Chapter	
02	**Sequence And Series**

JEE-FLASHBACK

JEE MAINS QUESTION

Q.1 If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \ldots\ldots\ldots +$ $10 (11)^9 = k(10)^9$, then k is equal to

[JEE(Main) 2014]

(1) 100   (2) 110   (3) $\dfrac{121}{10}$ (4) $\dfrac{441}{100}$

Q.2 Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is **[JEE(Main) 2014]**

(1) $2 - \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $\sqrt{2} + \sqrt{3}$ (4) $3 + \sqrt{2}$

Q.3 If m is the A. M. of two distinct real numbers l and n(l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2 G_2^4 + G_3^4$ equals : **[JEE(Main) 2015]**

(1) $4 l^2$ mm   (2) $4 lm^2$ n

(3) $4 lmn^2$ (4) $4 l^2 m^2 n^2$

Q.4 The sum of first 9 terms of the series

$$\dfrac{1^3}{1} + \dfrac{1^3 + 2^3}{1 + 3} + \dfrac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \ldots\ldots\text{ is :}$$

[JEE(Main) 2015]

(1) 71   (2) 96   (3) 142   (4) 192

Q.5 If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:

[JEE(Main) 2016]

(1) $\dfrac{4}{3}$ (2) 1   (3) $\dfrac{7}{4}$ (4) $\dfrac{8}{5}$

Q.6 If the sum of the first ten terms of the series

$$\left(1\dfrac{3}{5}\right)^2 + \left(2\dfrac{2}{5}\right)^2 + \left(3\dfrac{1}{5}\right)^2 + 4^2 + \left(4\dfrac{4}{5}\right)^2 + \ldots\ldots$$

is $\dfrac{16}{5}$ m, then m is equal to: **[JEE(Main) 2016]**

(1) 101   (2) 100   (3) 99   (4) 102

Q.7 For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$, Then

[JEE(Main) 2017]

(1) b , c and a are in G.P.

(2) b, c and a are in A.P.

(3) a, b and c are in A.P.

(4) a, b and c are in G.P.

Q.8 Let a,b,c $\in$ R. If $f(x) = ax^2 + bx + c$ is such that a + b + c = 3 and $f(x + y) = f(x) + f(y) + xy$, $\forall$ x,y $\in$ R, then $\sum\limits_{n=1}^{10} f(n)$ is equal to **[JEE(Main) 2017]**

(1) 330   (2) 165   (3) 190   (4) 225

Q.9 Let a_1, a_2, a_3,......, a_{49} be in A.P. such that $\sum\limits_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots\ldots + a_{17}^2 = 140$ m , then m is equal to :

[JEE(Main) 2018]

(1) 34   (2) 33   (3) 66   (4) 68

Q.10 Let A be the sum of the first 20 terms and B be sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2. 6^2 + \ldots$

If B − 2A = 100 λ, then λ is equal to :

[JEE(Main) 2018]

(1) 464   (2) 496   (3) 232   (4) 248

Q.11 The sum of the following series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+$$

$$\frac{15(1^2+2^2+.....+5^2)}{11}+.... \text{ up to 15 terms, is :}$$

[JEE(Main) 2019]

(1) 7510   (2) 7830   (3) 7520   (4) 7820

Q.12 Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is

[JEE MAIN 2020]

(1) 16   (2) 27   (3) 7   (4) $\frac{21}{2}$

Q.13 If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + …. Is (102)m, then m is equal to   **[JEE MAIN 2020]**

(1) 5   (2) 10   (3) 20   (4) 25

Q.14 If $|x|<1, |y|<1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$ is

[JEE MAIN 2020]

(1) $\frac{x+y-xy}{(1+x)(1+y)}$ (2) $\frac{x+y+xy}{(1+x)(1+y)}$

(3) $\frac{x+y-xy}{(1-x)(1-y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

Q.15 Let S be the sum of the first 9 terms of the series : $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3 + (k + a)a\} + \{x^4 + (k + 6)a\} + ……$ where $a \neq 0$ and $x \neq 1$.

