Chapter 01

Basic Maths and Logarithm

RANKER'S STUFF

SINGLE CORRECT TYPE QUESTION

Q.1 $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(1) 1/2 (2) 1 (3) 2 (4) 4

Q.2 If $5\, x^{\log_2 3} + 3^{\log_2 x} = 162$ then logarithm of x to the base 4 has the value equal to

(1) 2 (2) 1 (3) -1 (4) 3/2

Q.3 If $\log (x+y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$, then

(1) $x+y=0$ (2) $xy = 1$

(3) $x^2 + xy + y^2 = 0$ (4) $x-y=0$

Q.4 If $\log_2(\log_3(\log_4 x)) = 0$, $\log_4(\log_3(\log_2 y)) = 0$ and $\log_3(\log_4(\log_2 z)) = 0$, then the correct option is

(1) $x > y > z$ (2) $x > z > y$

(3) $z > x > y$ (4) $z > y > z$

Q.5 The value of $\log_2\left(\frac{1}{7^{\log_7 0.125}}\right)$, is

(1) 1 (2) 2 (3) 3 (4) 4

Q.6 Let x satisfies the equation $\log_3(\log_9 x) = \log_9(\log_3 x)$ then the product of the digits in x is

(1) 9 (2) 18 (3) 36 (4) 8

Q.7 If $\log_3(\log_2 a) + \log_{\frac{1}{3}}\left(\log_{\frac{1}{2}} b\right) = 1$, then the value of ab^3 is

(1) 9 (2) 3 (3) 1 (4) $\frac{1}{3}$

Q.8 The value of

$$\log_{\frac{4}{3}}\left(\frac{56+\sqrt{56+\sqrt{56+\sqrt{56+\ldots\ldots\infty}}}}{\sqrt{64\sqrt{64\sqrt{64\ldots\ldots\infty}}}}\right)$$ is

equal to

(1) 0 (2) 2 (3) 3 (4) 4

Q.9 The value of $\log_{\frac{1}{6}} 2 \cdot \log_5 36 \cdot \log_{17} 125 \cdot \log_{\frac{1}{\sqrt{2}}} 17$, is equal to

(1) -3 (2) -6 (3) 6 (4) 12

Q.10 Product of all the solution of equation $x^{\log_{10} x} = \left(100 + 2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}}\right)x$ is

(1) $\frac{1}{10}$ (2) 1 (3) 10 (4) 100

Q.11 If $\log_7 2 = m$, then the value of $\log_{49} 28$ is

(1) 2 (1 + 2m) (2) $\frac{1+2m}{2}$

(3) $\frac{2}{1+2m}$ (4) 1 + m

Q.12 If $\frac{a+\log_4 3}{a+\log_2 3} = \frac{a+\log_8 3}{a+\log_4 3} = b$, then b is equal to

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{3}{2}$

Q.13 Let x, y and z be positive real numbers such that $x^{\log_2 7} = 8$, $y^{\log_3 5} = 81$ and $z^{\log_5 216} = \sqrt[3]{5}$. The value of $x^{(\log_2 7)^2} + y^{(\log_3 5)^2} + z^{(\log_5 216)^2}$, is

(1) 526 (2) 750 (3) 874 (4) 974

Q.14 If x = 500, y = 100 and z = 5050, then the value of $\left(\log_{xyz} x^z\right)\left(1+\log_x yz\right)$ is equal to

(1) 500 (2) 100 (3) 5050 (4) 10

Q.15 Suppose n be an integer greater than 1, let $a_n = \frac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then (b – c) equals

(1) $\frac{1}{1001}$ (2) $\frac{1}{1002}$

(3) – 1 (4) – 2

Q.16 If $10^{\log_a(\log_b(\log_c x))} = 1$ and $10^{(\log_b(\log_c(\log_a x)))} = 1$ then, a is equal to

(1) $\frac{a}{b}$ (2) $c^{a/b}$ (3) ab (4) $c^{b/c}$

Q.17 If $x \in R$, then number of real solution of the equation $2^x + 2^{-x} = \log_5 24$ is

(1) 0 (2) 1

(3) 2 (4) more than 2

Q.18 If $x \geq y > 1$ then the maximum value of $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ is equal to

(1) – 2 (2) 0 (3) 2 (4) 4

Q.19 If $\log_5(3^x - 4^y) = 3$ and $3^{\frac{x}{2}} - 2^y = 5$, then $\frac{x}{y}$ is equal to

