Trigonometry DPPS-3 solution

(1) $\sin\theta = \frac{12}{13}$

$\because 0<\theta<\frac{\pi}{2} \Rightarrow \cos\theta = +\sqrt{1-\frac{144}{169}} = \frac{5}{13}$

$\cos\phi = -3/5$

$\because \phi \in \left(\pi, \frac{3\pi}{2}\right)$ then $\sin\phi = -\sqrt{1-\cos^2\phi}$

$\phi \to$ 3rd Quad

$\sin\phi = -\sqrt{1-\frac{9}{25}}$

$\sin\phi = -4/5$

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$= \frac{12}{13}\times\frac{-3}{5} + \frac{5}{13}\times\frac{-4}{5}$

$= \frac{-36-20}{65} = -\frac{56}{65}$

(2) $\sin\alpha = \frac{15}{17}$

$\because \frac{\pi}{2}<\alpha<\pi$ then $\cos\alpha = -\sqrt{1-\sin^2\alpha}$

$= -\sqrt{1-\frac{225}{289}} = \frac{-8}{17}$

$\tan\beta = \frac{12}{5} = \frac{P}{B}$

$H = 13$ $\qquad \sin\beta = -\frac{12}{13}$

$\because \pi<\beta<3\pi/2$

$\beta \in$ 3rd Quadrant $\quad \cos\beta = -\frac{5}{13}$

$\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$

$= \frac{-12}{13}\times\frac{-8}{17} + \frac{5}{13}\times\frac{15}{17}$

$= +\frac{96+75}{221} = \frac{171}{221}$

(3) $\sin\theta = 3\sin(\theta+2\alpha)$

$\frac{\sin\theta}{\sin(\theta+2\alpha)} = \frac{3}{1}$

$\frac{\sin\theta+\sin(\theta+2\alpha)}{\sin\theta-\sin(\theta+2\alpha)} = \frac{3+1}{3-1}$

$\frac{2\sin\left(\frac{\theta+\theta+2\alpha}{2}\right)\cdot\cos\left(\frac{\theta-\theta-2\alpha}{2}\right)}{2\cos\frac{\theta+\theta+2\alpha}{2}\cdot\sin\frac{\theta-(\theta+2\alpha)}{2}} = \frac{4}{2}$

$\frac{\cancel{2}\sin(\theta+\alpha)\cos\alpha}{\cancel{2}\cos(\theta+\alpha)\sin(-\alpha)} = 2$

$\frac{\tan(\theta+\alpha)\cos\alpha}{-\sin\alpha} = 2$

$\frac{\tan(\theta+\alpha)}{-\tan\alpha} = 2 \Rightarrow \tan(\theta+\alpha) = -2\tan\alpha$

$\tan(\theta+\alpha) + 2\tan\alpha = 0$

(4) $\cos 15^\circ - \sin 15^\circ$

$\sqrt{2}\left(\frac{\cos 15^\circ}{\sqrt{2}} - \frac{\sin 15^\circ}{\sqrt{2}}\right)$

$\sqrt{2}(\sin 45^\circ\cos 15^\circ - \cos 45^\circ\sin 15^\circ)$

$\sqrt{2}(\sin(45^\circ-15^\circ))$

$\sqrt{2}\sin 30^\circ$

$\sqrt{2}\times\frac{1}{2} = \frac{1}{\sqrt{2}}$

(5) $\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$

$2\cos\left(\frac{\frac{\pi}{4}+\cancel{x}+\frac{\pi}{4}-\cancel{x}}{2}\right)\cdot\cos\left[\frac{\frac{\pi}{4}+x-\left(\frac{\pi}{4}-x\right)}{2}\right]$

$2\cos\left(\frac{\pi}{4}\right)\cdot\cos(2x)$

$2\times\frac{1}{\sqrt{2}}\cos 2x = \sqrt{2}\cos 2x$

(6) $\cos\left(\frac{\alpha-\beta}{2}\right) = 2\cos\left(\frac{\alpha+\beta}{2}\right)$

$\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{2}{1}$

$\frac{\cos\left(\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)-\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{2+1}{2-1}$

$\frac{\cancel{2}\cos\alpha/2\cos\beta/2}{\cancel{2}\sin\frac{\frac{\alpha-\beta+\alpha+\beta}{2}}{2}\sin\frac{\frac{\alpha+\beta-\alpha+\beta}{2}}{2}} = 3$

$\boxed{\cot\alpha/2\,\cot\beta/2 = 3}$

$\tan\alpha/2\,\tan\beta/2 = \frac{1}{3}$

(7) $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$

$\frac{\cos\left(\frac{\pi}{4}+\theta\right)}{\sin\left(\frac{\pi}{4}+\theta\right)}\,\frac{\cos\left(\frac{\pi}{4}-\theta\right)}{\sin\left(\frac{\pi}{4}-\theta\right)} =$

$= \frac{\cos^2\frac{\pi}{4}-\sin^2\theta}{\sin^2\frac{\pi}{4}-\sin^2\theta} = \frac{\frac{1}{2}-\sin^2\theta}{\frac{1}{2}-\sin^2\theta} = 1$

(8) $\tan\theta_1 = K\cot\theta_2$

$$\frac{\sin\theta_1}{\cos\theta_1} = K\frac{\cos\theta_2}{\sin\theta_2}$$

$$\frac{1}{K} = \frac{\cos\theta_1\cos\theta_2}{\sin\theta_1\sin\theta_2}$$

$$\frac{1+K}{1-K} = \frac{\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2}{\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2}$$

$$\frac{1+K}{1-K} = \frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}$$

$$\boxed{\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)} = \frac{1-K}{1+K}}$$