If S = $\frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to

[JEE MAIN 2020]

(1) −5   (2) 1   (3) −3   (4) 3

Q.16 Let a_1, a_2, a_3, …. be an A.P. If $\frac{a_1 + a_2 + ... + a_{10}}{a_1 + a_2 + ... + a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to   **[JEE MAIN 2021]**

(1) $\frac{19}{21}$ (2) $\frac{100}{121}$

(3) $\frac{21}{19}$ (4) $\frac{121}{100}$

Q.17 Let a_1, a_2,….a_{21} be an AP such that $\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to   **[JEE MAIN 2021]**

(1) 57   (2) 48   (3) 36   (4) 72

Q.18 Let S_n denote the sum of first n- terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to   **[JEE MAIN 2021]**

(1) 1862   (2) 1852

(3) 1842   (4) 1872

Q.19 If the sum of an infinite GP a, ar, ar^2, ar^3, …. Is 15, and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, …. Is

[JEE MAIN 2021]

(1) $\frac{25}{2}$ (2) $\frac{9}{2}$

(3) $\frac{1}{2}$ (4) $\frac{5}{2}$

Q.20 If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ….$, is equal to

[JEE MAIN 2021]

(1) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(2) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(3) $\frac{1+x}{1-x} + \log_e(1-x)$

(4) $\frac{1-x}{1+x} + \log_e(1-x)$

Q.21 The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + ...$ is

[JEE MAIN 2021]

(1) $\frac{143}{144}$ (2) $\frac{99}{100}$

(3) 1   (4) $\frac{120}{121}$

MATHEMATICS

Q.22 Suppose a_1, a_2,……, a_n,…… be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is 5:17 and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to **[JEE MAIN 2022]**
(1) 290   (2) 380
(3) 460   (4) 510

Q.23 If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n}a_i = 192, \sum_{i=1}^{n/2}a_{2i} = 120$, and then n is equal to **[JEE MAIN 2022]**
(1) 48   (2) 96   (3) 92   (4) 104

Q.24 If a_1, a_2, a_3 …. and b_1, b_2, b_3 …. are in A.P., and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10}b_{10}$, then a_4b_4 is equal to **[JEE MAIN 2022]**
(1) $\frac{35}{27}$ (2) 1   (3) $\frac{27}{28}$ (4) $\frac{28}{27}$

Q.25 Let $f : N \rightarrow R$ be a function such that $f(x + y) = 2f(x) f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which $\sum_{k=1}^{10}f(\alpha+k) = \frac{512}{3}$ $(2^{20}-1)$ holds, is **[JEE MAIN 2022]**
(1) 2   (2) 3   (3) 4   (4) 6

Q.26 The sum $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}$ is equal to **[JEE MAIN 2022]**
(1) $\frac{7}{87}$ (2) $\frac{7}{29}$
(3) $\frac{14}{84}$ (4) $\frac{21}{29}$

Q.27 Consider two G.P.'s 2, 2^2, 2^3, …….. and 4, 4^2, 4^3, ….. of 60 and n terns respectively. If the geometric mean of all the 60 + n terms is sssa

$2^{\frac{225}{8}}$, then $\sum_{k=1}^{n}k(n-k)$ is equal to **[JEE MAIN 2022]**
(1) 560   (2) 1540
(3) 1330   (4) 2600

Q.28 Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _____ . **[JEE MAIN 2022]**

Q.29 The series of positive multiple of 3 is divided into sets : {3}, {6, 9, 12}, {15, 18, 21, 24, 27}, …… Then the sum of the elements in the 11th set is equal to **[JEE MAIN 2022]**

Q.30 Let the sum of infinite G.P., whose first term is a and the common ratio is r, be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an A.P., whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to **[JEE MAIN 2022]**
(1) $21a_{11}$ (2) $22a_{11}$
(3) $15a_{16}$ (4) $14a_{16}$

Q.31 If $\frac{1}{2.3^{10}}+\frac{1}{2^2.3^9}+....+\frac{1}{2^{10}.3}=\frac{K}{2^{10}.3^{10}}$, then the remainder when K is divided by 6 is **[JEE MAIN 2022]**
(1) 1   (2) 2   (3) 3   (4) 5

Q.32 Let x_1, x_2, x_3,……, x_{20} be geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data,

then the greatest integer less than or equal to $\bar{x}$ is ____ . **[JEE MAIN 2022]**

Q.33 If $\dfrac{6}{3^{12}} + \dfrac{10}{3^{11}} + \dfrac{20}{3^{10}} + \dfrac{40}{3^{9}} + ... + \dfrac{10240}{3} = 2^{n}.\,m$, where m is odd, then .m. n is equal to ____ .

[JEE MAIN 2022]

Q.34 Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\displaystyle\sum_{n=1}^{15} a_n . b_n$ is equal to ____ . **[JEE MAIN 2022]**

Q.35 If $\displaystyle\sum_{k=1}^{10} \dfrac{k}{k^4 + k^2 + 1} = \dfrac{m}{n}$, where m and n are co-prime, then m + n is equal to _____.

[JEE MAIN 2022]

Q.36 If gcd (m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012\,m^2 n$ then $m^2 - n^2$ is equal to **[JEE MAIN 2023]**
(1) 240   (2) 200   (3) 220   (4) 180

Q.37 If $(20)^{19} + 2(21)\,(20)^{18} + 3(21)^2\,(20)^{17} + ... + 20(21)^{19} = k(20)^{19}$, then k is equal to _____.

