exist without charge.

- **Invariant Property of charge:** Charge is frame independent quantity i.e. charge on a body does not change whatever be its speed. The mass of a body depends on its speed and increases with increase in speed.

  $m = \dfrac{m_0}{\sqrt{1 - v^2/C^2}}$, m_0 = Rest mass, v = speed of object, c = speed of light

- A charge at rest produces only electric field around itself; a charge having uniform velocity produces electric as well as magnetic field around itself while an accelerated charge produces electric field, magnetic field and emits electromagnetic radiation.
- **Additivity of charge**: Since charge is a scalar quantity so, to calculate the total charge of a system it is added algebraic (with proper sign) the all-individual charges in the system.

DETECTIVE MIND

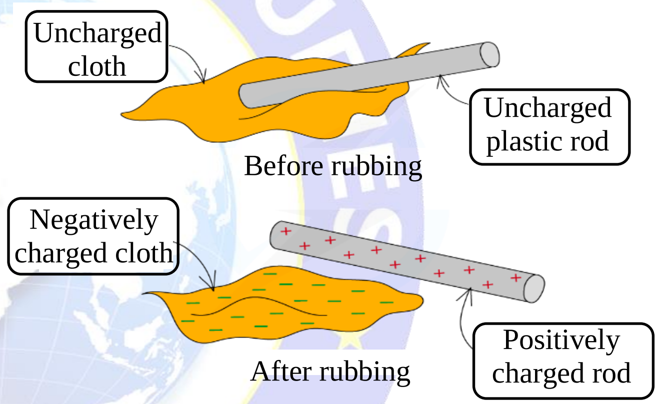
Charge cannot be created or destroyed. It can be converted from one from to another in any body. While charging a body we can transfer only electron, but not the proton. As the protons are present in the nucleus, to transfer the proton one will have to break the nucleus and after breaking it the existance of atom will be vanished out.

1.2 Charging of a body :

A body can be charged by means of (A) friction, (B) conduction, (C) induction

(A) Charging by Friction :

When we rub a body with the other, due to friction heat is generated between them this heat energy removes the loosely bounded electrons from one body and the liberated electrons may enter into the other body. In consequence, one body becomes positively charged by losing electrons and the other will get a net negative charge by gaining electrons. for example, when a glass rod is rubbed with silk, the glass gets a positive charge and the silk, becomes negatively charged.



Atmospheric electricity such as charge deposition in clouds occur due to its friction with air.

DETECTIVE MIND

For charging a conductor with frictional electrification it should be hold by the help of an insulated handle otherwise charge will flow from the conductor through other body to the ground.

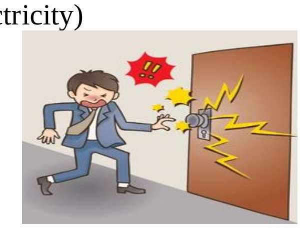
STATIC ELECTRICITY (OR SIMPLY STATIC CHARGE)

It is an electrical discharge caused by imbalance of electric charges within or on the surface of a material.

As due to air friction hazards of inflammable points appeared on the truck tank carrying highly inflammable products like oil or gas so a chain is hung back of the truck tank to ground the excess charges appeared on the surfaces of the tank. Another example of static electricity is hand shaking or touching something by hand. Due air friction charges are accumulated on the tip of fingers and during shaking the hand or touching something electric sparks are created caused by electric discharge (static electricity)

Chain hanghing in inflammable tank truck

Sparking in hand shaking

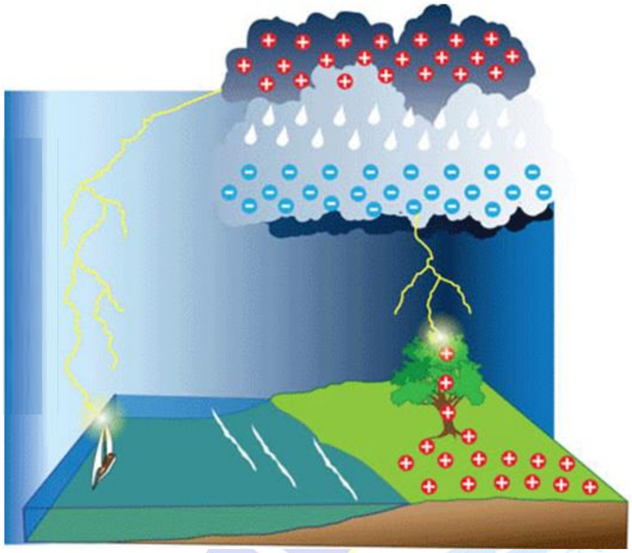


Sparking in touching the door

DETECTIVE MIND

Static charges are accumulated where curvature of the surfaces are greater.

Lightning: it is also on electrical discharge event. Lightning happens when the negative charges (electrons) in the bottom of the cloud are attracted to the positive charges (protons) in the ground.



Collision of ice particles polarize Electrical charges within the clouds

Negative charges at the bottom (due to heavy mass than that of positive charge) of clouds induce positive charges on the ground

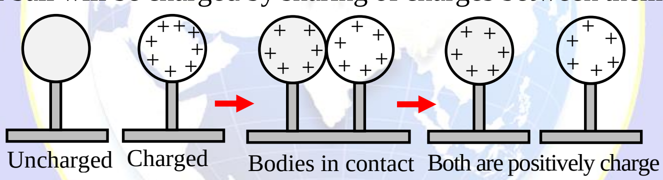
Negative leaders extend from the clouds

Positive streamers rise from the earth

The two join and create a channel through which lightning strikes

(B) Charging by Conduction

When we touch a charged body (a charged conductor) charge flows from the body to earth through our body. This process is called conduction (flow) of charge. If you touch (or connect) a neutral metallic ball to a charged metallic ball, the neutral ball will be charged by sharing of charges between them by the process of conduction.

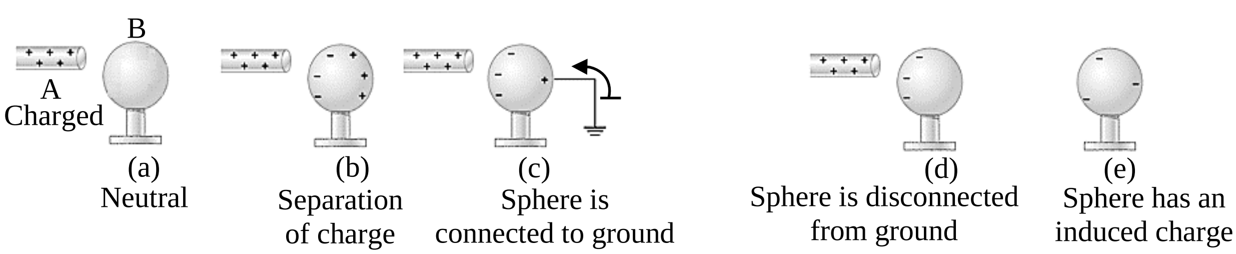


SPOT LIGHT

If two conducting spheres are connected then the net charge on the spheres get divided in the ratio of their radius. ($Q \propto R$)

(C) Charging by Induction

When a neutral body B is kept near a positively charged body A the neutral body induces a negative charge nearer to the charged body and equal positive charge at its opposite side. If the body B is a conductor, by connecting it with earth, excess electrons flow from earth to neutralize the induced positive charges leaving it as a negatively charged body. This process of charging a neutral body without touching it to a charged body is called "Induction".

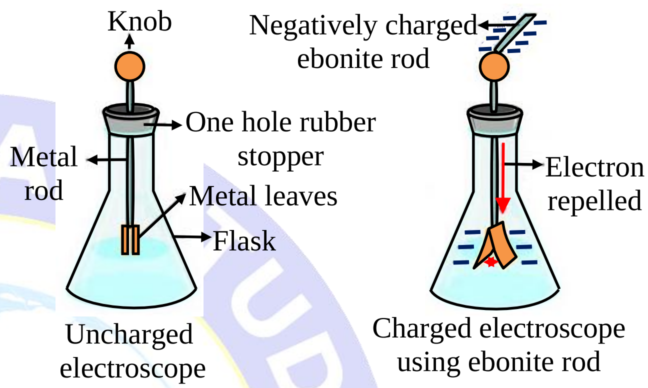


SPOT LIGHT

If there is attraction between two bodies then one of them may be neutral. But if there is repulsion between two point like bodies, both must be charged (similarly charged).
So **"repulsion is the sure test of electrification".**

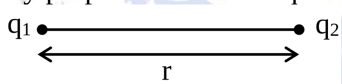
DETECTION OF CHARGE: ELECTROSCOPE (Gold Leaf)

An electroscope is one of the simplest device used to determine electric charge. An uncharged electroscope can detect only whether an object is electrically charged or not. If the electroscope is previously charged with a known nature of charge, it can then also determine the nature of the charge on the object. An electroscope consists of two strips of thin aluminum or gold foil(Leaf) fastened to a metal rod and connected to the knob at the top. The rod is supported and also separated from the frame by means of wood fittings. When a charged body touches the knob at the top of the electroscope, the leaves diverge, indicating that the electroscope is charged. Now bringing in contact another charge at the knob, if metal leaves are attracted towards each other. It means the nature of second charge (unknown nature of charge) is opposite to the previously given (known nature) charge.



2. COULOMB'S LAW (INVERSE SQUARE LAW):

The magnitude of electrostatic force between two point charges is directly proportional to the product of charges and inversely proportional to the square of the distance between them.



Charles Augustin de Coulomb (1736–1806)

i.e. $F \propto q_1q_2$ and $F \propto \frac{1}{r^2}$

$\Rightarrow \quad F \propto \frac{q_1q_2}{r^2} \qquad \Rightarrow F = \frac{Kq_1q_2}{r^2}$

Important points regarding Coulomb's law :

- It is applicable only for point charges.
- Measured the quantity of forces of attraction or repulsion between point charges and concluded the inverse square law.

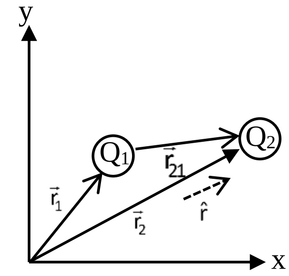
SPOT LIGHT

A torsion balance is a sensitive device to measure force. It was also used later by Cavendishto measure the very feeble gravitational force between two objects, to verify Newton's Law of Gravitation.

- The constant of proportionality(coulomb's constant) K in SI units in vacuum is expressed as $\frac{1}{4\pi\varepsilon_0}$ and in any other medium expressed as $\frac{1}{4\pi\varepsilon_M}[\varepsilon_M = \varepsilon_0.\varepsilon_r]$.
- If charges are dipped in a medium then electrostatic force on one charge is $\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$. ε_0 and ε_r are called permittivity of vacuum and absolute permittivity of the medium respectively. The ratio $\varepsilon_M / \varepsilon_0 = \varepsilon_r$ is called relative permittivity or dielectric constant of the medium, which is a dimensionless quantity.

- The value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2\ C^{-2}$

  $\Rightarrow \quad \varepsilon_0 = 8.855 \times 10^{-12}\ C^2/Nm^2$
- The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium, in which they lie.
- The force is conservative in nature i.e., work done by electrostatic force in moving a point charge along a closed loop of any shape is zero.



- Vector form

  $$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q_1Q_2}{|\vec{r}|^2}\hat{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q_1Q_2}{|\vec{r}|^3}\vec{r}$$

  Here $\vec{r}$ is position vector of the test charge (on which force is to be calculated) with respect to the source charge (due to which force is to be calculated) and $\vec{F}_{2\text{ on }1}$ and $\vec{F}_{1\text{ on }2}$ are action-reaction pair of forces.
- The force exerted by one charge on another is independent of presence of other charge.

DETECTIVE MIND

Coulomb's law is also applicable for the regular or irregular bodies if their dimensions are negligible with respect to the distance between them.

3. FORCES BETWEEN MULTIPLE CHARGES (PRINCIPLE OF SUPERPOSITION)

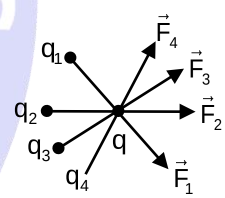
Force on any charge due to a number of other charges is the vector sum of all the forces.

The individual forces are unaffected due to the presence of other charges.

This is termed as the principle of superposition.

therefore force on a point test charge due to many charges is given by

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$



4. ELECTRIC FIELD :

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force. Electric field is generated by the electric charge.

4.1 Electric field intensity $\vec{E}$:

Electric field intensity at a point is equal to the electrostatic force experienced by a unit positive point charge, both in magnitude and direction.

If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$ due to some charges (called source charges), the electric field intensity at that point due to source charges is given by $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

or simply $\vec{E} = \frac{\vec{F}}{q_0}$

DETECTIVE MIND

Test charge (q_0) is taken small so that force on test charge can be small, So that its position can not change while finding the force.

4.2 Properties of electric field intensity $\vec{E}$:

- It is a vector quantity. Its direction is the same as the force experienced by unit positive charge.
- Its S.I. unit is Newton/Coulomb [N/C].

- Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is given by $\vec{F} = q\vec{E}$.

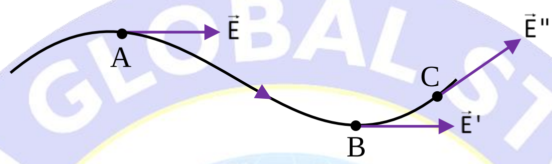
  Here if q = +ve force on charge q will be in the direction of electric field and if q = –ve a force on charge q will be in opposite direction of electric field.
- It obeys the superposition principle, that is, the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

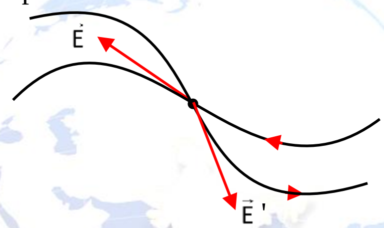
5. ELECTRIC FIELD LINES (EFL):

Electric field lines is imaginary lines .It is in general is a curve drawn in such away that the tangent to it at each point is in the direction of the electric field at that point.

(i) An Electric field line of force (Electric field line) is a hypothetical line such that tangent drawn at any point on it gives direction of electric field (i.e. net electric field) at that point.

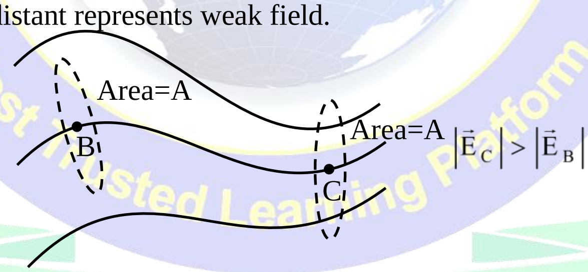


(ii) Two EFL can never intersect as at a point there cannot be two direction of electric field.

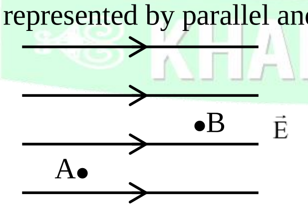


Here two direction are possible at a single point which is wrong

(iii) Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant represents weak field.



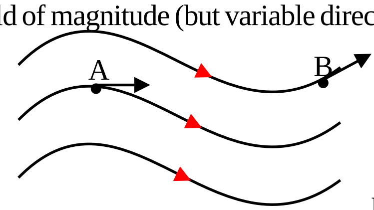
(iv) Uniform electric field is represented by parallel and equally spaced electric lines of forces.



$\vec{E}_A = \vec{E}_B$

(Magnitude and direction both are same)

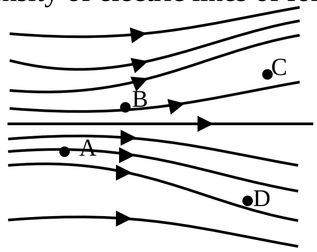
(v) Non-uniform electric field of magnitude (but variable directions) is represented by equally spaced electric field lines.



$|\vec{E}_A| = |\vec{E}_B|$

(Magnitudes are same but directions different)

(vi) Non-uniform electric field of different magnitude and different directions is represented by un-equally spaced with different relative density of electric lines of forces.

