[Trigonometry](DPPS-1)

① $2\cot^2\frac{\pi}{6} + 4\tan^2\frac{\pi}{6} - 3\csc\frac{\pi}{6}$

$2\cot^2 30^\circ + 4\tan^2 30^\circ - 3\csc 30^\circ$

$\cot 30^\circ = \sqrt{3}$ $\tan 30^\circ = \frac{1}{\sqrt{3}}$ $\csc 30^\circ = 2$

$2\times(\sqrt{3})^2 + 4\times\left(\frac{1}{\sqrt{3}}\right)^2 - 3\times 2$

$\cancel{6} + \frac{4}{3} - \cancel{6} = 4/3$

② $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$

$\cos A + \sqrt{6}\sin A = ?$

$(\sin A - \sqrt{6}\cos A)^2 + (\cos A + \sqrt{6}\sin A)^2$

$= 1+6$

$(\sqrt{7}\cos A)^2 + (\cos A + \sqrt{6}\sin A)^2 = 1+6$

$7\cos^2 A + (\cos A + \sqrt{6}\sin A)^2 = 7$

$(\cos A + \sqrt{6}\sin A)^2 = 7 - 7\cos^2 A$

$(\cos A + \sqrt{6}\sin A)^2 = 7\sin^2 A$

$\boxed{\cos A + \sqrt{6}\sin A = \pm\sqrt{7}\sin A}$

③ $\cos 20^\circ - \sin 20^\circ = P$

$(\cos 20)^2 + \sin^2 20^\circ - 2\sin 20\cos 20^\circ = p^2$

$1 - \sin 40^\circ = p^2$

$\sin 40^\circ = 1 - p^2$

$\cos 40^\circ = +\sqrt{1-\sin^2 40^\circ}$

$= \sqrt{1-(1-p^2)^2}$

$= \sqrt{1-(1+p^4-2p^2)}$

$= \sqrt{(\cancel{1}-\cancel{1}-p^4+2p^2)}$

$= \sqrt{2p^2-p^4}$

$= \sqrt{p^2(2-p^2)}$

$\boxed{\cos 40^\circ = P\sqrt{2-p^2}}$

④ $S_n = \cos^n\theta + \sin^n\theta$

$3S_4 - 2S_6$

$= 3(\cos^4\theta + \sin^4\theta) - 2(\cos^6\theta + \sin^6\theta)$

Ist method put $\theta = 0^\circ$

$= 3[\cos^4 0 + \sin^4 0^\circ] - 2[\cos^6 0^\circ + \sin^6 0]$

$= 3[1+0] - 2[1+0]$

$= 3-2 = 1$

2nd method

$= 3(1-2\sin^2\theta\cos^2\theta) - 2[1-3\sin^2\theta\cos^2\theta]$

$= 3 - \cancel{6\sin^2\theta\cos^2\theta} - 2 + \cancel{6\sin^2\theta\cos^2\theta}$

$= 1$

we know that

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

⑤ $\sin^2 17.5^\circ + \sin^2 72.5^\circ$

$\sin^2 17.5^\circ + \sin^2(90-17.5)$

$\sin^2 17.5 + \cos^2 17.5$

$= 1$

2nd method we know that

If $A+B = 90^\circ$ then

$\sin^2 A + \sin^2 B = 1$

$17.5 + 72.5 = 90^\circ$

Hence $\sin^2 17.5 + \sin^2 72.5 = 1$

⑥ $A+B = 45^\circ$

$\cot(A+B) = \cot 45^\circ$

$\frac{\cot A\cot B - 1}{\cot B + \cot A} = 1$

$\cot A\cot B - 1 = \cot B + \cot A$ —①

$(\cot A - 1)(\cot B - 1)$

$= \cot A\cot B - \cot A - \cot B + 1$

$= 1+1 = 2$ (from 1)

6 का Ans 2 होगा option गलत है।

(7) $\cos^2\left(\frac{\pi}{4}+\theta\right)-\sin^2\left(\frac{\pi}{4}-\theta\right)$

we know that

$\cos^2 A-\sin^2 B$

$=\cos(A+B)\cdot\cos(A-B)$

$\cos\left(\frac{\pi}{4}+\cancel{\theta}+\frac{\pi}{4}-\cancel{\theta}\right)\cdot\cos\left(\frac{\pi}{4}+\theta-\left(\frac{\pi}{4}-\theta\right)\right)$

$\cos\frac{\pi}{2}\cdot\cos(2\theta)=0$

(8) $3\cos x-4\sin x=f(x)$

$\max=\sqrt{3^2+4^2}=5$

(9) $\sin\theta=\frac{24}{25}=\frac{P}{H}$ $\theta\in$ 2nd Quad.

