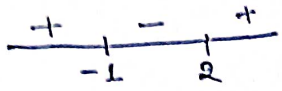
Basic maths DPPS 1. (Solution)

① $x^2 - x - 2 > 0$

$x^2 - 2x + x - 2 > 0$

$(x-2)(x+1) > 0$

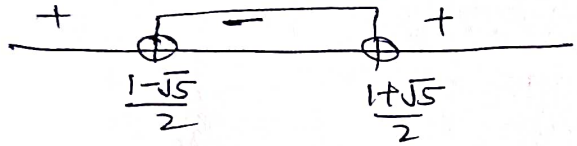


Ans $x \in (-\infty, -1) \cup (2, \infty)$

② $x^2 - x - 1 < 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

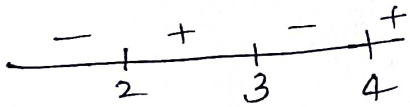
$\left(x - \left(\frac{1+\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{1-\sqrt{5}}{2}\right)\right) < 0$



$x \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ Ans

③ $(x-2)(x-4)(3-x) > 0$

$(x-2)(x-4)(x-3) < 0$ [-1 से multiply करने पर]



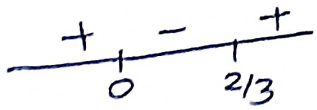
$x \in (-\infty, 2) \cup (3, 4)$ Ans

④ $\frac{2}{x} < 3$

$\frac{2}{x} - 3 < 0$

$\frac{2-3x}{x} < 0$

$\frac{3x-2}{x} > 0$



$x \in (-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$

⑤ $x > \sqrt{3-x}$

$\sqrt{1-x}$ is defined if $1-x \geq 0$

$\boxed{x \leq 1}$ and $\boxed{x > 0}$ also ——— ①

$\because$ x and $\sqrt{1-x}$ both are +ve then.

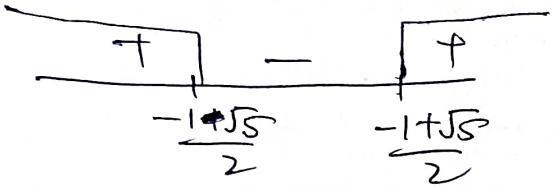
squaring both side.

$x^2 > 1-x$

$x^2+x-1 > 0$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$\left(x - \left(\frac{-1+\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{-1+\sqrt{5}}{2}\right)\right) > 0$



$\boxed{x \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)}$ ——— ②

Find Intersection of ① and ②

$x \in \left(\frac{1+\sqrt{5}}{2}, 1\right]$

⑥ $\sqrt{x-5} - \sqrt{9-x} > 1 \quad , x \in Z.$

$\sqrt{x-5}$ is defined if $x-5 \geq 0$ ie $x \geq 5$

$\sqrt{9-x}$ is defined if $9-x \geq 0$ ie $x \leq 9$

$\boxed{x \in [5, 9]}$

$x = 5, 6, 7, 8, 9$

For $x=5$ eqⁿ is not valid.

$x=6$ " " " " "

$x=7$ " " " "

$x=8$ " " " "

$x=9$ satisfied given eqⁿ

Hence $\boxed{x=9}$ Ans

⑦ $\sqrt{x-2} \geq -1$

$\sqrt{x-2}$ is defined if $x-2 \geq 0$ ie $x \geq 2$

$\because \sqrt{x-2}$ is always greater than -1

Hence Ans $x \in [2, \infty)$

(8) $\sqrt{x-1} > \sqrt{3-x}$

$x-1 \geq 0$ and $3-x \geq 0$

$x \in [1, 3]$ — (1)

Squaring both sides

$x-1 > 3-x$

$2x > 4$

$\boxed{x > 2}$ — (2)

(1) $\cap$ (2)

Ans $x \in (2, 3]$

(9) $x + \sqrt{x} \geq \sqrt{x} - 3$

Domain $\boxed{x \geq 0}$ — (1)

$\boxed{x \geq -3}$ — (2)

(1) $\cap$ (2) $\Rightarrow \boxed{x \geq 0}$

Ans $x \in [0, \infty)$

(10) $1-x = \sqrt{x^2-2x+1}$

$(1-x) = \sqrt{(x-1)^2}$

$(1-x) = |x-1|$

$|x-1| = -(x-1) \Rightarrow x-1 \leq 0$

$\boxed{x \leq 1}$

Ans $x \in (-\infty, 1]$

(11) $|3x-2| = x$

$3x-2 \geq 0$ i.e. $\boxed{x \geq \frac{2}{3}}$ — (1)

Case 1

$3x-2 = x$

$2x = 2$

$\boxed{x = 2/2} = 1$

$\boxed{x=1}$ — (2)

(1) ∩ (2) $\boxed{x=1}$

Case 2: $3x-2 < 0$ i.e $x < 2/3$ — (1)