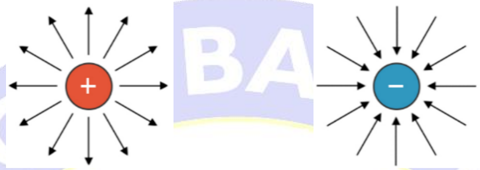


$\vec{E}_A \neq \vec{E}_B \neq \vec{E}_C \neq \vec{E}_D$

(Magnitudes and directions are different at all points)

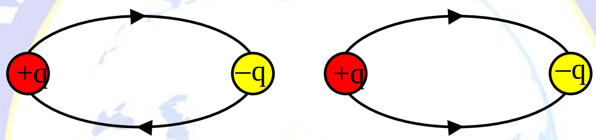
(vii) Lines of forces starts from (+ve) charge or infinity and ends on (-ve) charge or infinity.

Electric field line



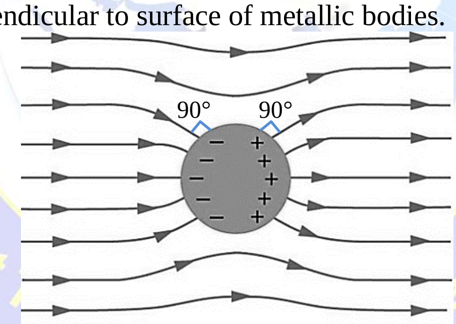
Positive charge Negative charge

(viii) Can never forms closed loops



Form close loop [✗] Does not form close loop [✓]

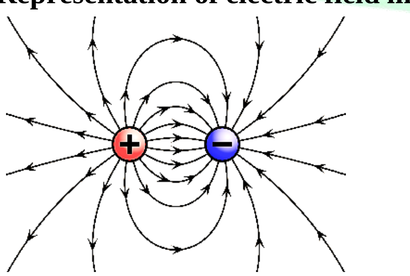
(ix) Electric field lines always perpendicular to surface of metallic bodies.



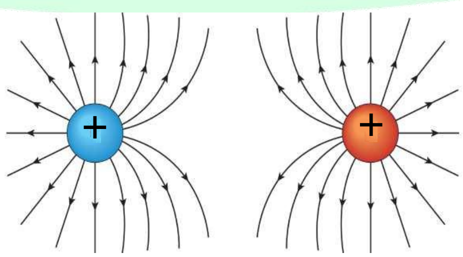
(x) If there is no electric field there will be no lines of force.

(xi) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalized MKS system, $1/\varepsilon_0$ electric lines are associated with unit charge, so if a body encloses a charge q, total lines of force associated with it (called flux) will be q/ε_0.

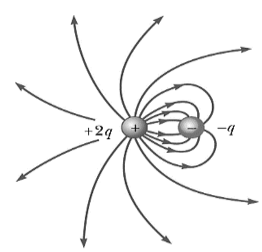
5.1 Representation of electric field lines:



EFL of two charges of equal magnitude but opposite nature



EFL of two positive charges of equal magnitude but same nature



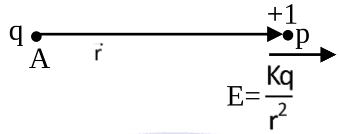
EFL of unequal magnitude of charges but opposite nature

DETECTIVE MIND

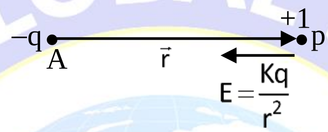
Drawback of electric field lines is, it does not give the quantitative magnitude of electric field it gives only relative magnitude of electric field

6. ELECTRIC FIELD DUE TO A POINT CHARGE:

➢ Electric field due to positive point charge, $E = \dfrac{kq}{r^2}$



➢ Electric field due to negative point charge, $E = \dfrac{kq}{r^2}$



➢ Vector form equation of electric field $\vec{E} = \dfrac{Kq}{r^2}\hat{r}$. Here charge must be placed with proper sign.

$\boxed{\vec{E} = \dfrac{kq}{r^2}\hat{r}}$ or $\boxed{\vec{E} = \dfrac{kq}{r^3}\vec{r}}$

Also, $E = |\vec{E}|$ or $\boxed{E = \dfrac{kq}{r^2}}$

- $\vec{r} = \vec{r}_P - \vec{r}_A$ = Position vector of P with respect to A. Put value of charge with sign.

DETECTIVE MIND

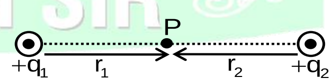
In the formula $E = \frac{Kq}{r^2}$ the charge q is taken without sign however, in the formula $\boxed{\vec{E} = \dfrac{kq}{r^2}\hat{r}}$ charge is taken with its proper sign.

6.1 Null Point: It is a point in space where the net electric field due to all near by charges is zero.

6.2 Null Point Identification:

Case: 1 For charges of same nature and unequal magnitude null point will lie on the line of joining of charges and **in between the two charges**

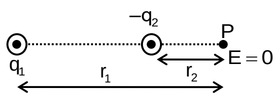
at point 'p' $\Rightarrow \dfrac{kq_1}{r_1^2} = \dfrac{kq_2}{r_2^2} \Rightarrow \boxed{\dfrac{r_1}{r_2} = \dfrac{\sqrt{q_1}}{\sqrt{q_2}}}$



Case: 2 For charges of opposite nature & different magnitude null point will lie on the line of joining of the charges, **closer to smaller magnitude charge & will be on exterior point**

here $|q_1| > |q_2|$

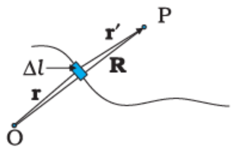
Here $\dfrac{r_1}{r_2} = \dfrac{\sqrt{q_1}}{\sqrt{q_2}}$



7. ELECTRIC FIELD INTENSITY DUE TO CONTINUOUS CHARGES

Continuous charge distribution

Line charge distribution

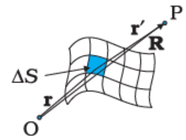


Charges are distributed over length

Linear charge density,

$(\lambda) = \dfrac{\Delta Q}{\Delta \ell} C/m$

Surface or superficial charge distribution

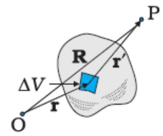


Charges are distributed over surface

Surface charge density,

$(\sigma) = \dfrac{\Delta Q}{\Delta S} C/m^2$

Volume charge distribution



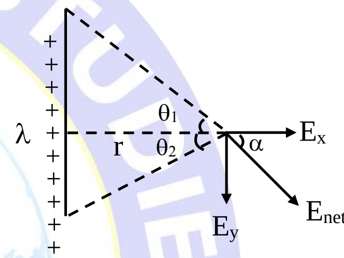
Charges are distributed over volume

Volume charge density,

$(\rho) = \dfrac{\Delta Q}{\Delta V} C/m^3$

7.1 Electric field due to uniformly charged wire

(A) Line charge of finite length :



$E_x = E_{\perp} = \dfrac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2]$

$E_y = E_{||} = \dfrac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]$

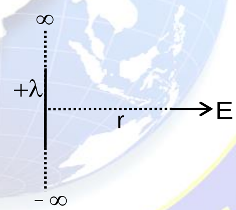
Net electric field at the point $E_{net} = \sqrt{E_x^2 + E_y^2}$

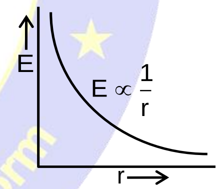
$\tan\alpha = \dfrac{E_y}{E_x}$

(B) Infinitely long line charge

In above eq. put $\theta_1 = \theta_2 = 90°$

$E_{net} = E_x = \dfrac{2K\lambda}{r}$; $E_y = 0$

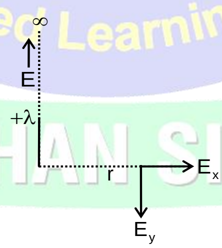




(C) For Semi- infinite wire

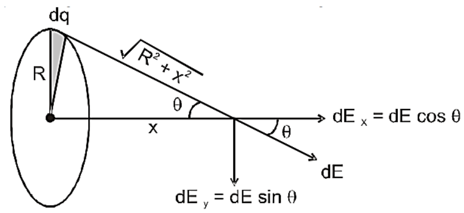
$\theta_1 = 90°$ and $\theta_2 = 0°$

So $E_x = \dfrac{K\lambda}{r}$, $E_y = \dfrac{K\lambda}{r} \Rightarrow E_{net} = \dfrac{\sqrt{2}k\lambda}{r}$

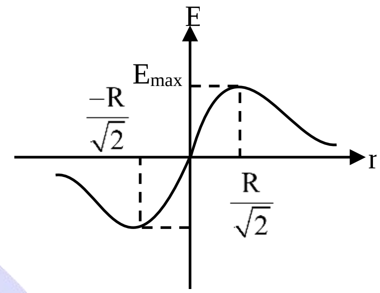


SPOT LIGHT
If the charges on the wire are negative, the direction of electric field will be reversed.

7.2 Electric field due to uniformly charged circular ring at an axial point :



As y-component of electric field due to all the elements will be cancelled out to each other so net electric field intensity at the point will be only due to X-component of each element.



$$E_{net} = \int dE_x = \int dE\cos\theta = \int_O^Q \frac{K(dq)}{R^2+x^2} \times \frac{x}{\sqrt{R^2+X^2}}$$

or $E_P = \dfrac{kQx}{\left(R^2+x^2\right)^{3/2}}$

At centre of ring x = 0 so $E_0 = 0$

E will be maximum when $\dfrac{dE}{dx} = 0$, that is at $x = \dfrac{R}{\sqrt{2}} \Rightarrow \qquad E_{max} = \dfrac{2KQ}{3\sqrt{3}R^2}$

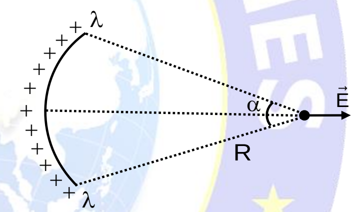
Case (A) : if x>>R, $E = \dfrac{KQ}{x^2}$, Hence the ring will behave like a point charge.

Case (B) : if x<<R, $E = \dfrac{KQx}{R^3}$, $E \propto x$

7.3 Segment of ring :

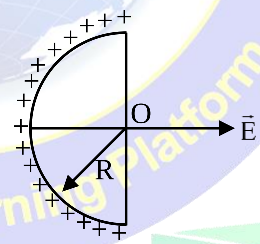
(A) Uniformly charged arc of ring (E at center) :

$$E_0 = \frac{2K\lambda}{R}\sin\frac{\alpha}{2}$$



(B) Uniformly charged semicircular ring :

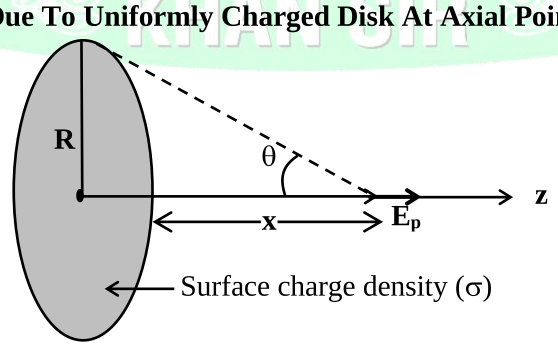
(E at center)



$$E_0 = \frac{2K\lambda}{R} = \frac{Q}{2\pi^2 \epsilon_0 R^2}$$

$\because \quad \lambda = \dfrac{Q}{\pi R}$

Electric Field Intensity Due To Uniformly Charged Disk At Axial Point



$$E_p = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right] = \frac{\sigma}{2\varepsilon_0}[1-\cos\theta],$$

along the axis, passing through the centre and $\perp$ to the plane of disc.

Case: (i) If x >> R

$$E_p = \frac{\sigma \pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

i.e. behaviour of the disc is like a point charge.

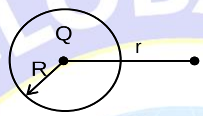
Case : (ii) If x << R

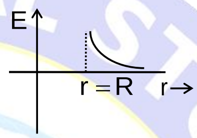
$$E_p = \frac{\sigma}{2\varepsilon_0}[1-0] = \frac{\sigma}{2\varepsilon_0}$$ i.e. behaviour of the disc is like infinite sheet.

7.4 Electric field due to uniformly charged spherical shell (Conducting or nonconducting)

E = 0 for r < R

$E = \frac{KQ}{r^2}$ for r ≥ R



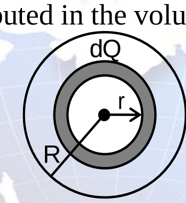


For the outside points & point on the surface the uniformly charged spherical shell behaves as a point charge placed at the centre

Electric field due to spherical shell at outside point is always along the radial direction.

7.5 Electric field due to uniformly charged solid non conducting sphere of radius R

If total charge Q which is uniformly distributed in the volume.



(A) Exterior Point (r ≥ R) :

Assume an elementary concentric shell of charge dq. Due to this shell the electric field at the point (r > R) will be $dE = \frac{Kdq}{r^2}$ [from above result of hollow sphere]

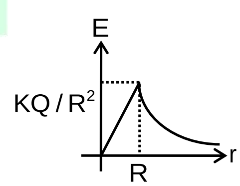
$$E_{net} = \int dE = \frac{KQ}{r^2}$$

(B) Interior Point

For r ≤ R

There will be no electric field due to shell of radius greater than r

$$E'_{net} = \frac{KQ'}{r^2} \text{ here } Q' = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$$



$$E'_{net} = \frac{KQ'}{r^2} = \frac{KQr}{R^3}$$ away from the centre.

DETECTIVE MIND

The electric field inside and outside the sphere is always in radial direction.

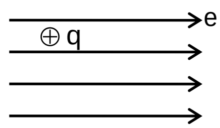
(C) In terms of charge density

$$E = \frac{\rho r}{3\varepsilon_0}$$

8. MOTION OF CHARGE IN A UNIFORM ELECTRIC FIELD

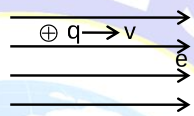
8.1 A charge is released from rest in a uniform electric field, then

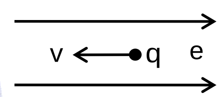
- Force on charge = qe
- Acceleration of charge = $\frac{qe}{m}$
- Velocity after time 't' = $\frac{qe}{m}t$
- displacement after time 't' = $\frac{1}{2}\frac{qe}{m}t^2$
- Path of particle will be straight line



8.2 A charge enters in a uniform electric field with a velocity v parallel to the electric field.

- Path of particle will be straight line
- Acceleration of particle will be = $\frac{qe}{m}$
- Velocity after time 't' = v + $\frac{qe}{m}t$
- Displacement after time 't' =vt + $\frac{1}{2}\frac{qe}{m}t^2$
- If the negative charge enters along the field or positive charge enters opposite to the field then the speed of particle will first decrease then increase.



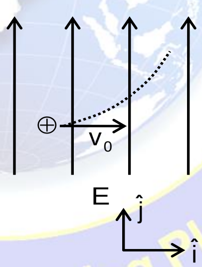


8.3 A charge enters in a uniform electric field perpendicularly

- Path of particle will be parabolic
- acceleration of particle = $\frac{qe}{m}\hat{j}$
- velocity of particle after time 't'

  $= v_0\hat{i} + \frac{qe}{m}(t)\hat{j} = v_0\hat{i} + \frac{qe}{m}t\hat{j}$
- Displacement of particle after time 't'

  $= v_0 t\hat{i} + \frac{qe}{2m}t^2\hat{j}$



SOLVED EXAMPLES

Ex.1 How many electrons must be removed from a body to make it electrified by 6.4C of charge?

(1) 2×10^{19} (2) 4×10^{19} (3) 2×10^{20} (4) 6×10^{20}

Sol. From Q = ne

$$\Rightarrow n = \frac{Q}{e} = \frac{6.4}{1.6\times10^{-19}} \qquad \Rightarrow n = 4 \times 10^{19}$$

Ex.2 10^{10} electron are ejected per second from a body, then after how much time, the body will acquire a charge of –8 μC ?

