Chapter 03

Quadratic Equation

JEE-FLASHBACK

JEE MAINS QUESTION

Q.1 If $a \in R$ and the equation $-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval : **[JEE(Main) 2014]**

(1) $(-2, -1)$ (2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-1, 0) \cup (0, 1)$ (4) $(1, 2)$

Q.2 Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q , r are in the A.P. and $\frac{1}{\alpha}+\frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is : **[JEE(Main)2014]**

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

Q.3 Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \ge 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[JEE(Main)2015]**

(1) 6 (2) – 6 (3) 3 (4) –3

Q.4 The sum of all real values of x satisfying the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is :- **[JEE-MAIN-2016]**

(1) 5 (2) 3 (3) –4 (4) 6

Q.5 If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is- **[JEE MAIN-2017]**

(1) $(-3, 3)$ (2) $(-3, \infty)$

(3) $(3, \infty)$ (4) $(-\infty, -3)$

Q.6 The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is : **[JEE(Main) 2019]**

(1) 3 (2) 4 (3) 5 (4) 2

Q.7 If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is : **[JEE(Main) 2019]**

(1) $-2 + \sqrt{2}$ (2) $4 - 3\sqrt{2}$

(3) $2 - \sqrt{3}$ (4) $4 - 2\sqrt{3}$

Q.8 Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\cdot\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is : **[JEE-MAIN-2020]**

(1) 10 (2) $5\sqrt{2}$ (3) 5 (4) $10\sqrt{2}$

Q.9 Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \ge 1$, then which one of the following statements is not true ? **[JEE-MAIN-2020]**

(1) $p_3 = p_5 - p_4$

(2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_5 = 11$

(4) $p_5 = p_2 \cdot p_3$

Q.10 Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$ Then S. **[JEE-MAIN-2020]**

(1) Contains exactly two elements

(2) contains at least four elements

(3) is an empty sets

(4) is a singleton

Q.11 Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : **[JEE-MAIN-2020]**

(1) 25 (2) 28 (3) 26 (4) 24

Q.12 Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3, ..., then: **[JEE-MAIN-2020]**

(1) $6S_6 + 5S_5 = 2S_4$ (2) $5S_6 + 6S_5 = 2S_4$

(3) $6S_6 + 5S_5 + 2S_4 = 0$ (4) $5S_6 + 6S_5 + 2S_4 = 0$

Q.13 Let f(x) be a quadratic polynomial such that f(–1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in : **[JEE-MAIN-2020]**

(1) (0, 1) (2) (–3, –1)

(3) (–1, 0) (4) (1, 3)

Q.14 If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal to : **[JEE-MAIN-2020]**

(1) $\frac{9}{4}(9 + q^2)$ (2) $\frac{9}{4}(9 - p^2)$

(3) $\frac{9}{4}(9 - q^2)$ (4) $\frac{9}{4}(9 + p^2)$

Q.15 Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio (2q + p) : (2q – p) is : **[JEE-MAIN-2020]**

(1) 3 : 1 (2) 5 : 3

(3) 9 : 7 (4) 33 : 31

Q.16 Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to – **[JEE-MAIN-2020]**

(1) 9 (2) 36 (3) 18 (4) 27

Q.17 If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$ is equal to : **[JEE-MAIN-2020]**

(1) $\frac{3}{8}$ (2) $\frac{1}{24}$ (3) $\frac{27}{16}$ (4) $\frac{27}{32}$

Q.18 If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}}+\left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is **[JEE-MAIN-2020]**

(1) 1 (2) 3 (3) 4 (4) 2

Q.19 If α and β are the roots of the equation 2x(2x + 1) = 1, then β is equal to **[JEE-MAIN-2020]**

(1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$

(3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

Q.20 Let p and q be two positive numbers such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation: **[JEE-MAIN-2021]**

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Q.21 Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is : **[JEE-MAIN-2021]**

(1) $\frac{71}{256}$ (2) $\frac{69}{256}$ (3) $-\frac{69}{256}$ (4) $-\frac{71}{256}$

Q.22 The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is : **[JEE-MAIN-2021]**

(1) 3 (2) 2 (3) 0 (4) 4

Q.23 If $\alpha, \beta \in$ R are such that 1– 2i (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to : **[JEE-MAIN-2021]**

(1) –3 (2) –7 (3) 7 (4) 3

Q.24 Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$. then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

[JEE-MAIN-2021]

(1) 2 (2) 1 (3) 4 (4) 3

Q.25 The number of real solution of the equation, $x^2 - |x| - 12 = 0$ is : **[JEE-MAIN-2021]**