[JEE MAIN 2023]

Q.38 Let $S_k = \dfrac{1 + 2 + ... + K}{K}$ and $\displaystyle\sum_{j=1}^{n} S_j^2 = \dfrac{n}{A}\left(Bn^2 + Cn + D\right)$ where A, B, C, D $\in \mathbb{N}$ and A has least value. then

[JEE MAIN 2023]

(1) A + C + D is not divisible by B
(2) A + B = 5(D − C)
(3) A + B + C + D is divisible by D
(4) A + B is divisible by D

Q.39 Let a_n be n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 + …. and $S_n = \displaystyle\sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to **[JEE MAIN 2023]**

(1) 11310   (2) 11260
(3) 11290   (4) 11280

Q.40 Let 0 < z < y < x be three real numbers such that $\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$ are in an arithmetic progression and x, $\sqrt{2}y$, z are in a geometric progression. If xy + yz + zx = $\dfrac{3}{\sqrt{2}}$ xyz, then $3(x + y + z)^2$ is equal to _____. **[JEE MAIN 2023]**

Q.41 Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to **[JEE MAIN 2023]**
(1) 241   (2) 231   (3) 210   (4) 220

Q.42 The sum of all those terms, of the arithmetic progression 3, 8, 13, …, 373, which are not divisible by 3, is equal to _____.

[JEE MAIN 2023]

Q.43 If $S_n = 4 + 11 + 21 + 34 + 50 + ...$ to n terms, then $\dfrac{1}{60}\left(S_{29} - S_9\right)$ is equal to **[JEE MAIN 2023]**
(1) 223   (2) 226   (3) 220   (4) 227

Q.44 Suppose a_1 , a_2 , 2, a_3 , a_4 be in an arithmetico geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\dfrac{49}{2}$, then a_4 is equal to _____. **[JEE MAIN 2023]**

Q.45 Let $S = 109 + \dfrac{108}{5} + \dfrac{107}{5^2} + ... + \dfrac{2}{5^{107}} + \dfrac{1}{5^{108}}$. Then the value of $\left(16S - (25)^{-54}\right)$ is equal to_____. **[JEE MAIN 2023]**

MATHEMATICS

JEE ADVANCED QUESTION

Q.1 Let $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s) **[JEE (Advanced) 2013]**
(1) 1056   (2) 1088
(3) 1120   (4) 1332

Q.2 Let a,b,c be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is $b + 2$, then the value of $\frac{a^2+a-14}{a+1}$ is **[JEE (Advanced) 2014]**

Q.3 Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is
[JEE (Advanced) 2015]

Q.4 Let $b_i > 1$ for $i = 1,2,....,101$. Suppose $\log_e b_1$, $\log_e b_2,...,\log_e b_{101}$ are in Arithmetic progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2,...,a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$, then **[JEE (Advanced) 2016]**
(1) $s > t$ and $a_{101} > b_{101}$
(2) $s > t$ and $a_{101} < b_{101}$
(3) $s < t$ and $a_{101} > b_{101}$
(4) $s < t$ and $a_{101} < b_{101}$

Q.5 The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?
[JEE(Advanced) 2017]

Q.6 Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set X $\cup$ Y is _____. **[JEE(Advanced) 2018]**

Q.7 Let m be the minimum possible value of $\log_3 (3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1,y_2,y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^2) + \log_3(M^2)$ is _____.
[JEE (Advance) 2020]

Q.8 Let $a_1,a_2,a_3, ...$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1,b_2,b_3, ...$ be a sequence of positive integers in geometric progression with common ration 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality $2(a_1 + a_2 +...+ a_n) = b_1 + b_2 +...+ b_n$ holds for some positive integer n, is ____.
[JEE (Advance) 2020]

Q.9 Let $a_1, a_2, a_3, ...$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, ...$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are true?
[JEE(Advanced) 2022]
(1) $T_{20} = 1604$ (2) $\sum_{k=1}^{20} T_k = 10510$
(3) $T_{30} = 3454$ (4) $\sum_{k=1}^{30} T_k = 35610$

Q.10 Let $l_1,l_2,...,l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1,w_2,...w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each $i = 1,2...,100$, let R_i be a rectangle with length l_i width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is _____ **[JEE(Advanced) 2022]**

ANSWER KEY

JEE-FLASHBACK

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	2	2	2	1	1	2	1	1	4	4	1	3	3	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	4	1	3	2	4	2	2	4	3	2	3	38	6993	1
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	4	142	12	27560	166	1	400	4	3	150	2	9525	1	16	2175

JEE-ADVANCED QUESTION

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1,4	4	9	2	6	3748	8	1	2,3	18900