(1) 5×10^5 (2) 5×10^4 (3) 4×10^5 (4) 5×10^3

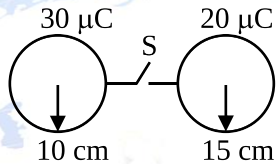
Sol. Charge appear per second on body $Q = \left(\frac{N}{t}\right)e$

$Q = 10^{10} \times (-1.6 \times 10^{-19})$

$Q = -1.6 \times 10^{-9}$

$$t = \frac{-8\times10^{-6}}{-1.6\times10^{-9}} = 5\times10^3 \text{ sec}$$

Ex.3 Find the charge transferred between the spheres when switch S is closed.



(1) Zero (2) 30μC (3) 10μC (4) 20μC

Sol. Total charge of the system Q =30μC+20μC= 50μC

Then after conduction $\frac{Q_1'}{Q_2'} = \frac{R_1}{R_2} = \frac{10}{15} = \frac{2}{3}$

$$Q_1' = \left(\frac{2}{2+3}\right)50\mu C = 20\mu C$$

$$Q_2' = \left(\frac{3}{2+3}\right)50\mu C = 30\mu C$$

Charge transferred = 10μC

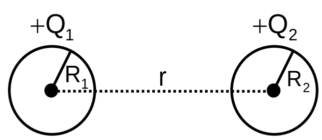
Ex.4 Two point charges $+q_1$ and $+q_2$ are placed r meter apart in vacuum. Force on q_1 due to q_2 is F. If now a third charge $+q_3$ is placed at mid point of line joining the charges, then force on q_1 due to q_2

(1) Increases (2) Decreases (3) Remains same (4) Can't say

Sol. $F_{12} = \frac{kq_1q_2}{r^2}$

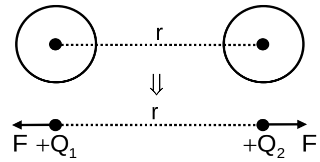
Force on charge q_1 due to charge q_2 is is independent of presence of charge q_3

Ex.5 Find force acting between two shells of radius R_1 and R_2, which have uniformly distributed charges Q_1 and Q_2 respectively and distance between their centre is r.



Sol. The shells can be replaced by point charges kept at centre so force between them

$$F = \frac{KQ_1Q_2}{r^2}$$



Ex.6 Two particles having charges q_1 and q_2 when kept at a certain distance, exert a force F on each other. If the distance between the two particles is reduced to half and the charge on each particle is doubled then what will be the force between the particles :

(1) F **(2) 4F** **(3) 8F** **(4) 16F**

Sol. $\because \quad F = \dfrac{kq_1q_2}{r^2}$ If $q'_1 = 2q_1,\ q'_2 = 2q_2,\ r' = \dfrac{r}{2}$

then $F' = \dfrac{kq'_1q'_2}{r'_2} = \dfrac{k(2q_1)(2q_2)}{\left(\dfrac{r}{2}\right)^2}$

$F' = \dfrac{16kq_1q_2}{r^2} \Rightarrow F' = 16F$

Ex.7 A particle of mass m carrying charge q_1 is revolving around a fixed charge $-q_2$ in a circular path of radius r. Calculate its speed.

(1) $\sqrt{\dfrac{q_1q_2}{2\pi\varepsilon_0 mr}}$ **(2)** $\sqrt{\dfrac{q_1q_2}{4\pi\varepsilon_0 mr}}$ **(3)** $\sqrt{\dfrac{4\pi\varepsilon_0 mr}{q_1q_2}}$ **(4)** $\sqrt{\dfrac{2\pi\varepsilon_0 mr}{q_1q_2}}$

Sol. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2} = mr\omega^2 = \dfrac{4\pi^2 mr}{T^2}$

$T^2 = \dfrac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2}$ or $T = 4\pi r\sqrt{\dfrac{\pi\varepsilon_0 mr}{q_1q_2}}$

and also we can say that $\dfrac{q_1q_2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r}$

$\Rightarrow \quad v = \sqrt{\dfrac{q_1q_2}{4\pi\varepsilon_0 mr}}$

Ex.8 Two charged pendulums of charges $\dfrac{2}{3}$ μC & $\dfrac{10}{3}$ mC are in equilibrium when they are 1 m Apart. If the pendulums are identical then what will be the mass of each pendulum if the angle between the strings in equilibrium is 60°.

(1) $2\sqrt{3}$ kg **(2)** $4\sqrt{3}$ kg **(3)** $\dfrac{2}{\sqrt{3}}$ kg **(4)** $\dfrac{4}{\sqrt{3}}$ kg

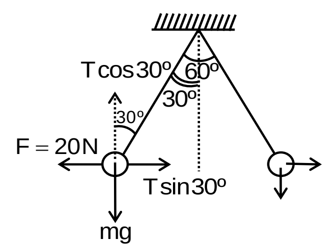
Sol. Force between the charges

$F = \dfrac{(9\times10^9)\left(\dfrac{2}{3}\times10^{-6}\right)\left(\dfrac{10}{3}\times10^{-3}\right)}{1^2}$

F = 20 N

For balancing

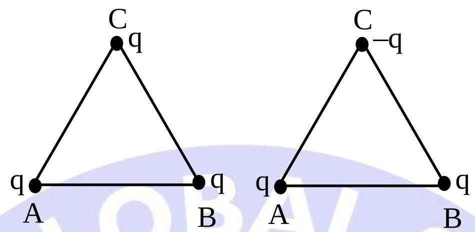
T sin 30° = 20 T cos 30° = mg



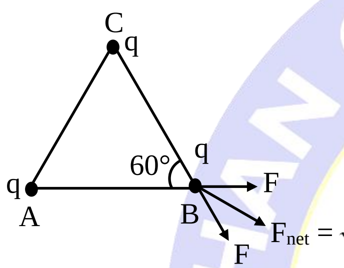
$$\Rightarrow \quad \frac{T\sin 30^\circ}{T\cos 30^\circ} = \frac{20}{mg}$$

$$\tan 30^\circ = \frac{20}{mg} \Rightarrow m = 2\sqrt{3}\ \text{Kg}$$

Ex.9 Find net force on the charge, placed at B, in the given arrangements

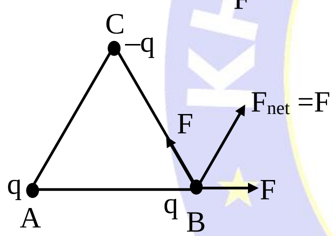


Sol.



$$F = \frac{kq^2}{r^2}$$

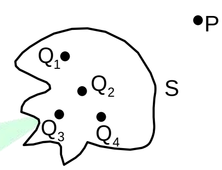
$$F_{net} = \frac{\sqrt{3}kq^2}{r^2}$$



$$F = \frac{kq^2}{r^2}$$

$$F_{net} = \frac{kq^2}{r^2}$$

Ex.10 Electrostatic force experienced by –3μC charge placed at point 'P' due to a system 'S' of fixed point charges as shown in figure is $\vec{F} = (21\hat{i} + 9\hat{j})$ μN.



(i) Find out electric field intensity at point P due to S.

(ii) If now 2μC charge is placed and –3 μC is removed at point P then force experienced by it will be.

Sol. (i) $\vec{F} = q\vec{E} \Rightarrow (21\hat{i} + 9\hat{j})\mu N = -3\mu C(\vec{E})$

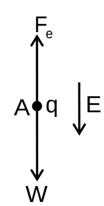
$\Rightarrow \vec{E} = -7\hat{i} - 3\hat{j}$

(ii) $\vec{F}_{2\mu C} = +2(\vec{E}) = 2(-7\hat{i} - 3\hat{j}) = (-14\hat{i} - 6\hat{j})\mu N$

Ex.11 Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge –10 μc and mass 10 mg. (take g = 10 ms^2)

(1) 5 N/C **(2) 10 N/C**

(3) 15 N/C **(4) 20 N/C**



Sol. $|\vec{F}_q| = |\vec{W}|$ i.e., $|q|E = mg$

i.e., $E = \frac{mg}{|q|}$ = 10 N/C in downward direction

Ex.12 Find out electric field intensity at point A (0, 1m, 2m) due to a point charge –20μC situated at point B($\sqrt{2}$ m, 0, 1m).

(1) $-22.5 \times 10^3 (-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$ N/C **(2)** $-45 \times 10^3 (-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$ N/C

(3) $-22.5 \times 10^3 (\sqrt{2}\hat{i} - \hat{j} - \hat{k})$ N/C **(4)** $-45 \times 10^3 (\sqrt{2}\hat{i} - \hat{j} - \hat{k})$ N/C

Sol. $\vec{E} = \dfrac{KQ}{|\vec{r}|^3}\vec{r}$

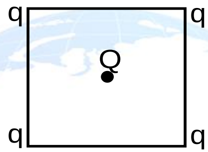
$\Rightarrow$ $\vec{r}$ = Position vector of A – Position vector of B $\vec{r} = \vec{r}_A - \vec{r}_B = (-\sqrt{2}\,\hat{i} + \hat{j} + \hat{k})$

$|\vec{r}| = \sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2} = 2$

$\vec{E} = \dfrac{9\times10^9 \times (-20\times10^{-6})}{8}(-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$

$= -22.5 \times 10^3 (-\sqrt{2}\hat{i} + \hat{j} + \hat{k})$ N/C

Ex.13 What has to be the value of Q for the system to be in equilibrium

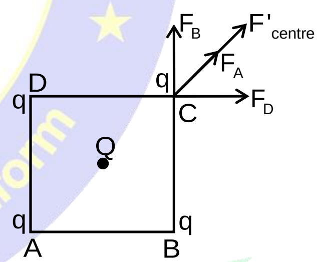


(1) $-\left[\dfrac{q}{4} + \dfrac{q}{\sqrt{2}}\right]$ **(2)** $-\left[\dfrac{q}{2\sqrt{2}} + \dfrac{q}{\sqrt{2}}\right]$ **(3)** $-\left[q + \dfrac{q}{\sqrt{2}}\right]$ **(4)** $-\left[\dfrac{q}{4} + \dfrac{q}{2\sqrt{2}}\right]$

Sol. Charge Q is at centre where E = 0

So Q is in equilibrium for any charge 'q' to be in equilibrium

$F_D = F_B = \dfrac{Kq^2}{a^2}$; $F_A = \dfrac{Kq^2}{2a^2}$, $F_{centre} = \dfrac{2KQq}{a^2}$



For equilibrium $= \dfrac{2KQq}{a^2}$

$= -\left(\dfrac{Kq^2}{2a^2} + \sqrt{2}\dfrac{Kq^2}{a^2}\right) \Rightarrow 2Q = -\left(\dfrac{q}{2} + \sqrt{2}q\right)$;

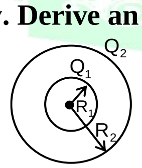
$Q = -\left[\dfrac{q}{4} + \dfrac{q}{\sqrt{2}}\right]$

Ex.14 Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.

(i) $r < R_1$

(ii) $R_1 \le r < R_2$

(iii) $r \ge R_2$



Sol. (i) for $r < R_1$, therefore point lies inside both the spheres

$E_{net} = E_{Inner} + E_{outer} = 0 + 0$

(ii) for $R_1 \le r < R_2$, therefore point lies outside inner sphere but inside outer sphere:

$E_{net} = E_{inner} + E_{outer} = \dfrac{KQ_1}{r^2}\hat{r} + 0 = \dfrac{KQ_1}{r^2}\hat{r}$

(iii) for $r \ge R_2$ point lies outside inner as well as outer sphere therefore.

$$E_{Net} = E_{outer} + E_{outer} = \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} = \frac{K(Q_1+Q_2)}{r^2}\hat{r}$$

Ex.15 Find electric field on the axis of a uniformly charged ring (Q,R) at a distance x = $2\sqrt{2}$ R.

(1) $\frac{2\sqrt{2}kQ}{R^2}$ **(2)** $\frac{2\sqrt{2}kQ}{9R^2}$ **(3)** $\frac{3\sqrt{3}kQ}{8R^2}$ **(4)** $\frac{2\sqrt{2}kQ}{27R^2}$

Sol. $\because E = \frac{kQx}{\left(R^2+x^2\right)^{3/2}}$

$$x = 2\sqrt{2}R$$

$$E = \frac{kQ2\sqrt{2}R}{\left(R^2+(2\sqrt{2}R)^2\right)^{3/2}} = \frac{kQ2\sqrt{2}R}{\left(9R^2\right)^{3/2}}$$

$$E = \frac{kQ2\sqrt{2}R}{27R^3} = \frac{2\sqrt{2}kQ}{27R^2}$$

Ex.16 An electron is allowed to fall through some distance in uniform electric field in time t_1. Now the direction of electric field is reversed and proton is allowed to fall through same distance in time t_2. Then relation between t_1 and t_2.

(1) $t_1 = t_2$ **(2) $t_1 > t_2$** **(3) $t_1 < t_2$** **(4) Can't say**

Sol. $\because$ $s = \frac{1}{2}at^2$

$$t \propto \frac{1}{\sqrt{a}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{a_p}{a_e}} = \sqrt{\frac{m_e}{m_p}} = \frac{1}{\sqrt{1837}} \Rightarrow t_1 < t_2$$

Ex.17 A particle of mass 1g and charge 3μC is projected with velocity 5 m/s in direction perpendicular to a uniform electric field of 1000 V/m. Find

(a) its speed after 4 second **(b) deviation after 4 second.**

Sol. (a) $v_x = u = 5$ m/s; $v_y = \left(\frac{qE}{m}\right)t$

$$v_y = \left(\frac{3\times10^{-6}\times10^3}{10^{-3}}\right)\times 4 \Rightarrow v_y = 12 m/s$$

Speed after 4 second $v = \sqrt{v_x^2+v_y^2} = \sqrt{5^2+12^2} = 13 m/s$

(b) Deviation $\rightarrow$ y , $y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2 = \frac{1}{2}\times3\times4^2 \Rightarrow y = 24$ m

9. ELECTRIC FLUX (ϕ) :

It is the measure of number of electric line of force passing through some surfaces

Electric flux

Qualitative definition of flux

↓

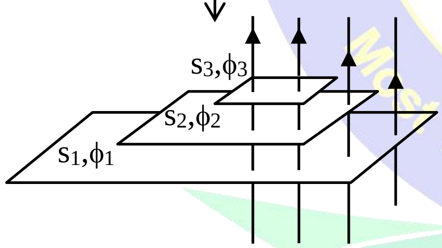
It is the measure of number of electric lines of forces passing through some surfaces.

↓

If more number of EFL passes through the surface more will be the flux

↓

For example :
Say the flux through surface s_1 is ϕ_1 that through surface s_2 is ϕ_2 and that through surface s_3 is ϕ_3 then



$\phi_1 > \phi_2 > \phi_3$

Quantitative definition of flux → It is the dot product of the electric field and surface area through which EFL are passing, ($\phi = \vec{E} \cdot d\vec{s}$)

For open surface

↓

$\phi = \int d\phi = \int \vec{E} \cdot d\vec{s}$

$\phi_c = \vec{E}.\vec{S} = E\ S\cos\theta$

$\vec{s}$ = always normal to surface and pointed out wards

For close surface

↓

$\phi_c = \oint \vec{E}.d\vec{s}$ (By defination of flux)

$\phi_c = \dfrac{q_{net}}{\epsilon_0}$ or $4\pi k q_{net}$ (Gauss's law)

$d\vec{s}$ = always normal to surface and pointed out wards

Important facts about flux

(i) It is a scalar quantity

(ii) Units (V-m) and N-m^2/C, Dimensions :[$ML^3T^{-3}A^{-1}$]

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

(iv) The value of ϕ is zero in the following circumstances:

(a) If a dipole is (or many dipoles are) enclosed by a closed surface

(b) Magnitude of (+ve) and (-ve) charges are equal inside a closed surface

(c) If no charge is enclosed by the closed surface

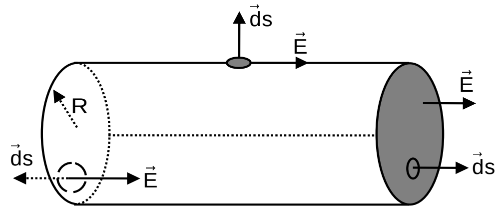
(d) Incoming flux (- ve) = out going flux (+ve)

SPOT LIGHT

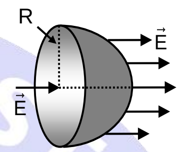
Area vector: It is a vector whose magnitude is equal to area of the surface an direction is perpendicular to the surface

9.1 Some Important application of flux linked with surface when electric field is uniform

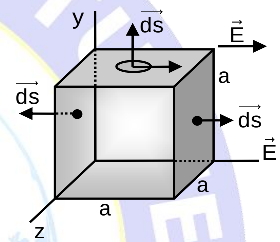
(A) $\phi_{in} = -\pi R^2 E$ and $\phi_{out} = \pi R^2 E$ So, $\phi_{total} = 0$



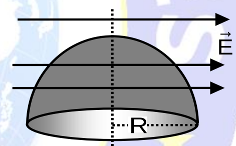
(B) $\phi_{in} = \phi_{circular} = -\pi R^2 E$ and $\phi_{out} = \phi_{curved} = \pi R^2 E$ So, $\phi_{total} = 0$



(C) $\phi_{in} = -a^2 E$ and $\phi_{out} = a^2 E$ So, $\phi_{total} = 0$



(D) $\phi_{in} = -\frac{1}{2}\pi R^2 E$ and $\phi_{out} = \frac{1}{2}\pi R^2 E$ So, $\phi_{total} = 0$



DETECTIVE MIND

Electric flux linked with any closed surface in a uniform electric field is always zero.