(1) $\frac{2(\log_2 5)-2}{1+\log_2 5}$ (2) $\frac{(\log_3 5)+2}{1+\log_2 5}$

(3) $\frac{2(\log_3 5)+2}{1+\log_2 5}$ (4) $\frac{2(\log_3 5)+1}{1+\log_2 5}$

Q.20 Let $x = 4^{\log_2 \sqrt{9^{k-1}+7}}$ and $y = \frac{1}{32^{\log_2 \sqrt[5]{3^{k-1}+1}}}$ and xy = 4, then the sum of the cubes of the real value(s) of k is

(1) 1 (2) 5 (3) 8 (4) 9

Q.21 Number of real solution(s) of the equation $9^{\log_3(\ln x)} = \ln x - (\ln^2 x) + 1$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

Q.22 If a, b, c are distinct positive number but no one among them is equal to one and $\log_b a \log_c a + \log_a b \log_c b + \log_a c \log_b c = 3$, then value of abc is

(1) 2 (2) 3 (3) 0 (4) 1

Q.23 Let α, β, are two real solution of equation $(\log_{10} x)^2 + \log_{10} x^2 = (\log_{10} 2)^2 - 1$, then $\sqrt{\frac{1}{\alpha\beta}}$ equal to

(1) 20 (2) 3 (3) 10 (4) 1

Q.24 Let a, b, c, d are positive integer such that $\log_a b = 3/2$ and $\log_c d = 5/4$. If a – c = 9, then value of (b – d) is equal to

(1) 20 (2) 93 (3) 10 (4) 1

Q.25 The values of a for which the equation $2(\log_3 x)^2 - |\log_3 x| + a = 0$ posses four real solution

(1) $-2 < a < 0$ (2) $0 < a < \frac{1}{8}$

(3) $0 < a < 5$ (4) $-\frac{1}{8} < a < 0$

NUMERICAL VALUE TYPE QUESTIONS

Q.26 Suppose x, y, z > 0 & different from one and $\ell n\ x + \ell n\ y + \ell n\ z = 0$, then value of $x^{\frac{1}{\ell ny}+\frac{1}{\ell nz}} \cdot y^{\frac{1}{\ell nz}+\frac{1}{\ell nx}} \cdot z^{\frac{1}{\ell nx}+\frac{1}{\ell ny}}$ is e^{-k} then k equals _____.

Q.27 If $\log\left(\frac{x^2}{y^3}\right) = 1$ & $\log(x^2y^3) = 7$ then $\log|xy|$ is equal to_____.

Q.28 The number of positive integers satisfying $x + \log_{10}(2^x + 1) = x\log_{10}5 + \log_{10}6$ is _____.

Q.29 If $x, y \in R^+$ satisfies $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$, then the value of xy is_____.

Q.30 Find all positive values of 'a' for which the equation $\log(ax) = 2\log(x + 1)$ has the unique root_____.

Q.31 If x, y, z be positive real numbers such that $\log_{2x} z = 3$, $\log_{5y} z = 6$ and $\log_{xy} z = 2/3$ then the value of z is in the form of m/n in lowest form then n – m is equal to_____.

Q.32 The number $N = \frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$ when simplified reduces to a natural number N. find N

Q.33 Let P = $\log_5(\log_5 3)$. If $3^{C+5^{-P}} = 405$ then C equals_____.

Q.34 Number of solution for $|3x^2 - 2| = [-2\pi]$ (where [·] denotes greatest integer) is _____.

Q.35 The number of positive integers satisfying the equation $x + \log_{10}(2^x + 1) = x\log_{10}5 + \log_{10}6$ is _____.

STATEMENT TYPE QUESTIONS

Each of the questions given below consist of Statement -I and Statement- II. Use the following key to choose the appropriate answer.

(A) Both Statement-I Statement-II are true, and Statement-II is the correct explanation of Statement-I.
(B) Both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I
(C) Statement-I is true but Statement-II is false
(D) Statement-I is false but Statement-II is true.

Q.36 **Statement-I:** If $N = \left(\frac{1}{0{\cdot}4}\right)^{20}$ then N contains 7 digits before decimal
Statement-II: Characteristic of logarithm of N to base 10 is 7.
(1) A (2) B (3) C (4) D

Q.37 **Statement-I:** The equation $(\log x)^2 - \log x^3 + 2 = 0$ has only one solution.
Statement-II: $\log x^2 = 2\log x$ if $x > 0$.
(1) A (2) B (3) C (4) D

Q.38 **Statement-I:** Number of cyphers after decimal before a significant figure comes is $N = 2^{-100}$ is 30
Statement-II: Number of cyphers after decimal before a significant figure comes in $N = 2^{-10}$ is 3
(1) A (2) B (3) C (4) D