$B=\sqrt{625-576}=7$

$\sec\theta=-\frac{25}{7}$ (-ve) $\tan\theta=-\frac{24}{7}$ (-ve)

$\sec\theta+\tan\theta=-\frac{25}{7}-\frac{24}{7}=-7$

(10) $\sec\theta+\tan\theta=e^x$ —(1)

then $\sec\theta-\tan\theta=\frac{1}{e^x}=e^{-x}$ —(2)

(1)+(2)

$2\sec\theta=e^x+e^{-x}$

$\sec\theta=\frac{e^x+e^{-x}}{2}$

$\cos\theta=\frac{2}{e^x+e^{-x}}$

(11) $\sec\theta+\tan\theta=P$ —(1)

$\sec\theta-\tan\theta=\frac{1}{P}$ —(2)

(1)−(2)

$2\tan\theta=P-\frac{1}{P}$

$\tan\theta=\frac{P^2-1}{2P}$

(12) $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$

$\Rightarrow\theta_1=\theta_2=\theta_3=90^\circ$

Hence $\cos\theta_1+\cos\theta_2+\cos\theta_3$

$=\cos 90^\circ+\cos 90^\circ+\cos 90^\circ$

$=0+0+0$

$=0$

(13) $\frac{1}{4}\left[\sqrt{3}\cos 23^\circ-\sin 23^\circ\right]$

$\frac{1}{2}\left[\frac{\sqrt{3}}{2}\cos 23^\circ-\frac{1}{2}\sin 23^\circ\right]$

$\frac{1}{2}\left[\sin 60^\circ\cos 23^\circ-\cos 60^\circ\sin 23^\circ\right]$

$\frac{1}{2}\left[\sin(60^\circ)\cos 23^\circ-\cos 60^\circ\sin 23^\circ\right]$

$\frac{1}{2}\sin(60-23^\circ)$

$\frac{1}{2}\sin 47^\circ$

(14) $\frac{1}{\sin 10}-\frac{\sqrt{3}}{\cos 10^\circ}$

$\frac{2\left(\frac{1}{2}\cos 10-\frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\sin 10\cos 10^\circ}$

$\frac{2\times 2\left(\sin 30^\circ\cos 10^\circ-\cos 30^\circ\sin 10^\circ\right)}{2\sin 10^\circ\cos 10^\circ}$

$\frac{4\sin(30-10^\circ)}{\sin 20^\circ}=\frac{4\cancel{\sin 20}}{\cancel{\sin 20}}=4$

(15) $\sqrt{3}\operatorname{cosec} 20^\circ-\sec 20^\circ$

$=\frac{\sqrt{3}}{\sin 20^\circ}-\frac{1}{\cos 20^\circ}$

$=\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)}{\sin 20^\circ\cos 20^\circ}$

$=\frac{2\times 2\left(\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ\right)}{2\sin 20^\circ\cos 20^\circ}$

$=\frac{4\sin(60-20^\circ)}{\sin 40^\circ}=\frac{4\cancel{\sin 40^\circ}}{\cancel{\sin 40}}=4$

(16) $\tan(-1575^\circ)$

$-\tan(1575^\circ)$

$-\tan\left(4\cdot 2\pi+\frac{3\pi}{4}\right)$

$-\tan(4\times 360+135^\circ)$

$-\tan(135^\circ)$

$-\tan(90+45^\circ)$

$=-(-\cot 45^\circ)$

$=1$

(17)

$1 - \sin 10^\circ \sin 50^\circ \sin 70^\circ$

$1 - \sin 10^\circ \sin(60-10^\circ) \sin(60+10^\circ)$

$1 - \frac{1}{4}\sin 10^\circ \times 3$ $\left[\because \sin A \sin(60-A) \sin(60+A) = \frac{1}{4}\sin 3A\right]$

$1 - \frac{1}{4}\sin 30^\circ$

$1 - \frac{1}{4} \times \frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$

(18) $\operatorname{cosec} A + \cot A = \frac{2}{3}$ — (1)

then $\operatorname{cosec} A - \cot A = \frac{3}{2}$ — (2)

(1) + (2)

$2\operatorname{cosec} A = \frac{2}{3} + \frac{3}{2} = \frac{13}{6}$

$\operatorname{cosec} A = \frac{13}{12} = \frac{H}{P}$ $B = 5$

$\cos A = \frac{B}{H} = \frac{5}{13}$ (a is true DPP मे Answer galat है।)

(19) $5\sin x + 4\cos x = 3$

let $4\sin x - 5\cos x = K$

$(5\sin x + 4\cos x)^2 + (4\sin x - 5\cos x)^2 = 4^2 + 5^2$

$9 + K^2 = 16 + 25$

$K^2 = 32$

$\boxed{K = 4\sqrt{2}}$

(20) $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$\frac{\sin A/\cos A}{1 - \frac{\cos A}{\sin A}} + \frac{\cos A/\sin A}{1 - \sin A/\cos A}$

$\frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$

$\frac{\sin^2 A}{\cos A(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$

$\frac{1}{(\sin A - \cos A)}\left[\frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}\right]$

$\frac{1}{\sin A - \cos A}\left[\frac{\sin^3 A - \cos^3 A}{\sin A \cos A}\right]$

$\frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \cos^2 A + \sin A\cos A)}{\cancel{\sin A - \cos A}\,(\sin A \cos A)} = \frac{1 + \sin A \cos A}{\sin A \cos A} = \sec A \operatorname{cosec} A + 1$