10. GAUSS THEOREM

The total flux linked with a closed surface is $\frac{1}{\varepsilon_0}$ times the charge enclosed by the closed surface (gaussian surface) i.e. $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$, $\boxed{\phi_E = \frac{q}{\varepsilon_0}}$ q = total charge enclosed, ε_0 = Electric constant

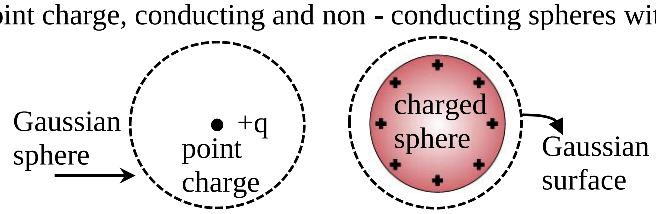
DETECTIVE MIND

Gauss law is used to calculate for symmetrical charge distribution and valid for all vector fields obeying inverse square law. However, it is applicable for all kinds of charge distributions either it is symmetrical or nonsymmetrical.

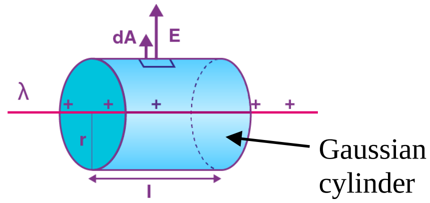
10.1 Gaussian surface

(i) Is imaginary closed surface.

(ii) Is spherical for a point charge, conducting and non - conducting spheres with same centre.



(iii) Is cylindrical for infinite line of charges, uniformly charged long cylindrical bodies.



DETECTIVE MIND

Gaussian surfaces are chosen in such a way that either the magnitude of electric field at its all point of surface be same or flux linked through that surface be zero.

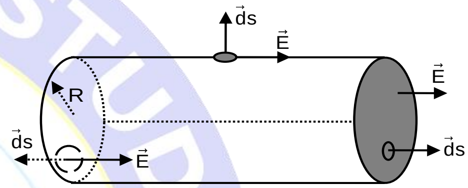
We select a Gaussian surface such that determination of $\oint \vec{E}.\vec{ds}$ can be done in simplest way (by symmetry)

Note:(i) Flux through gaussian surface is independent of its shape.

(ii) Flux through gaussian surface depends only on total charge present inside the gaussian surface.

(iii) Flux through gaussian surface is independent of position of charges inside the gaussian surface.

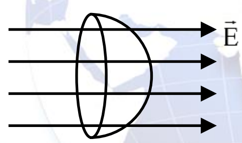
(iv) Electric field intensity at the gaussian surface is due to all the charges present inside as well as outside the gaussian surface.



(v) In a closed surface incoming flux is taken negative while outgoing flux is taken positive, because outward direction is taken as positive.

$\phi_{incoming} = -\pi R^2 E$ and $\phi_{outgoing} = \pi R^2 E$

(vi) In a gaussian surface $\phi = 0$ does not imply $E = 0$ at every point of the surface but $E = 0$ at every point implies $\phi = 0$.



$\phi_{\text{hemisphere}} = 0$ But $\vec{E} \neq 0$

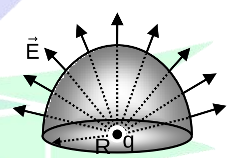
Without electric field

$E = 0$ represents that $\phi_{\text{through each point of the gaussian surface}} = 0$

10.2 Calculation of flux using symmetry

(1) Charge placed at centre of sphere

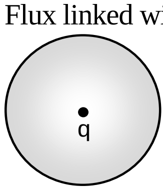
(a) Flux linked with hemisphere



$$E = \frac{kq}{R^2},\ \phi = 2\pi R^2 \times \frac{q}{4\pi\varepsilon_0 R^2} = \frac{q}{2\varepsilon_0}$$

Note : here electric field is radial

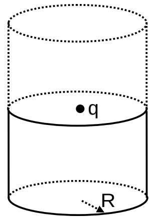
(b) Flux linked with complete sphere



$$\phi_{Total} = \frac{q}{\varepsilon_0}$$

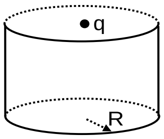
(2) Charge placed at centre of cylinder

(a) Flux linked with complete cylinder



$$\phi_{Total} = \frac{q}{\varepsilon_0}$$

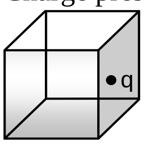
(b) Flux linked with half of cylinder



$$\phi_{cylinder} = \frac{q}{2\varepsilon_0}$$

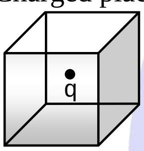
(3) Charge placed in a cube

(i) Charge present at any one of the face of a cube



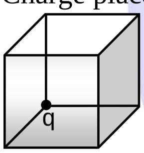
$$\phi_{cube} = \frac{q}{2\varepsilon_0}$$

(ii) Charged placed at centre



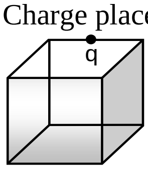
$$\phi_{cube} = \frac{q}{\varepsilon_0}$$

(iii) Charge placed at any one corner



$$\phi_{cube} = \frac{q}{8\varepsilon_0}$$

(iv) Charge placed at anyone arm of cube



$$\phi_{cube} = \frac{q}{4\varepsilon_0}$$

11. CALCULATION OF ELECTRIC FIELD (APPLICATION OF GAUSS LAW):

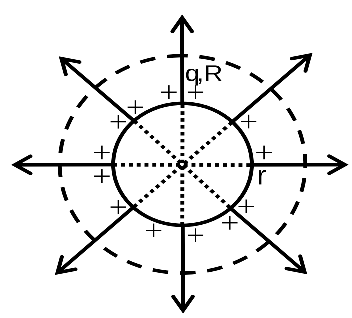
From gauss`s theorem, we can say

$$\oint \vec{E}.d\vec{s} = \phi_{net} = \frac{q_{in}}{\varepsilon_0}$$

11.1 Electric field due to a uniformly charged spherical shell:

(A) At Exterior Point

Since, electric field due to a shell will be radially outwards. So let's choose a spherical Gaussian surface. Applying Gauss`s theorem for this spherical Gaussian surface



$$\Rightarrow \int \vec{E}\cdot d\vec{s} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{q}{\varepsilon_o} \Rightarrow \int E ds \cos 0° = \frac{q}{\varepsilon_0}$$

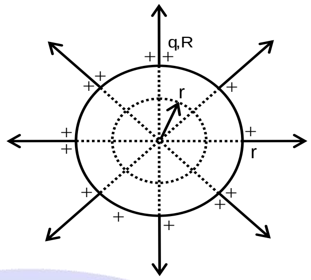
$$\Rightarrow E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$$

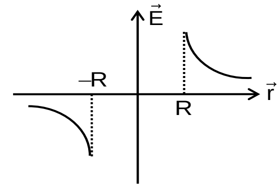
$\Rightarrow \quad E_{out} = \dfrac{q}{4\pi\varepsilon_o r^2} = \dfrac{kq}{r^2} \Rightarrow E_{out} \propto \dfrac{1}{r^2}$

(B) At Interior Point

Let's choose a spherical gaussian surface inside the shell.

Applying Gauss`s theorem for this surface





$\int \vec{E}.d\vec{s} = \phi_{net} = \dfrac{q_{in}}{\varepsilon_0} = 0$

Since angle between $\vec{E}$ and $d\vec{s}$ is 0°

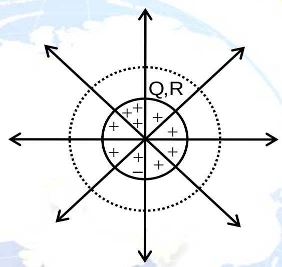
$\vec{E}\cdot d\vec{s} = E(4\pi r^2) = 0 \qquad E_{in} = 0$

11.2 Electric field due to solid sphere (having uniformly distributed charge Q and radius R) :

(A) At Exterior Point

Direction of electric field is radially outwards, so we will choose a spherical gaussian surface Applying Gauss`s theorem

$\int \vec{E}.d\vec{s} = \phi_{net} = \dfrac{q_{in}}{\varepsilon_o} = \dfrac{Q}{\varepsilon_o}$



Since angle between and is 0°

$\vec{E}.d\vec{s} = E(4\pi r^2) = \dfrac{Q}{\varepsilon_0};$

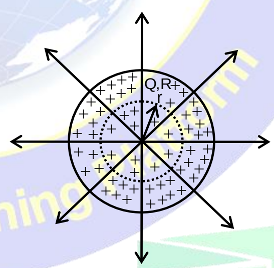
$E_{out} = \dfrac{Q}{4\pi\varepsilon_o r^2} = \dfrac{kQ}{r^2} \Rightarrow E_{out} \propto \dfrac{1}{r^2}$

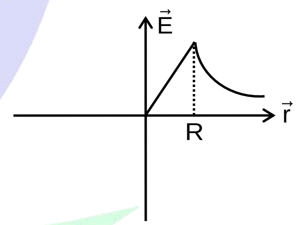
(B) At interior point

For this choose a spherical gaussian surface inside the solid sphere. Applying gauss`s theorem for this surface

$\oint \vec{E}.d\vec{s} = \phi_{net} = \dfrac{Q_{in}}{\varepsilon_0} = \dfrac{\dfrac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{\varepsilon_0} = \dfrac{Qr^3}{\varepsilon_o R^3}$

$E = \dfrac{Qr}{4\pi\varepsilon_o R^3} \Rightarrow E_{in} = \dfrac{kQ}{R^3} r \Rightarrow E \propto r$





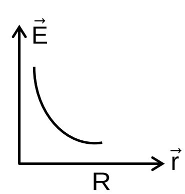
11.3 Electric field due to infinite line charge (having uniformly distributed charge of charge density λ):

Electric field due to infinite wire is radial so we will choose cylindrical Gaussian surface as shown is figure.

$\phi_{net} = \phi_1 + \phi_2 + \phi_3$

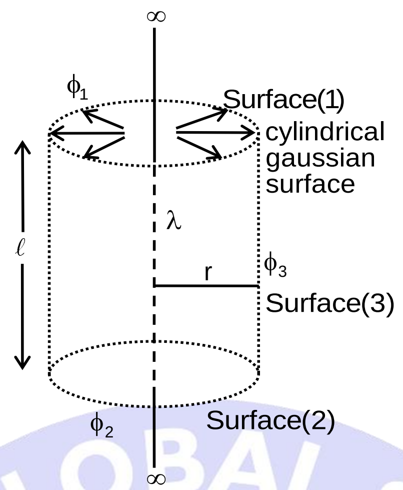
$\because \quad \phi_1 = \phi_2 = 0(\theta = 90°) = \dfrac{q_{in}}{\varepsilon_o} = \dfrac{\lambda\ell}{\varepsilon_o}$

$\phi_3 = \int \vec{E}.d\vec{s} = \int E\, ds\cos\theta = E\int ds = E(2\pi r\ell)$

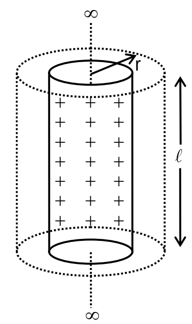


$\Rightarrow \quad E\,(2\pi r\ell) = \dfrac{\lambda\ell}{\varepsilon_o}$

$\Rightarrow \quad E = \dfrac{\lambda}{2\pi\varepsilon_o r} = \dfrac{2k\lambda}{r} \propto \dfrac{1}{r} \qquad \Rightarrow \quad E \propto \dfrac{1}{r}$



11.4 Electric field due to infinity long charged tube (having uniform surface charge density σ and radius R):



(A) E outside the tube : - lets choose a cylindrical gaussian surface. Applying gauss's theorem

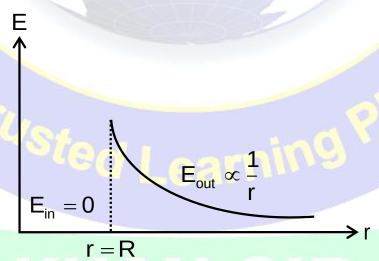
$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\sigma 2\pi R\ell}{\varepsilon_o}$$

$$\Rightarrow \quad E_{out} \times 2\pi r\ell = \frac{\sigma 2\pi R\ell}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\sigma R}{r\varepsilon_o} \Rightarrow E \propto \frac{1}{r}$$

(B) E inside the tube: let's choose a cylindrical gaussian surface inside the tube. Applying gauss's theorem

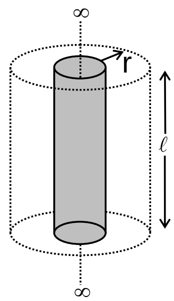
$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = 0 \quad \text{So} \quad E_{in} = 0$$



11.5 E due to infinitely long solid cylinder of radius R having uniformly distributed charge in volume (charge density ρ) :

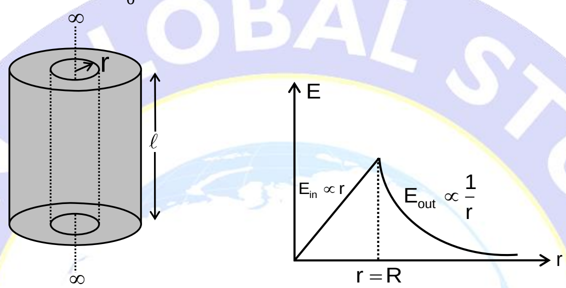
(A) E at outside point :-Let's choose a cylindrical gaussian surface.
Applying gauss's theorem

$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi R^2 \ell}{\varepsilon_o} \Rightarrow E_{out} = \frac{\rho R^2}{2r\varepsilon_o}$$



(B) E at inside point : let's choose a cylindrical gaussian surface inside the solid cylinder.
Applying gauss`s theorem

$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi r^2 \ell}{\varepsilon_o} \Rightarrow E_{in} = \frac{\rho r}{2\varepsilon_o}$$



11.6 Electric field due to a conducting and nonconducting uniformly charge infinite sheets

Suppose Q charge is given to

Conducting plate | Non – conducting plate

Electric field for both the cases

$$E = \frac{Q}{2A\varepsilon_o}$$

$$E = \frac{\sigma_{conducting}}{\varepsilon_o} \qquad E = \frac{\sigma_{non-conducting}}{2\varepsilon_o}$$

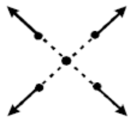
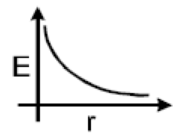
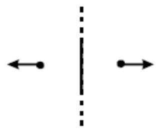
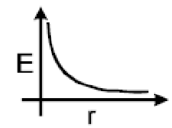
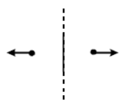
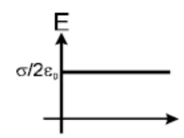
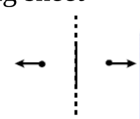
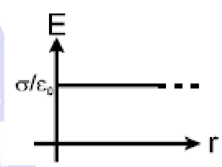
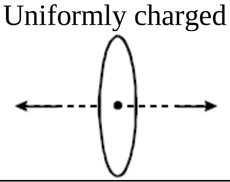
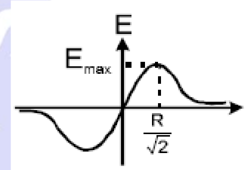
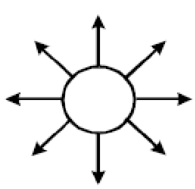
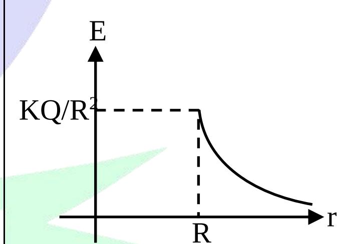
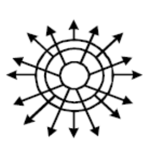
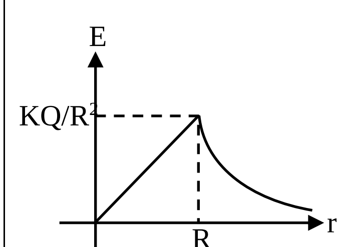
where $\sigma_{conducting} = \frac{Q}{2A}$

Because Q is distributed in '2A' area.

where $\sigma_{non-conducting} = \frac{Q}{A}$

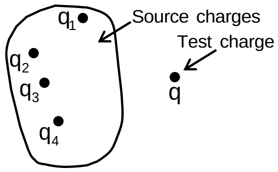
Because Q is distributed in 'A' area.