MORE THAN ONE CORRECT TYPE QUESTIONS

Q.39 Which of the following when simplified reduces to unity ?
(1) $\log_{1.5}\log_4{}^{\log_{\sqrt{3}}}81$
(2) $\log_2\sqrt{6} + \log_2\sqrt{\frac{2}{3}}$
(3) $-\frac{1}{6}\log_{\frac{\sqrt{3}}{2}}\left(\frac{64}{27}\right)$
(4) $\log_{3.5}(1 + 2 + 3 \div 6)$

Q.40 The x-values satisfying the equation $|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$ is/are
(1) $\frac{1}{\sqrt{3}}$ (2) 1 (3) 2 (4) 81

Q.41 If x = 9 is solution of $\ell n(x^2+15a^2) - \ell n(a-2) = \ell n\left(\frac{8ax}{a-2}\right)$ then
(1) $a = \frac{3}{5}$ (2) a = 3 (3) x = 15 (4) x = 2

Q.42 If $y = \log_{7-a}(2x^2 + 2x + a + 3)$ is defined $\forall x \in R$ then possible integer value of a is/are
(1) 4 (2) – 3 (3) – 2 (4) 5

COMPREHENSION TYPE QUESTIONS

Q.43 Sometimes to solve an equation we may use the identity $a^{\log_a b} = b$, $b > 0$, $a > 0$, $a \neq 1$.

(i) The number of solution of $x^{\log_x(x+3)^2} = 16$ is -
(1) 0 (2) 1 (3) 2 (4) ∞

(ii) Solution set of the equation $\frac{1}{4}x^{\log_2\sqrt{x}} = (2^{1/4})(\log_2 x)^2$ is -
(1) ϕ (2) $\{4^{-\sqrt{2}}\}$
(3) $\{4^{\sqrt{2}}\}$ (4) None of these

(iii) Solution set of $x^{\log_4 x} = 2^{3(\log_4 x+3)}$ is -
(1) {64, 1/8} (2) {64}
(3) {1/8} (4) ϕ

Q.44 Given: $|x-1| + |x-2| = \lambda$ where $\lambda \in R$ then answer the following

(i) If $\lambda = 6$ then x belongs to

(1) $\left[-\frac{3}{2},\frac{9}{2}\right]$ (2) $\left(-\frac{3}{2},\frac{9}{2}\right)$

(3) $\left\{-\frac{3}{2},\frac{9}{2}\right\}$ (4) None

(ii) If $\lambda = 0$ then x belongs to

(1) {3} (2) {–1, 6} (3) {1, 2} (4) ϕ

(iii) If $\lambda = 1$ then x belongs to

(1) $x \in (1, 2)$ (2) $x \in [1, 2)$

(3) $x \in (1, 2]$ (4) $x \in [1, 2]$

MATCH THE COLUMN TYPE QUESTIONS

Q.45 Match the following :

Column I	Column II
(1) If $\log_3 (5 + 8\log_{49}(5 + 4 \log_{49}7))$ $= \lvert k \rvert$ then value of k is	(P) 0
(2) No. of roots of equation $\log_x (x^2 - 1) = 0$ is	(Q) 1
(3) If $x = \sqrt{9+\sqrt{77}}$ then value of $\frac{1}{11}\left(x+\frac{2}{x}\right)^2$ is	(R) 2
(4) $\sqrt{(\log_{.5} 4)^2}$	(S) –2

Q.46 Match the following :

Column-I	Column-II
(1) $\log_{\sin x} \log_3 \log_{0.2} x < 0$	(P) $x \in [-1, 1]$
(2) $\frac{(e^x - 1)(2x-3)(x^2+x+2)}{(\sin x - 2)x(x+1)} \le 0$	(Q) $x \in [-3, 6]$
(3) $\lvert 2 - \lvert [x] - 1 \rvert \rvert \le 2$, [.] represents greatest integer function	(R) $x \in (0, 1/125)$
(4) $\lvert \sin^{-1}(3x - 4x^3) \rvert \le \pi/2$	(S) $x \in (-\infty, -1) \cup \left[\frac{3}{2}, \infty\right)$

ANSWER KEY

RANKER'S STUFF

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	4	4	2	3	4	3	1	4	3	2	3	4	3	3
Qus.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	4	1	2	3	4	2	4	3	2	2	3	3	1	512	4
Qus.	31	32	33	34	35	36	37	38	39	40	41	42	43(i)	43(ii)	43(iii)
Ans.	9	0	4	0	1	4	4	2	1,2,3,4	3,4	2,3	1,3,4	1	3	1
Qus.	44(i)	44(ii)	44(iii)												
Ans.	1	4	4												

Q.45 (1)→ R,S; (2) → Q; (3)→ R; (4) → R

Q.46 (1)→ R, (2) → S, (3) → Q, (4) → P