(1) 2 (2) 3 (3) 1 (4) 4

Q.26 Let $\alpha = \max_{x \in R} \left\{ 8^{2\sin 3x} . 4^{4\cos 3x} \right\}$ and $\beta = \min_{x \in R} \left\{ 8^{2\sin 3x} . 4^{4\cos 3x} \right\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of c – b is equal to :

[JEE-MAIN-2021]

(1) 42 (2) 47 (3)43 (4) 50

Q.27 Let α, β be two roots of the equation $x^2 + (20)^{1/4} x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to **[JEE-MAIN-2021]**

(1) 10 (2) 100 (3) 50 (4) 160

Q.28 The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is _____. **[JEE-MAIN-2021]**

Q.29 The number of the real roots of the equation $(x + 1)^2 + |x - 5| = \frac{27}{4}$ is _____.

[JEE-MAIN-2021]

Q.30 If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to

[JEE MAIN 2022]

(1) 18 (2) 24 (3) 36 (96)

Q.31 Let $a, b \in$ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to **[JEE MAIN 2022]**

(1) 37 (2) 58 (3) 68 (4) 92

Q.32 Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $\left|\alpha^8 + \beta^8\right|$ is equal to :

[JEE MAIN 2022]

(1) 50 (2) 250 (3) 1250 (4) 1500

Q.33 If the sum of all the roots of the equation $e^{2x} - 11e^x - 45^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to_____. **[JEE MAIN 2022]**

Q.34 If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x = 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to : **[JEE MAIN 2022]**

(1) -4 (2) -1 (3) 1 (4) 4

Q.35 The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is

[JEE MAIN 2022]

(1) 4 (2) 5 (3) 6 (4) 8

Q.36 Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation

$X^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

[JEE MAIN 2022]

(1) non-real complex numbers

(2) real and both negative

(3) real and both positive

(4) real and exactly one of them is positive

Q.37 For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is _____.

[JEE MAIN 2022]

Q.38 Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to **[JEE MAIN 2023]**

(1) $\frac{155}{8}$ (2) 21 (3) $\frac{169}{8}$ (4) 19

Q.39 Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and

$x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer $\le x$. Then $m^2 + mn + n^2$ is equal to _____. **[JEE MAIN 2023]**

Q.40 If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is equal to____ **[JEE MAIN 2023]**

JEE ADVANCED QUESTION

Q.1 The quadratic equation p(x) = 0 with real coefficients has purely imaginary roots. Then the equation p(p(x)) = 0 has **[JEE (Advanced) 2014]**

(1) only purely imaginary roots
(2) all real roots
(3) two real and two purely imaginary roots
(4) neither real nor purely imaginary roots

Q.2 Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **[JEE (Advanced) 2015]**

(1) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (2) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(3) $\left(0, \frac{1}{\sqrt{5}}\right)$ (4) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Q.3 Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals **[JEE (Advanced) 2016]**

(1) $2(\sec\theta - \tan\theta)$ (2) $2\sec\theta$
(C) $-2\tan\theta$ (4) 0

Comprehension (Q.4 & Q.5)

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \ne \beta$. For n = 0,1,2,...., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

Q.4 a_{12} = **[JEE(Advanced) 2017]**

(1) $a_{11} + 2a_{10}$ (2) $2a_{11} + a_{10}$
(3) $a_{11} - a_{10}$ (4) $a_{11} + a_{10}$

Q.5 If $a_4 = 28$, then p + 2q = **[JEE(Advanced) 2017]**

(1) 14 (2) 7
(3) 21 (4) 12

Q.6 Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \ge 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \ge 2$.

Then the following options is/are correct? **[JEE(Advanced) 2019]**

(1) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(2) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$

(3) $a_1 + a_2 + ...+ a_n = a_{n+2} - 1$ for all $n \ge 1$

(4) $x^2 \ge 1$

Q.7 Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 + 20x - 2020$. Then the value of ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d) is **[JEE(Advanced) 2020]**

(1) 0 (2) 8000
(3) 8080 (4) 16000

Q.8 For $x \in \mathbb{R}$, the number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is_____. **[JEE(Advanced) 2021]**

ANSWER KEY

JEE-FLASHBACK

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	2	3	2	1	1	2	1	4	4	1	2	3	2	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	3	4	3	4	4	1	2	1	1	1	3	8	2	2
Que.	31	32	33	34	35	36	37	38	39	40					
Ans.	2	1	45	2	3	2	4	4	9	51					

JEE-ADVANCED QUESTION

Que.	1	2	3	4	5	6	7	8
Ans:	4	1,4	3	4	4	1,2,3	4	4