List of formula for Electric Field Intensity due to various types of charge distribution :

Name/Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{KQ}{\lvert\vec{r}\rvert^2}.\hat{r}$	• q is source charge. • $\vec{r}$ is vector drawn from source charge to the test point. outwards due to +charges & inwards due to –charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda\hat{r}}{r}$	• q is linear charge density (assumed uniform) • r is perpendicular distance of point from line charge. • $\hat{r}$ is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	• σ is surface charge density. (assumed uniform) • $\hat{n}$ is unit normal vector.	
lnftnltely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	• σ is the surface charge. density (assumed uniform) • $\hat{n}$ is the unit vector perpendicular is the surface.	
Uniformly charged ring 	$E = \frac{KQx}{\left(R^2 + x^2\right)^{3/2}}$ $E_{centre} = 0$	• Q is total charge of the ring • x = distance of point on the axis from centre of the ring. • electric field is always along the axis.	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere 	(i) for r ≥R $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for r < R E = 0	• R is radius of the sphere. • $\vec{r}$ is vector drawn from center of sphere to the point. • Sphere behave like a point charge, placed at centre for points outside the sphere. • $\vec{E}$ is always along radial direction. • Q is total charge (= $\sigma 4\pi R^2$). (σ= surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material) 	(i) for r ≥R $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for r ≤R $\vec{E} = \frac{kQ}{R^3}\vec{r}$	• $\vec{r}$ is vector drawn from centre of sphere to the point • Sphere acts like a point charge placed at the centre for points outside the sphere • $\vec{E}$ is always along radial direction. • Q is total charge $(\rho.\frac{4}{3}\pi R^3)$. (ρ = volume charge density) • lnside the sphere $E \propto r$. • Outside the sphere $E \propto 1/r^2$.	

12. ELECTROSTATIC POTENTIAL ENERGY :

12.1 Electrostatic potential energy of a point charge due to many charges:



The electrostatic potential energy of a point charge at a point in electric field is the work done in taking the charge from reference point (generally at infinity) to that point without acceleration (or keeping KE const. or $K_i = K_f$). Its Mathematical formula is

$$U = W_{ext.(\infty \to P)} = -W_{elec.(\infty \to P)}$$

Here q is the charge whose potential energy is being calculated and V is the potential at its position due to the source charges.

Note : Always put q and V with sign.

Properties :

(i) Electric potential energy is a scalar quantity but may be positive, negative or zero.

(ii) Its unit is same as unit of work or energy that is joule (in S.I. system).
Some times energy is also given in electron-volts. $1eV = 1.6 \times 10^{-19}$ J

(iii) Electric potential energy depends on reference point. (Generally Potential Energy at $r = \infty$ is taken zero)

13. ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

(This concept is useful when more than one charges move)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without accelerating it.

13.1 Types of system of charge

(i) Point charge system

(ii) Continuous charge system

13.2 Derivation for a system of point charges:

(a) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebraic sum of all the works.

Let W_1 = work done in bringing first charge

W_2 = work done in bringing second charge against force due to 1st charge.

W_3 = work done in bringing third charge against force due to 1st and 2nd charge.

$$PE = W_1 + W_2 + W_3 + \ldots\ldots$$

(This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

(b) Method of calculation (to be used in problems) U = sum of the interaction energies of the charges.

$= (U_{12} + U_{13} + \ldots\ldots + U_{1n}) + (U_{23} + U_{24} + \ldots\ldots + U_{2n}) + (U_{34} + U_{35} + \ldots\ldots.. + U_{3n}) \ldots.$

(c) Method of calculation useful for symmetrical point charge systems. Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges $= (U_{12} + U_{13} + \ldots\ldots.. + U_{1n})$

U_2 = PE of second charge due to all other charges $= (U_{21} + U_{23} + \ldots\ldots.. + U_{2n})$

then U = PE of the system $= \frac{U_1 + U_2 + U_n}{2}$

13.3 Electric potential energy for continuous charge system :

This energy is also known as self energy.

(i) P.E. (Self Energy) of a uniformly Charged spherical shell :- $U_{self} = \frac{KQ^2}{2R}$

(ii) Self energy of uniformly charged solid sphere: For a solid sphere P.E. is $U_{self} = \frac{3}{5}\frac{KQ^2}{R}$

Energy density :

Def: Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following

Energy density = $\frac{1}{2}\varepsilon_m E^2$

where E = electric field intensity at that point

$\varepsilon_m = \varepsilon_0\varepsilon_r$ electric permittivity of medium

14. ELECTRIC POTENTIAL :

In electrostatic field the electric potential (due to some point source charges) at a point P is defined as the work done by external agent in taking a unit positive point charge from a reference point (generally taken at infinity) to that point P without changing its kinetic energy.

OR

Electric potential at a point is also equal to the negative of the work done by the electric field in taking the unit point charge from reference point (i.e. infinity) to that point.

Mathematical representation:

If $W_{ext(\infty\to P)}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{W_{ext(\infty\to P)}}{q}\right|_{\Delta K=0} = \frac{-W_{elec(\infty\to p)}}{q}$$

$$= \frac{\Delta U}{q} = \frac{U_p - U_\infty}{q} = \frac{U_p}{q}$$

$W_{ext(\infty\to P)}$ can also be called as the work done by external agent against the electric force on a unit positive charge due to the source charge.

14.1 Properties :

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulmb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

(iii) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinite. (taking $V_\infty = 0$)

(iv) Potential decreases in the direction of electric field.

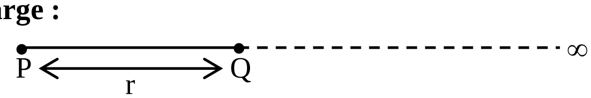
(v) $V = V_1 + V_2 + V_3 + ...$

(vi) Potential at reference point may or may not be zero.

14.2 Use of potential :

If the potential at some point (in terms of numerical value or in terms of formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula $W_{elec(p\to\infty)} = qV_p$

14.3 Potential due to a point charge :



$$V = \frac{W_{ext(\infty\to p)}}{q_o} = \frac{-\int_{\infty}^{r}(q_0\vec{E})\cdot d\vec{r}}{q_o} = -\int_{\infty}^{r}\vec{E}\cdot d\vec{r}$$

$$\Rightarrow \quad V = -\int_{\infty}^{r}\frac{KQ}{r^2}(-dr)\cos 180^\circ = \frac{KQ}{r}$$

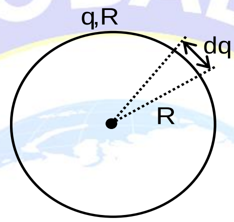
14.4 Potential due to a ring :

(A) Potential at the centre of uniformly or non uniformly charged ring :

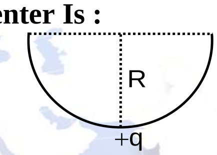
Potential due to the small element dq

$$dV = \frac{Kdq}{R} \Rightarrow \text{Net potential} V = \int\frac{Kdq}{R}$$

$$V = \frac{K}{R}\int dq = \frac{Kq}{R}$$



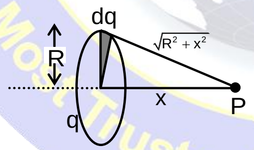
(B) Potential Due To Half Ring At Center Is :

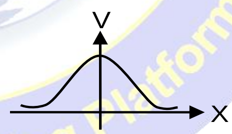


$$V = \frac{Kq}{R}$$

(C) Potential At The Axis Of A Ring:

$$V = \frac{Kq}{\sqrt{R^2 + x^2}}$$





14.5 Potential Due To Uniformly Charged Spherical shell:

As we know $V = -\int_{r\to\infty}^{r=r}\vec{E}\cdot d\vec{r}$

(A) At outside point (r > R):

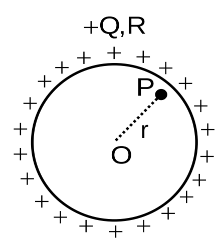
$$V_{out} = -\int_{r\to\infty}^{r=r}\left(\frac{KQ}{r^2}\right)dr \qquad \Rightarrow \qquad V_{out} = \frac{KQ}{r}$$

For outside point, the hollow sphere act like a point charge.

(B) Potential at inside point (r < R) :

Suppose we want to find potential at point P, inside the sphere.

Potential difference between Point P and O :



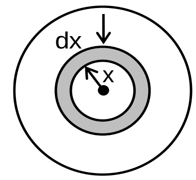
$$V_P - V_O = -\int_{O}^{P}\vec{E}_{in}\cdot d\vec{r} \text{ Where } E_{in} = 0$$

So, $V_P - V_O = 0 \quad \Rightarrow V_P = V_O = \frac{KQ}{R} \qquad \Rightarrow V_{IN} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$

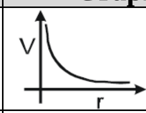
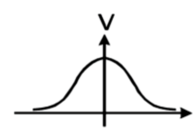
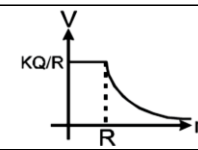
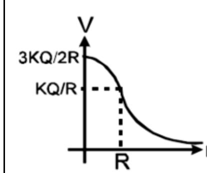
14.6 Potential Due To Uniformly Charged Solid Non Conducting Sphere :

- for $r \geq R$(outside) $V = \dfrac{KQ}{r}$
- for $r \leq R$(inside) $V = \dfrac{KQ}{2R^3}(3R^2 - r^2)$

$\Rightarrow$ Here, $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$



Electric Potential due to various charge distributions are given in table.

Name/Type	Formula	Note	Graph
Point charge	$\dfrac{Kq}{r}$	• q is source charge. • r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre $\dfrac{KQ}{R}$ at the axis $\dfrac{KQ}{\sqrt{R^2 + x^2}}$	• Q is source charge. • x is the distance of the point on the axis	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere	for $r \geq R$ $V = \dfrac{kQ}{r}$ for $r \leq R$ $V = \dfrac{kQ}{R}$	• R is radius of sphere • r is the distance from centre of sphere to the point • Q is total charge = $\sigma 4\pi R^2$.	
Uniformly charged solid nonconducting	for $r \geq R$ $V = \dfrac{kQ}{r}$ for $r \leq R$ $\dfrac{KQ(3R^2 - r^2)}{2R^3}$ $= \dfrac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	• R is radius of sphere • r is distance from centre to the point. • $V_{centre} = \dfrac{3}{2}V_{surface}$ • Q is total charge = $\rho\dfrac{4}{3}\pi R^3$ • Inside sphere potential varies parabolically • outside potential varies hyperbolically.	

15. POTENTIAL DIFFERENCE :

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from B to A without acceleration (or keeping Kinetic Energy constant or $K_i = K_f$).

➢ **Mathematical representation:**

If $W_{ext(A\to B)}$ = work done by external agent against electric field in taking the unit charge from A to B

$$V_B - V_A = \left.\frac{W_{ext(A\to B)}}{q}\right)_{\Delta K=0} = \frac{-W_{elec(A\to B)}}{q} = \frac{U_B - U_A}{q} = \frac{-\int_A^B \vec{F}_e.d\vec{r}}{q} = -\int_A^B \vec{E}.d\vec{r}$$

Note : Take W and q both with sign

15.1 Properties :

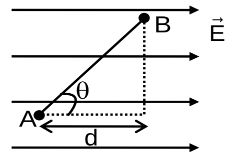
(i) The difference of potential between two points is called potential difference. It is also called voltage.

(ii) Potential difference is a scalar quantity. Its S.I. unit is volt.

(iii) If V_A and V_B be the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is $W_{ext(A\to B)} = q\,(V_B - V_A)$ or $W_{elec(A\to B)} = q\,(V_A - V_B)$.

(iv) Potential difference between two points is independent of reference point.

15.2 Potential difference in a uniform electric field :



$$V_B - V_A = -\,\vec{E}\cdot\overrightarrow{AB}$$

$$\Rightarrow\quad V_B - V_A = -\,|E|\,|AB|\cos\theta$$

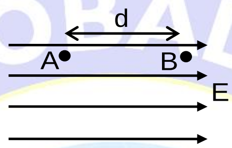
$$= -\,|E|\,d = -\,Ed$$

d = effective distance between A and B along electric field. or we can also say that

$$E = \frac{\Delta V}{\Delta d}$$

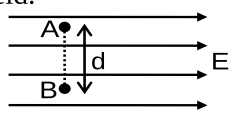
Special Cases :

Case 1. Line AB is parallel to electric field.



$$\therefore\quad V_A - V_B = Ed$$

Case 2. Line AB is perpendicular to electric field.



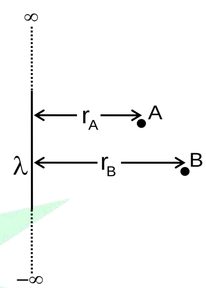
$$\therefore\quad V_A - V_B = 0$$

$$\Rightarrow\quad V_A = V_B$$

DETECTIVE MIND

If a charge is moving in the direction that it it would normally move, its electric potential energy is decreasing.

15.3 Potential difference due to infinitely long wire :

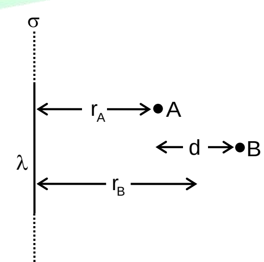


θ = angle between $\vec{E}$ & $d\vec{r} = 0$

$$V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{2K\lambda}{r}\,\hat{r}\cdot d\vec{r}$$

$$\Rightarrow\quad V_{AB} = -2K\lambda\,\ell n\left(\frac{r_B}{r_A}\right)$$

15.4 Potential difference due to infinitely long thin sheet:



$$V_{BA} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{\sigma}{2\varepsilon_0}\,\hat{r}\cdot d\vec{r}$$

$$\Rightarrow\quad V_{BA} = -\,\frac{\sigma}{2\varepsilon_0}\,(r_B - r_A) = -\,\frac{\sigma d}{2\varepsilon_0}$$

16. EQUIPOTENTIAL SURFACE

If potential of a surface (imaginary or physically existing) is same throughout then such surface is known as an equipotential surface.

16.1 Properties of equipotential surfaces :

(i) When a charge is shifted from one point to another point on an equipotential surface then work done against electrostatic forces is zero.