$-(3x-2) = x$

$-3x+2 = x$

$4x \geq 2$

$\boxed{x \geq \frac{1}{2}}$ — (2)

(1) ∩ (2)

$\boxed{x \geq \frac{1}{2}}$

Soln $x=1$ $x \geq \frac{1}{2}$

$\{1, \frac{1}{2}\}$

(12) $|x-3| + |x-2| = 1$

↓

$|3-x| + |x-2| = 1$

let $a = 3-x$

$b = x-2$

We know that $|a| + |b| = |a+b| \Rightarrow \boxed{a \cdot b \geq 0}$

$|3-x| + |x-2| = |3-x+x-2| = 1$

Hence $(3-x)(x-2) \geq 0$

$(x-3)(x-2) \leq 0$

+ | − | +

2 3

$\boxed{x \in [2,3]}$

(13) $x^2 - 4|x| + 3 < 0$

$x^2 = |x|^2$

$|x|^2 - 4|x| + 3 < 0$

$|x|^2 - 3|x| - |x| + 3 < 0$

$|x|(|x|-3) - 1(|x|-3) < 0$

$(|x|-1)(|x|-3) < 0$

+ | − | +

1 3

$1 < |x| < 3$

$x \in (-3, -1) \cup (1, 3)$ Ans

(14) $|3x-2| < 4$

$-4 < 3x-2 < 4$

$-4+2 < 3x-2+2 < 4+2$

$-2 < 3x < 6$

$-\frac{2}{3} < x < 2$

$x \in \left(-\frac{2}{3}, 2\right)$ Ans

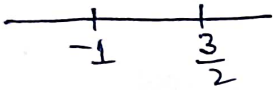
(15) $1 \le |x-2| \le 3$

$x-2 \in [-3,-1] \cup [1,3]$

$x \in [-3+2, -1+2] \cup [1+2, 3+2]$

$x \in [-1, 1] \cup [3, 5]$ Ans

(16) $|x+1| + |2x-3| = 4$



Case① if $x \ge 3/2$ $\quad x+1+2x-3=4$

$3x = 6$

$x = 2$

$x = 2$

Case 2 $\quad -1 \le x < 3/2 \quad x+1-(2x-3)=4$

$x+1-2x+3=4$

$-x = 0$

$x = 0$

Case 3: $x < -1 \quad -(x+1)-(2x-3)=4$

$-x-1-2x+3=4$

$-3x = 2$

$x = -2/3 \rightarrow$ not acceptable

Hence $\{0, 2\} \rightarrow$ Soln

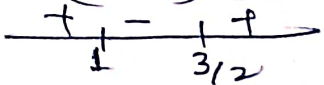
(17) $|2x-3| + |x-1| = |x-2|$

$|2x-3| + |1-x| = |2x-3+1-x| = |x-2|$

$|a| + |b| \ge |a+b| \Rightarrow a \cdot b \ge 0$

$\Rightarrow (2x-3)(1-x) \ge 0$

$(x-3/2)(x-1) \le 0$



$x \in [1, 3/2]$ Ans

(18) $|x| = 2x-1$

Case 1 $x \ge 0$ then $|x| = x$

$x = 2x-1$

$x = 1$ ✓

Case 2: $x < 0$ then $|x| = -x$

$-x = 2x-1 \quad 3x = 1$

$x = 1/3$ Not acceptable

Hence $x = 1$ → only Soln

(19) $|x^2-x-2| + |x+6| = |x^2-2x-8|$

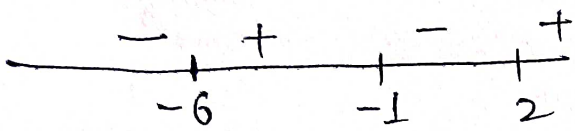
$|x^2-x-2| + |-x-6| = |x^2-x-2-x-6| = |x^2-2x-8|$

$|a| + |b| = |a+b| \Rightarrow a \cdot b \geq 0$

$(x^2-x-2)(-x-6) \geq 0$

$(x^2-2x+x-2)(x+6) \leq 0$

$(x-2)(x+1)(x+6) \leq 0$



$x \in (-\infty\ -6) \cup (-1, 2)$ Ans

(20) $|x^2-1| + |x^2-4| > 3$

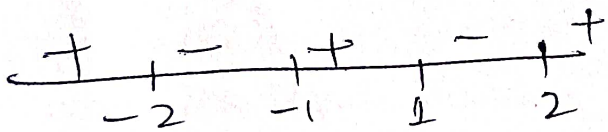
$|x|+|y| > |x+y|$ if $x \cdot y < 0$

$|x^2-1| + |4-x^2| > |x^2-1+4-x^2| = 3$

then $(x^2-1)(4-x^2) < 0$

$(x^2-1)(x^2-4) > 0$

$(x+1)(x-1)(x+2)(x-2) > 0$



$x \in (-\infty\ -2) \cup (-1, 1) \cup (2, \infty)$ Ans