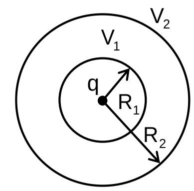
(ii) Electric field is always perpendicular to equipotential surfaces.

(iii) Two equipotential surfaces do not cross each other.

16.2 Examples of equipotential surfaces :

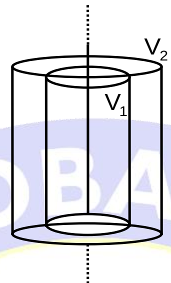
(A) Point charge :

Equipotential surfaces are concentric and spherical as shown in figure. In figure we can see that sphere of radius R_1 has potential V_1 throughout its surface and similarly for other concentric sphere potential is same.

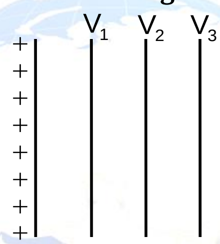


(B) Line charge :

Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.



(C) Uniformly charged large conducting / non conducting sheets : Equipotential surfaces are parallel planes.



DETECTIVE MIND

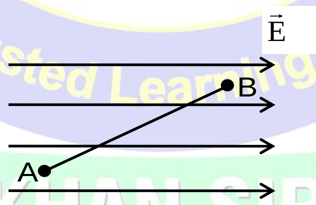
In uniform electric field equipotential surfaces are always parallel planes.

17. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL

(A) For uniform electric field :

(i) Potential difference between two points A and B

$V_B - V_A = -\vec{E}\cdot\overrightarrow{AB}$.



(B) Non uniform electric field

(i) $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

$$\Rightarrow \quad \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\left[\hat{i}\frac{\partial v}{\partial x} + \hat{j}\frac{\partial v}{\partial y} + \hat{k}\frac{\partial v}{\partial z}\right]$$

Where $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

(C) If electric potential and electric field depends only on one coordinate, say r :

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$

(ii) $\int dV = -\int \vec{E}\cdot d\vec{r} \qquad \Rightarrow V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.d\vec{r}$

(iii) The potential of a point

$\Rightarrow V = -\int_{\infty}^{r} \vec{E}.d\vec{r}$

SOLVED EXAMPLES

Ex.18 If $\vec{E} = (2\hat{i} + 3\hat{j} + \hat{k})$ N/C and $\vec{A} = (2\hat{i} - \hat{j} + \hat{k})$ m² then find electric flux.

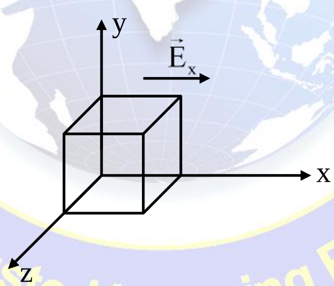
(1) 2 Nm²/C **(2) 4 Nm²/C** **(3) 5 Nm²/C** **(4) 8 Nm²/C**

Sol. $\phi = \vec{E}.\vec{A}$

$\phi = (2\hat{i} + 3\hat{j} + \hat{k}).(2\hat{i} - \hat{j} + \hat{k})$

$\phi = 4 - 3 + 1 = 2$ Nm²/C

Ex.19 If $\vec{E}_x = x^2\hat{i}$ and E_y & E_z are zero find, Net flux passing trough cube of edge 1m.



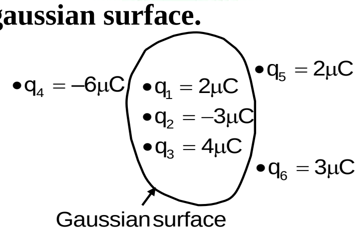
(1) 1 unit **(2) 2 unit** **(3) 4 unit** **(4) 0**

Sol. $\phi_{in} = -Ex_1\,(a)^2 = -(0)^2\,(1)^2 = 0$

$\phi_{out} = Ex_2\,(a)^2 = (1)^2\,(1)^2 = 1$

$\phi_{net} = 1$

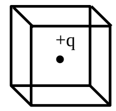
Ex.20 Find out flux through the given gaussian surface.



(1) $\frac{5\times10^{-6}}{\varepsilon_0}$ **Nm²/C** **(2)** $\frac{2\times10^{-6}}{\varepsilon_0}$ **Nm²/C** **(3)** $\frac{3\times10^{-6}}{\varepsilon_0}$ **Nm²/C** **(4)** $\frac{4\times10^{-6}}{\varepsilon_0}$ **Nm²/C**

Sol. $\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3\times10^{-6}}{\varepsilon_0}\,Nm^2/C$

Ex.21 Find the flux through each face of cube and also flux through each corner.

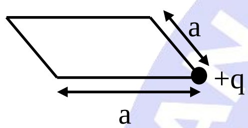


Sol. $\phi_{cube} = \frac{q}{\varepsilon_0}$

Due to symmetry, flux passing through each face will be same $\Rightarrow \phi_{face} = \frac{q}{6\varepsilon_0}$

Again due to symmetry, flux through each corner, $\Rightarrow \phi_{corner} = \frac{q}{8\varepsilon_0}$

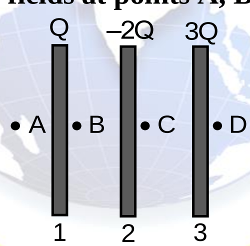
Ex.22 Find flux passing through sheet.



(1) $\frac{q}{\varepsilon_0}$ **(2)** $\frac{q}{16\varepsilon_0}$ **(3)** $\frac{q}{4\varepsilon_0}$ **(4)** 0

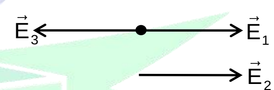
Sol. The electric field and area vector of the sheet will be perpendicular so flux through it will be zero.

Ex.23 Three large conducting sheets placed parallel to each other at a finite distance contain charges Q, –2Q and 3Q respectively. Find the electric fields at points A, B, C and D.



Sol. (i) Electric field at A is

$\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



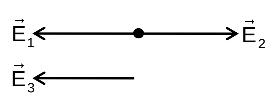
$\because$ $\vec{E}_1 = \frac{Q}{2\varepsilon_0 A}(-\hat{i}),\ \vec{E}_2 = \frac{2Q}{2\varepsilon_0 A}(\hat{i})$, and $\vec{E}_3 = \frac{3Q}{2\varepsilon_0 A}(-\hat{i})$

$\therefore$ $\vec{E}_A = \frac{Q}{2\varepsilon_0 A}(-\hat{i} + 2\hat{i} - 3\hat{i})$

$\Rightarrow$ $\vec{E}_A = \frac{Q}{\varepsilon_0 A}(-\hat{i})$

(ii) Electric field at B is

$\vec{E}_B = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



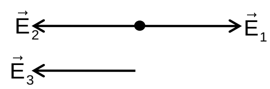
$\because$ $\vec{E}_1 = \frac{Q}{2\varepsilon_0 A}(\hat{i}),\ \vec{E}_2 = \frac{2Q}{2\varepsilon_0 A}(\hat{i})$

and $\vec{E}_3 = \frac{3Q}{2\varepsilon_0 A}(-\hat{i})$

$\therefore \quad \vec{E}_B = \dfrac{Q+2Q-3Q}{2\varepsilon_o A}(\hat{i}) = 0$

(iii) Electric field at C is

$\vec{E}_C = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



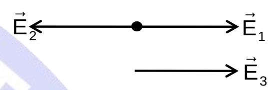
$\because \quad \vec{E}_1 = \dfrac{Q}{2\varepsilon_0 A}(\hat{i})$, $\quad \vec{E}_2 = \dfrac{2Q}{2\varepsilon_0 A}(-\hat{i})$

and $\vec{E}_3 = \dfrac{3Q}{2\varepsilon_0 A}(-\hat{i})$

$\therefore \quad \vec{E}_C = \dfrac{Q-2Q-3Q}{2\varepsilon_0 A}(\hat{i}) = \dfrac{2Q}{\varepsilon_0 A}(-\hat{i})$

(iv) Electric field at D is

$\vec{E}_D = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



$\because \quad \vec{E}_1 = \dfrac{Q}{2\varepsilon_0 A}(\hat{i})$, $\vec{E}_2 = \dfrac{2Q}{2\varepsilon_0 A}(-\hat{i})$

and $\quad \vec{E}_3 = \dfrac{3Q}{2\varepsilon_0 A}(\hat{i})$

$\therefore \quad \vec{E}_3 = \dfrac{(Q-2Q+3Q)}{2\varepsilon_0 A}(\hat{i}) = \dfrac{Q}{\varepsilon_0 A}(\hat{i})$

Ex.24 A charge of 10μC is given to a metallic sphere of radius 10 cm. Find E at a distance.

(1) r = 5cm **(2) r = 10cm**

(3) r =30cm, from centre.

Sol. (1) 5cm (r <R), so E = 0 (for conductor E= 0 at inside point)

(2) 10cm (r = R), $Es = \dfrac{kQ}{R^2}$

$E = \dfrac{9\times10^9 \times 10\times10^{-6}}{(0.1)^2} = 9\times10^6$ N/C

(3) 30cm (r = 3R),

$E'' = \dfrac{kQ}{9R^2}$ so $E'' = \dfrac{9\times10^6}{9} = 10^6$ N/C

Ex.25 Electric field at a distance 30 cm from a uniformly charged non conducting sphere of radius 15 cm is E. Find E at a distance :

(1) r = 5 cm **(2) r = 15 cm**

(3) r =20 cm, from centre.

Sol. (1) 5 cm (r <R), $E_{out} = \dfrac{kQ}{r^2}$

$E_{out} = \dfrac{kQ}{r^2} = E$ (given) or $kQ = (30)^2E$

$E_{in} = \dfrac{kQ}{R^3}(r) = \dfrac{(30)^2 E}{(15)^3}(5) = \dfrac{4}{3}E$

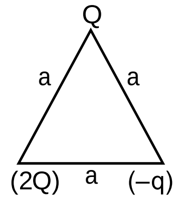
(2) 15cm (r = R), $Es = \dfrac{kQ}{R^2}$

$Es = \dfrac{(30)^2 E}{(15)^2} = 4E$

(3) 20cm (r > R),

$$E_{out} = \frac{kQ}{r^2} = \frac{(30)^2 E}{(20)^2} = \frac{9E}{4}$$

Ex.26 If electric potential energy of given system is positive then prove that 2Q > 3q



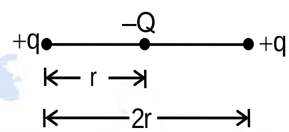
Sol. U(system) = Sum of potential energy of all pairs

$$= \frac{k(Q)(2Q)}{a} - \frac{k2Qq}{a} - \frac{kQq}{a} = \frac{kQ}{a}(2Q - 3q)$$

Given U is positive means U > 0

$$\Rightarrow \frac{kQ}{a}(2Q - 3q) > 0 \quad \Rightarrow \quad 2Q > 3q$$

Ex.27 Figure shows an arrangement of three point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\frac{q}{Q}$.



(1) $\frac{1}{2\sqrt{2}}$ **(2)** $\frac{3}{2}$ **(3)** $\frac{\sqrt{2}}{\sqrt{3}}$ **(4)** $\frac{4}{1}$

Sol. $U_{sys} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(+q)(+q)}{2r} + \frac{(-Q)q}{r}\right]$

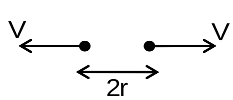
$= 0 - Q + \frac{q}{2} - Q = 0 \Rightarrow 2Q = \frac{q}{2} \Rightarrow . \frac{q}{Q} .= \frac{4}{1}$

Ex.28 Two point charges each of mass m and charge q are released when they are at a distance r from each other. What is the speed of each charge particle when they are at a distance 2r?

(1) $\sqrt{\frac{Kq^2}{rm}}$ **(2)** $\sqrt{\frac{Kq^2}{2rm}}$ **(3)** $\sqrt{\frac{2rm}{Kq^2}}$ **(4)** $\sqrt{\frac{rm}{Kq^2}}$

Sol. According to momentum conservation both the charge particles will move with same speed now applying energy conservation.

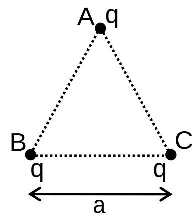
$k_1 + k_2 + u_i = u_f + k_f$



$$0 + 0 + \frac{Kq^2}{r} = 2\left[\frac{1}{2}mv^2\right] + \frac{Kq^2}{2r}$$

$$\Rightarrow \quad v = \sqrt{\frac{Kq^2}{2rm}}$$

Ex.29 Three equal charges q are placed at the corners of an equilateral triangle of side a.



(i) Find out potential energy of charge system.

(ii) Calculate work required to decrease the side of triangle to a/2.

(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.

Sol. (i) $U = U_{12} + U_{13} + U_{23} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$

(ii) Work required to decrease the sides to a/2 $W = U_f - U_i = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$

(iii) Work done by electrostatic forces = change is kinetic energy of particles

$U_i - U_f = K_f - K_i \quad \Rightarrow \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0 = \frac{3}{2}\frac{kq^2}{a} = \frac{3}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\frac{Kq^2}{am}}$

Ex.30 Find out energy stored in an imaginary cubical volume of side a in front of an infinitely large nonconducting sheet of uniform charge density σ.

(1) $\frac{\sigma^3 a^2}{4\varepsilon_0}$ **(2)** $\frac{\sigma^3 a^2}{8\varepsilon_0}$ **(3)** $\frac{\sigma^2 a^3}{4\varepsilon_0}$ **(4)** $\frac{\sigma^2 a^3}{8\varepsilon_0}$

Sol. Energy stored

$U = \int \frac{1}{2}\varepsilon_0 E^2 dV$ where dV is small volume

$= \frac{1}{2}\varepsilon_0 E^2 \int dV$

$\because$ E is constant $U = \frac{1}{2}\varepsilon_0 \frac{\sigma^2}{4\varepsilon_0^2} a^3 = \frac{\sigma^2 a^3}{8\varepsilon_0}$

Ex.31 When charge 10 μC is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is –10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

(1) –20 μJ **(2) –80 μJ** **(3) –120 μJ** **(4) –160 μJ**

Sol. $(W_{ext})_{\infty \to p} = (-w_{ele})_{\infty \to p} = (w_{ele})_{p \to \infty} = 10\ \mu J$

because ΔKE = 0

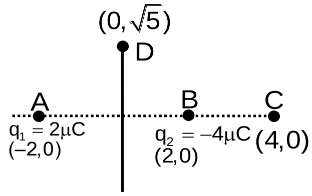
$V_p = 1V$

So if now the charge is doubled and taken from infinity then

$1 = \frac{(W_{ext})_{\infty \to p}}{20\mu C} \Rightarrow (W_{ext})_{\infty \to P} = 20\ \mu J$

$\Rightarrow \quad (W_{ele})_{\infty \to P} = -20\ \mu J$

Ex.32 Two point charges 2μC and – 4μC are situated at points (–2m, 0m) and (2m, 0m) respectively. Find out potential at point C(4m, 0m) and D(0m, $\sqrt{5}$ m).

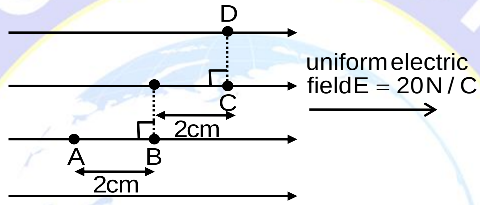


Sol. Potential at point C

$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{6} + \frac{K(-4\mu C)}{2} = \frac{9\times10^9\times2\times10^{-6}}{6} - \frac{9\times10^9\times4\times10^{-6}}{2} = -15000V$$

Similarly, $V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} + \frac{K(-4\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{K(2\mu C)}{3} + \frac{K(-4\mu C)}{3} = -6000\ V$

Ex.33 Find out following



(i) $V_A - V_B$ **(ii) $V_B - V_C$** **(iii) $V_C - V_A$** **(iv) $V_D - V_C$**

(v) $V_A - V_D$

(vi) Arrange the order of potential for points A, B, C and D.

Sol. (i) $|\Delta V_{AB}| = 20 \times 2 \times 10^{-2} = 0.4$ so, $V_A - V_B = 0.4$ V

(ii) $|\Delta V_{BC}| = 20\times2\times10^{-2} = 0.4$ so, $V_B - V_C = 0.4$ V

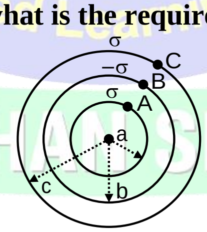
(iii) $|\Delta V_{CA}| = 20\times4\times10^{-2} = 0.8$ so, $V_C - V_A = -0.8$ V

(iv) $|\Delta V_{DC}| = 20 \times 0 = 0$ so, $V_D - V_C = 0$

(v) $|\Delta V_{AD}| = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_A - V_D = 0.8$ V

(vi) The order of potential $V_A > V_B > V_C = V_D$

Ex.34 Find V_A, V_B, V_C, and If $V_A = V_C$ then what is the required condition?



Sol. $V_A = \frac{\sigma a}{\epsilon_0} - \frac{\sigma b}{\epsilon_0} + \frac{\sigma c}{\epsilon_0} = \frac{\sigma}{\epsilon_0}(a - b + c)$

$V_B = \frac{\sigma a^2}{\epsilon_0 b} - \frac{\sigma b}{\epsilon_0} + \frac{\sigma c}{\epsilon_0}$ $\qquad V_C = \frac{\sigma a^2}{\epsilon_0 c} - \frac{\sigma b^2}{\epsilon_0 c} + \frac{\sigma c}{\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2 + c^2}{c}\right)$

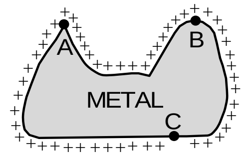
$\Rightarrow$ Now if $V_A = V_C \Rightarrow (a - b + c) = \frac{a^2 - b^2 + c^2}{c}$

$\Rightarrow$ $c(a - b) = a^2 - b^2 \Rightarrow c(a - b) = (a+b)(a-b)$

$\Rightarrow$ **$c = a + b$**

Ex.35 Compare field, potential and surface charge density at A, B, and C.

Sol : Surface of a metal is an equipotential surface (EPS) so charge on irregular shaped metal distributes to create same potential on surface



$\Rightarrow$ $V_A = V_B = V_C$ Now radius of curvature (R) for straight line = ∞

$R_C > R_B > R_A$ Now $\frac{Q_A}{R_A} = \frac{Q_B}{R_B} = \frac{Q_C}{R_C}$ or ($Q \propto R$)

$E = \frac{KQ}{R^2} \Rightarrow E = \frac{V}{R} \Rightarrow E \propto \frac{1}{R}$

Electric Field $E \propto \frac{Q}{R^2}$ or $E \propto \frac{1}{R} \Rightarrow E_A > E_B > E_C$

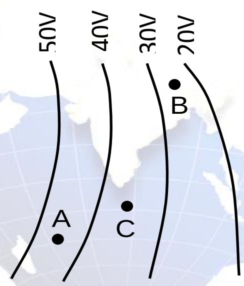
surface charge density $\sigma \propto \frac{Q}{R^2}$

$\Rightarrow$ $\sigma \propto \frac{V}{R} \propto \frac{1}{R}$ $\qquad \Rightarrow \sigma_A > \sigma_B > \sigma_C$

So, if there is any possibility of charge leakage, it starts from point A. (corona discharge)

Remember that E and σ at sharp points is more while potential is same

Ex.36 Figure shows some equipotential surface produced by some charges. At which point the value of electric field is greatest?



Sol. E is larger where equipotential surfaces are closer. ELOF are $\perp$ to equipotential surfaces. In the figure we can see that for point B they are closer so E at point B is maximum

Ex.37 For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$ find the potential at (x, y) if V at origin is 5 volts.

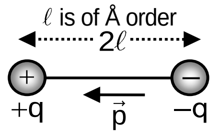
(1) $-x^2 - \frac{3y^2}{2} + 5$ **(2)** $x^2 + \frac{3y^2}{2} + 5$ **(3)** $x^2 - \frac{3y^2}{2} + 5$ **(4)** $x^2 - \frac{3y^2}{2} - 5$

Sol. $\int_5^V dV = -\int_0^r \vec{E} \cdot \vec{dr} = -\int_0^x E_x dx - \int_0^y E_y dy \Rightarrow V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2}$

$\Rightarrow$ $V = -x^2 - \frac{3y^2}{2} + 5$

18. ELECTRIC DIPOLE :

An electric dipole is pair of equal and opposite point charges q and –q separated by a distance of 2ℓ.



18.1 Dipole Moment $(\vec{P})$:

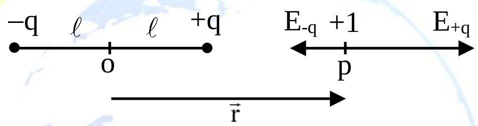
Dipole moment $= q \times 2\ell$

(i) Vector quantity directed from (-ve) charge to (+ve) charge

(ii) Dimension : [LTA] and Unit: coulomb × metre (or Cm)

(iii) Practical unit is Debye 1 Debye = 3.3×10^{-30} C × m

18.2 Electric field due to dipole at at axial point (end on position) at distance r (at point p) from the centre of dipole



$$E_p = (E_{+q} - E_{-q}) = \frac{kq}{(r-\ell)^2} - \frac{kq}{(r+\ell)^2} = kq\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

$$= kq\left[\frac{1}{r^2\left(1-\frac{\ell}{r}\right)^2} - \frac{1}{r^2\left(1+\frac{\ell}{r}\right)^2}\right] = \frac{kq}{r^2}\left[\left(1-\frac{\ell}{r}\right)^{-2} - \left(1+\frac{\ell}{r}\right)^{-2}\right]$$

Since $\ell \ll r$ in dipole. So, applying binomial approximation.

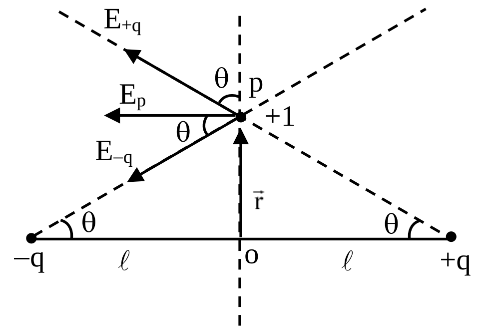
$$= \frac{kq}{r^2}\left[1+\frac{2\ell}{r} - 1 + \frac{2\ell}{r}\right] \quad E_p = \frac{kq}{r^2}\left[\frac{4\ell}{r}\right] = \frac{2kq(2\ell)}{r^3}$$

$$\therefore \boxed{E_p = \frac{2kp}{r^3}}$$

Direction of electric field is along dipole moment $\vec{p}$

So, in vector form $\boxed{\vec{E}_p = \frac{2k\vec{p}}{r^3}}$

18.3 Electric field due to dipole at equatorial point (broad on position/perpendicular bisector) at distance r (at point p) from the centre of dipole



$$E_p = E_{+q} + E_{-q}$$

$$= E_{+q}\cos\theta + E_{-q}\cos\theta = (E_{+q} + E_{-q})\cos\theta$$

$$= \left[\frac{kq}{\left(\sqrt{r^2+\ell^2}\right)^2} + \frac{kq}{\left(\sqrt{r^2+\ell^2}\right)^2}\right]\frac{\ell}{\sqrt{r^2+\ell^2}}$$

$$E_p = \frac{kq(2\ell)}{\left(r^2+\ell^2\right)^{3/2}}$$

Since $\ell \ll r$ so applying binomial approximation.

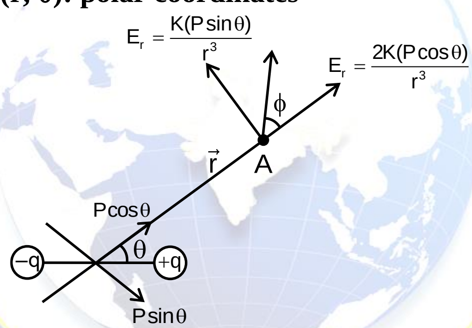
$$\boxed{E_p = \frac{kp}{r^3}}$$ and direction is opposite to dipole moment $(\vec{p})$.

In vector form $\boxed{\vec{E}_p = \frac{-k\vec{p}}{r^3}}$

DETECTIVE MIND

In electrostatics condition net electric field inside the conductor is zero.

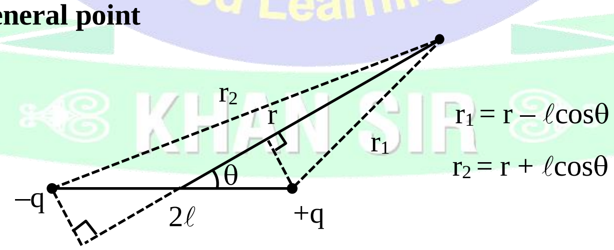
18.4 Electric field at general point (r, θ): polar coordinates



$$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}\ ; \ \tan\phi = \frac{\tan\theta}{2};$$

Here θ is angle between position vector $(\vec{r})$ and dipole moment vector $(\vec{p})$ in anti clockwise sense.

18.5 Electric potential at a general point



$$V_{net} = V_{+q} + V_{-q} = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

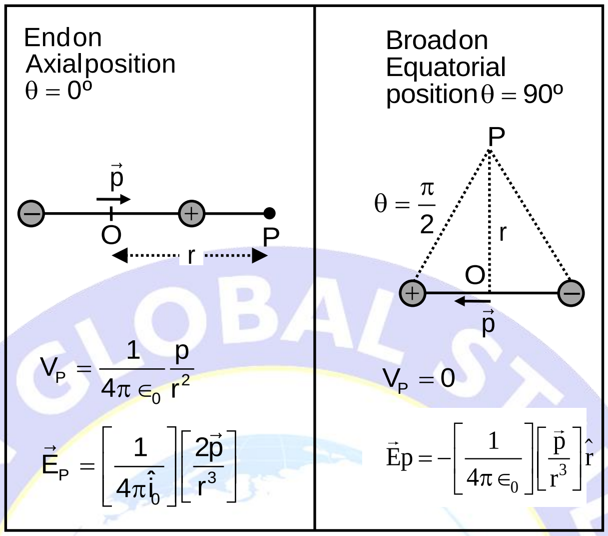
$$V_{net} = kq\left(\frac{1}{r-\ell\cos\theta} - \frac{1}{r+\ell\cos\theta}\right)$$

$$V_{net} = \frac{kq2\ell\cos\theta}{r^2-\ell^2\cos^2\theta} = \frac{kp\cos\theta}{r^2-\ell^2\cos^2\theta}$$

For $r >> \ell$

$$V_{net} = \frac{kp\cos\theta}{r^2}$$

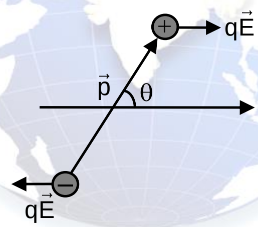
$$V_{net} = \frac{k\vec{p}.\vec{r}}{r^3}$$



18.6 Dipole Placed in uniform Electric Field: $\vec{p} = 2q\vec{\ell}$

(A) Net force: $\vec{F}_{net} = \left[q\vec{E} + (-q)\vec{E}\right] = 0$

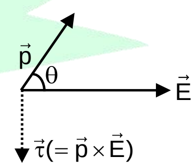
(B) Torque: $\vec{\tau} = \vec{r}\times\vec{F} = 2\ell \times q\vec{E} = 2q\vec{\ell}\times\vec{E} = \vec{p}\times\vec{E} = pE\sin\theta$



(C) Work: Work done in rotating an electric dipole from θ_1 to θ_2 [in uniform field]

$dW = \vec{\tau}\cdot d\vec{\theta} \Rightarrow W = \int dW = \int \tau d\theta$

$$W_{\theta_1\to\theta_2} = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$$

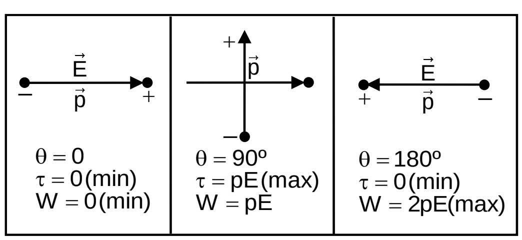


$W_{0\to 180^\circ} = pE[1-(-1)] = 2pE$

$W_{0\to 90^\circ} = pE(1-0) = pE$

If a dipole is rotated from field direction ($\theta = 0°$) to θ then,

$W = pE(1 - \cos\theta)$



Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e.

$U = W_\theta - W_{90^\circ} = pE(1-\cos\theta) - pE = -pE\cos\theta = -\vec{p}.\vec{E}$

$\vec{E}$ is a conservative field so whatever work is done in rotating a dipole from $\vec{E}p = \left[\frac{1}{4\pi\epsilon_0}\right]\left[\frac{2\vec{p}}{r^3}\right]\hat{r}$ θ_1 to θ_2 is just equal to change in electrostatic potential energy.

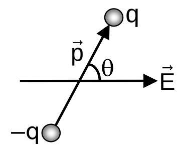
$W_{\theta_1\to\theta_2} = U_{\theta_2} - U_{\theta_1} = pE(\cos\theta_1 - \cos\theta_2)$

Angular SHM : When a dipole suspended in a uniform field it will align itself parallel to the field. Now if it is given a small angular displacement θ about its equilibrium position the restoring torque will be.

$\tau = pE\sin\theta$

If θ is small then $\tau = -pE\theta$

$\tau \propto (-\theta)$(Angular S.H.M)



But $\tau = I\alpha$ so $I\alpha = -pE\theta$ $\Rightarrow$ $\alpha = \frac{pE}{I}(-\theta) = -\omega^2\theta$

(where α – angular acceleration & I – moment of Inertia)

$\omega = \sqrt{\frac{pE}{I}}$; $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{pE}}$

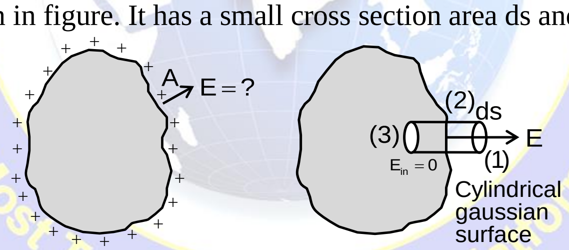
19. CONDUCTOR:-

Conductor are the materials through which electric charge can flow easily. Most of the conductors of electric charge.

19.1 Finding field due to a conductor :

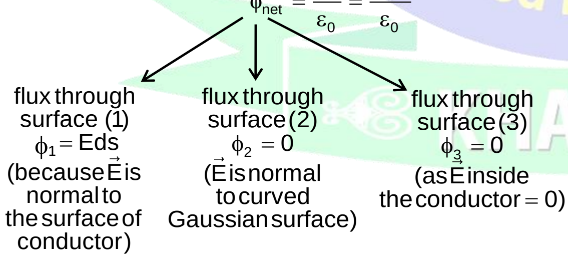
Suppose we have a conductor, and at any 'A', local surface charge density = σ. We have to find electric field just outside the conductor surface.

For this lets consider a small cylindrical gaussian surface, which is partly inside and partly outside the conductor surface, as shown in figure. It has a small cross section area ds and negligible height.



Applying gauss's theorem for this surface

$\phi_{net} = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma ds}{\varepsilon_0}$



So, $Eds = \frac{\sigma ds}{\varepsilon_0}$ $\Rightarrow$ $E = \frac{\sigma}{\varepsilon_0}$

DETECTIVE MIND

Electric field just outside the surface of conductor $E = \frac{\sigma}{\varepsilon_0}$ direction will be normal to the surface. In vector form $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ (here $\hat{n}$ unit vector normal to the conductor surface)

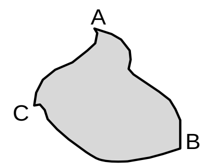
19.2 Properties of conductor under the electrostatics

(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.
(ii) In electrostatics, conductors are always equipotential surface
(iii) Charge always resides on outer surface of conductor.
(iv) If there is cavity inside the charged conductor with having no charge then the charge given to the conductor will reside on the outer surface of the conductor.
(v) Electric field is always perpendicular to conducting surface.
(vi) Electric lines of force never enter into conductors.
(vii) Electric field intensity near the conducting surface is given by formula

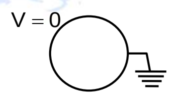
$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\hat{n}$$

$$\vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\hat{n} \text{ and } \vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$



(viii) When a conductor is grounded its potential becomes zero.



(ix) When an isolated conductor is grounded then its charge becomes zero.
(x) When two conductors are connected there will be charge flow till their potential becomes equal.
(xi) Electric pressure at the surface of a conductor is given by formula $P = \frac{\sigma^2}{2\varepsilon_0}$ where σ is the local surface charge density.

Cavity inside conductor: Cavity inside a conductor is the space surrounded by conductor such that one cannot reach that space without touching the conductor.

DETECTIVE MIND
If there is no net charge inside the cavity any excess charge on the conductor (metal) resides on the outer surface of the conductor.

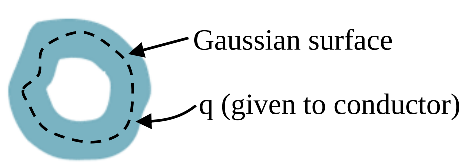
Verification:

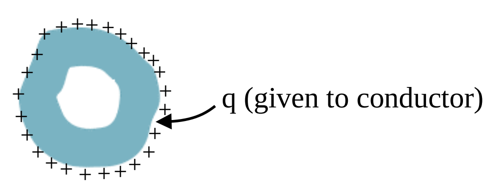
$$\vec{E}.d\vec{s} = \frac{q_{enclose}}{\varepsilon_0}$$

Since electric field $(\vec{E})$ inside conductor = 0

$\therefore \vec{E}.d\vec{s} = 0$ [i.e. flux (ϕ) through gaussian surface = 0]

$\Rightarrow 0 = \frac{q_{enclose}}{\varepsilon_0}$ $q_{enclose} = 0$





Here q enclosed is zero, it means total charge given to the conductor has gone on to the outer surface.

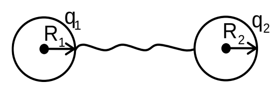
SPOT LIGHT

Charge on outer surface of the conductor (be it excess charge or induced charge or both) cancels the electric field due to any charge in the vicinity (surrounding) of the conductor at any point inside outer surface of the conductor. If charge in the vicinity is changed then conductor redistributes its charges in such a way that it can cancel the effect of electric field due to charge in vicinity, called shielding effect.

Electrostatic shielding : If there is no charge inside cavity any excess charge on the metal appears on its outer surface. Also charge on the outer surface cancels the electric field of the charge outside the metal at any point the outer surface thus, electric field inside cavity in this case is zero. So if an electrically sensitive device is put in the cavity it will remain safe for long time.

20. SHARING OF CHARGES :

Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and separated by large distance, are joined by a conducting wire



Let final charges on spheres are q_1 and q_2 respectively.

Potential on both spherical shell become equal after joining, therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2} \quad ....(i)$$

and $q_1 + q_2 = Q_1 + Q_2$ ....(ii)

from (i) and (ii)

$$q_1 = \frac{(Q_1+Q_2)R_1}{R_1+R_2};\ q_2 = \frac{(Q_1+Q_2)R_2}{R_1+R_2};$$

ratio of charges $\frac{q_1}{q_2} = \frac{R_1}{R_2} \Rightarrow \frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$

Ratio of surface charge densities $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

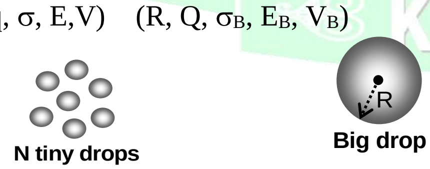
Ratio of final charges $\frac{q_1}{q_2} = \frac{R_1}{R_2}$

21. MIXING OF IDENTICAL CHARGED TINY DROPS :

Let , number of tiny drops = N

[For each **tiny** drop] [For **Big** drop]

(r, q, σ, E, V) $(R, Q, \sigma_B, E_B, V_B)$



(i) Charge conservation $Q = Nq$

(ii) Volume conservation $N\,\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

Hence $R = N^{1/3} r$, $Q = Nq$

$\therefore$ $\sigma_B = N^{1/3}\sigma$, $E_B = N^{1/3} E$, $V_B = N^{2/3} V$

SOLVED EXAMPLES

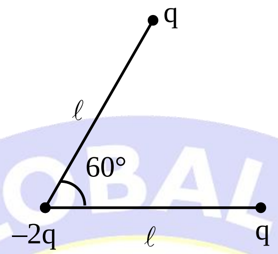
Ex.38 An electric dipole formed by +5μC and -5μC charges at 2mm distance. The dipole moment of the system is-

(1) 2×10^{-8} Cm **(2) 5×10^{-6} Cm** **(3) 4×10^{-5} Cm** **(4) 10^{-8} Cm**

Sol. $\vec{p} = q(2a)$

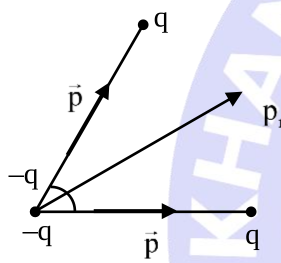
$\vec{p} = 5 \times 10^{-6} \times (2 \times 10^{-3}) = 10^{-8}$ Cm

Ex.39 Find electric dipole moment of the given arrangement.



(1) $\sqrt{3}q\ell$ **(2)** $\sqrt{5}q\ell$ **(3)** $2q\ell$ **(4)** $2\sqrt{3}q\ell$

Sol.

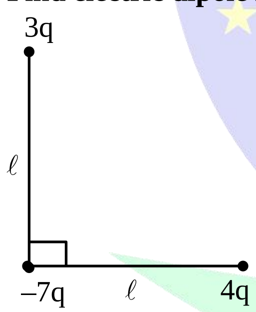


$p_{net} = \sqrt{3}\vec{p}$

$\vec{p} = q\ell$

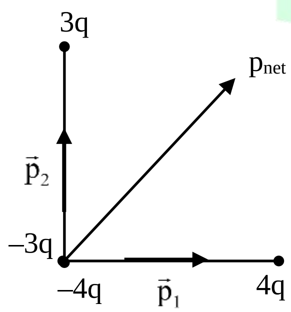
$\vec{p}_{net} = \sqrt{3}q\ell$

Ex.40 Find electric dipole moment of the given arrangement.



(1) $\sqrt{3}q\ell$ **(2)** $\sqrt{5}q\ell$ **(3)** $5q\ell$ **(4)** $2\sqrt{3}q\ell$

Sol.



$p_1 = 4q\ell$

$p_2 = 3q\ell$

$p_{net} = \sqrt{p_1^2 + p_2^2}$

$= \sqrt{(4q\ell)^2 + (3q\ell)^2}$

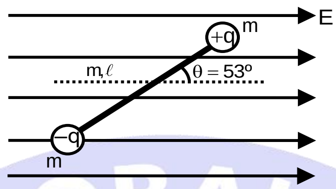
$= 5q\ell$

Ex.41 The electric field due to a short dipole at a distance r, on the axial line, from its mid point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio.

(1) $2^{2/3}$ **(2) $2^{4/3}$** **(3) $2^{3/4}$** **(4) $2^{1/3}$**

Sol. $\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$ or $\frac{2}{r^3} = \frac{1}{r'^3}$ or $\frac{r^3}{r'^3} = 2$ or $\frac{r}{r'} = 2^{1/3}$

Ex.42 Two point masses of mass m and equal and opposite charge of magnitude q are attached on the corners of a non-conducting uniform rod of mass m and the system is released from rest in uniform electric field E as shown in figure from θ = 53°



(i) Find angular acceleration of the rod, just after releasing

(ii) What will be angular velocity of the rod when it passes through stable equilibrium.

(iii) Find work required to rotate the dipole by 180°.

Sol. (i) $\tau_{net} = PE\sin 53° = I\alpha$

$$\alpha = \frac{(q\ell)E\left(\frac{4}{5}\right)}{\frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2} = \frac{48qE}{35m\ell}$$

(ii) From energy conservation : $K_i + U_i = K_f + U_f$

$$0 + (-PE\cos 53°) = \frac{1}{2}I\omega^2 + (-PE\cos 0°) \quad \text{where } I = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{48qE}{35m\ell}}$$

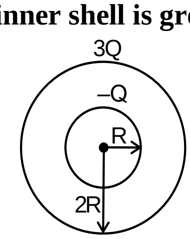
(iii) $W_{ext} = U_f - U_i$

$W_{ext} = (-PE\cos(180° + 53°)) - (-PE\cos 53°)$

$W_{ext} = (q\ell)E\left(\frac{3}{5}\right) + (q\ell)E\left(\frac{3}{5}\right)$

$\Rightarrow W_{ext} = \left(\frac{6}{5}\right)q\ell E$

Ex.43 Two conducting hollow spherical shells of radii R and 2R carry charges – Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded?



(1) $\frac{Q}{2}$ **(2)** $\frac{3Q}{2}$ **(3)** $\frac{Q}{4}$ **(4)** $\frac{3Q}{4}$

Sol. When inner shell is grounded to the Earth then the potential of inner shell will become zero because potential of the Earth is taken to be zero.

$$\frac{Kx}{R} + \frac{K3Q}{2R} = 0,\ x = \frac{-3Q}{2}$$

the charge that has increased $= \frac{-3Q}{2} - (-Q)$

$= \frac{-Q}{2}$

Hence charge flows into the Earth $= \frac{Q}{2}$

Ex.44 A charge of 10^{-9}C is placed on each 27 identical drops of radius 2 cm. They are then combined to form a bigger drop. Find its potential?

(1) 81×10^2 V **(2) 40.5×10^2 V** **(3) 81×10^3 V** **(4) 20.25×10^2 V**

Sol. $V_B = n^{2/3} \times V$

$$V_B = (27)^{2/3} \times 9 \times 10^9 \times \frac{10^{-9}}{2 \times 10^{-2}}$$

$$V_B = 9 \times 9 \times 10^9 \times \frac{10^{-9}}{2 \times 10^{-2}}$$

$$V_B = \frac{81}{2} \times 10^2 \Rightarrow V_B = 40.5 \times 10^2 \text{ V}$$

QUICK FOLLOW UP

Electric Flux (ϕ)

It is the measure of number of electric field lines through some surface

Gauss'law; $(\phi) = \frac{q_{\text{enclosed by the surface}}}{\varepsilon_0}$

$(\phi) = \oint \vec{E}\cdot d\vec{S} = \frac{q_{\text{enclosed by the surface}}}{\varepsilon_0}$

Electric dipole

Electric Dipole Moment p = q(2ℓ)

It is a vector quantity

Electric field at general point (r,θ) polar coordinates

$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta};\ \tan\phi = \frac{\tan\theta}{2};$

$E_{\text{axial point}} = \frac{2Kp}{r^3}$

$E_{\text{quatorial point}} = \frac{Kp}{r^3}$

$V_{\text{p general point}} = \frac{kp\cos\theta}{r^2}$

$V_{\text{axial point}} = \frac{Kp}{r^2}$

$V_{\text{quatorial point}} = 0$

$U_{\text{electric dipole}} = -\vec{p}.\vec{E}\ ;\ \tau_{\text{dipole}} = \vec{p} \times \vec{E}$

Conductor

A material having free electrons in its valence shell is called conductor.

- Inside a conductor, the net electrostatic field is zero
- At the surface of a charged conductor, the electrostatic field must be normal to the surface at every point
- The interior of a conductor can have no excess charge in the static situation i.e. excess charge reside only on the outer surface of conductor.
- Electric field at the surface of a Charged conductor

$E = \sigma\ \in_0 \left(\sigma \propto \frac{1}{\text{radius of curvature}}\right)$

Coulomb's law

Force between two charged particles

$F = \frac{Kq_1q_2}{r^2}$ (magnitude)

$\vec{F} = \frac{Kq_1q_2\vec{r}}{r^3} = \frac{Kq_1q_2\hat{r}}{r^2}$ (vector form)

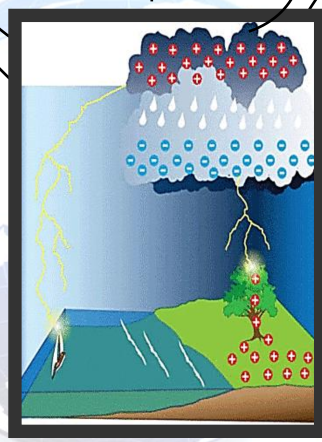
$K = \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9\ Nm^2c^{-2}$

ε_0 = Permitivity of Free Space= $8.854 \times 10^{-12} C^2/Nm^2$

Properties Of Charge

Charge is an intrinsic property of matter.

- It resides on mass.
- Charge is quantized and quantum of charge is ± e
- It is frame independent
- Charge is conserved



Electric charge

Electric Potential energy

Work done By External charge to move from postion 1 to 2 in static Electric Field E.

$W_{ext} = \int_1^2 \vec{F}.d\vec{r} = -q\int_1^2 \vec{E}.d\vec{r}$

$U_{\text{due to point charge}} = \frac{Kq_1q_2}{r}$,q_1q_2 is kept with sin

Electric potential Energy of a system of charges:

$U_{total} = \frac{Kq_1q_2}{r_1r_2} + \frac{Kq_2q_3}{r_1r_2} + + \frac{Kq_3q_4}{r_1r_2} + ...$

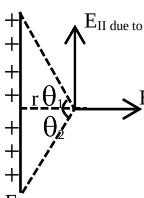
U_{self} (uniformly charged shell)= $\frac{KQ^2}{2R}$

U_{self} (uniformly charged solid sphere) = $\frac{3}{5}\frac{KQ^2}{R}$

Electric field

$E_{\text{due to point charge}} = \frac{Kq}{r^2}$ (magnitude)

$\vec{E}_{\text{due to point charge}} = \frac{Kq}{r^3}\vec{r} = \frac{Kq}{r^2}\hat{r}$ (vector Form)



$E_{\text{II due to line charge}} = \frac{K\lambda}{r}[\cos\theta_1 - \cos\theta_2]$

$E_{\perp\text{ due to line charge}} = \frac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2]$

$E_{\text{due to ring on axis at distance}} = \frac{KQX}{(R^2+X^2)^{\frac{3}{2}}}$

$E_{\perp\text{due to a plane Infinite sheet}} = \frac{\sigma}{2\varepsilon_0}$

$E_{\perp\text{charged conducting plate}} = \frac{\sigma}{\varepsilon_0}$

Electric field due to charged spherical shell or conducting sphere $E(r<R) = 0$; $E(r>R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$; $E(r=R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

Electric field due to a solid non-conducting sphere –

$E(r<R) = \frac{KQr}{3R^3}$; $E(r>R) = \frac{KQ}{r^2}$ $E(r=R) = \frac{KQ}{R^2}$

Relation between electric field & electric potential

$dv = -\vec{E}.d\vec{r}$

If $\vec{E}$ = constant

$\Delta V = -\vec{E}.\Delta\vec{r}$

Electric Potential

Electric potential

$V_p = \frac{W_{ext}(\infty \to P)}{q} = -\int_\infty^P \vec{E}.d\vec{r}$

$V_{\text{due to point charge}} = \frac{Kq}{r}$, q is taken with sign

$V_{\text{due to ring at centre}} = \frac{Kq}{r}$ (uniformly or nonuniformly)

	Hollow conducting	Solid Non-Conducting
(i) Inside (r < R)	$V_p = \frac{Kq}{R}$	$V_p = \frac{Kq}{2R^3}[3R^2 - r^2]$
(ii) Outside (r > R)	$V_p = \frac{Kq}{r}$	$V_p = \frac{Kq}{r}$
(iii) At surface (r = R)	$V_p = \frac{Kq}{R}$	$V_p = \frac{Kq}{R}$

Electric field line

- Electric Field lines are imaginary.
- Tangent at EFL gives direction of electric field
- Never intersect Each other.
- Aways normal to conducting surface.
- Lines originating from +ve charge
- Terminating at -ve charge
- Never form closed loop.