(9) $2\sin\left(\theta+\frac{\pi}{3}\right) = \cos\left(\theta-\frac{\pi}{6}\right)$

$$2\left[\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}\right] = \cos\theta\cos\frac{\pi}{6}+\sin\theta\sin\frac{\pi}{6}$$

$$2\left[\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta\right] = \cos\theta\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\sin\theta$$

$$2\sin\theta+2\sqrt{3}\cos\theta = \sqrt{3}\cos\theta+\sin\theta$$

$$\sin\theta = -\sqrt{3}\cos\theta$$

$$\boxed{\tan\theta = -\sqrt{3}}$$

(10) $\cos^2 45^\circ - \sin^2 15^\circ$

$$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$$

$$\cos(45^\circ+15^\circ)\ \cos(45^\circ-15^\circ)$$

$$\cos 60^\circ \times \cos 30^\circ = \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

(11) $\cos^2\left(\frac{\pi}{6}+\theta\right) - \sin^2\left(\frac{\pi}{6}-\theta\right)$

$$\cos\left(\frac{\pi}{6}+\cancel{\theta}+\frac{\pi}{6}-\cancel{\theta}\right)\cdot\cos\left(\cancel{\frac{\pi}{6}}+\theta-\cancel{\frac{\pi}{6}}+\theta\right)$$

$$\cos(60^\circ)\cos 2\theta$$

$$\frac{1}{2}\cos 2\theta$$

(12) $\tan(A+B) = p$

$\tan(A-B) = q$

$$\tan(2A) = \tan(A+B+A-B)$$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}$$

$$= \frac{p+q}{1-pq}$$

(13) $\cos(\alpha+\beta) = \frac{3}{5}$ $\quad\sin(\alpha+\beta) = +\frac{4}{5}$

$0<\alpha<\frac{\pi}{4}$, $0<\beta<\frac{\pi}{4}$ $\Rightarrow$ $0<\alpha+\beta<\pi/2$

$\sin(\alpha-\beta) = \frac{5}{13}$ $\quad -\frac{\pi}{4}<\alpha-\beta<\frac{\pi}{4}$

then $\cos(\alpha-\beta) = \frac{12}{13}$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{4}{3}$$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{5/13}{12/13} = \frac{5}{12}$$

$$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \frac{\frac{48+15}{36}}{\frac{36-20}{36}}$$

$$\tan 2\alpha = \frac{63}{16}$$

(14) $\cos(\alpha+\beta) = \frac{4}{5}$, $0<\alpha<\frac{\pi}{4}$, $0<\beta<\pi/4$

$0<\alpha+\beta<\pi/4$

$\sin(\alpha+\beta) = 3/5$

~~$\tan\beta$~~ $-\frac{\pi}{4}<\alpha-\beta<\pi/4 \Rightarrow$

$\sin(\alpha-\beta) = \frac{5}{13}$ $\quad\cos(\alpha-\beta) = \frac{12}{13}$

Since $\sin(\alpha-\beta) = +ve$ then $0<\alpha-\beta<\pi/4$

$\tan(\alpha-\beta) = \frac{5}{12}$ $\quad\tan(\alpha+\beta) = 3/4$

$$\tan(\alpha+\beta+\alpha-\beta) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{5}{12}+3/4}{1-5/12\times 3/4} = \frac{20+36/48}{33/48} = 56/33$$

(15) $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$

$= \dfrac{\cos 9^\circ\left[1 + \frac{\sin 9^\circ}{\cos 9^\circ}\right]}{\cos 9^\circ\left[1 - \frac{\sin 9^\circ}{\cos 9^\circ}\right]}$

$= \dfrac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \tan 9^\circ}$

$= \tan(45^\circ + 9^\circ) = \tan 54^\circ$

(16) if $A - B = \pi/4$

$(1 + \tan A)(1 - \tan B)$

$1 - \tan B + \tan A - \tan A \tan B$ — (1)

$A - B = \pi/4$

$\tan(A - B) = \tan\frac{\pi}{4}$

$\dfrac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\tan A - \tan B = 1 + \tan A \tan B$ — (2)

put $\tan A - \tan B = 1 + \tan A \tan B$ in (1)

$1 + 1 + \cancel{\tan A \tan B} - \cancel{\tan A \tan B} = 2$

(17) $\sin\left(\frac{31\pi}{3}\right)$

$\sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(18) $x = 5\pi/6$

$2\sin 3 \times \frac{5\pi}{6} + 3\cos 3 \times \frac{5\pi}{6}$

$2\sin\left(\frac{5\pi}{2}\right) + 3\cos\left(\frac{5\pi}{2}\right)$

$2\sin\left(2\pi + \frac{\pi}{2}\right) + 3\cos\left(2\pi + \frac{\pi}{2}\right)$

$2\sin\pi/2 + 3\cos\pi/2$

$= 2 \times 1 + 3 \times 0 = 2$

(19) $\tan 75^\circ - \cot 75^\circ$

$\tan 75^\circ = \tan(90 - 15^\circ) = \cot 15^\circ$

$\cot(75^\circ) = \cot(90 - 15^\circ) = \tan 15^\circ$

$\cot 15^\circ - \tan 15^\circ$

$= 2 + \sqrt{3} - (2 - \sqrt{3})$

$= 2\sqrt{3}$

(20) $\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin 359^\circ$

if $A + B = 360^\circ$

then $\sin A + \sin B = 0$

$(\sin 1^\circ + \sin 359^\circ) + (\sin 2^\circ + \sin 358^\circ)$

$+ (\sin 3^\circ + \sin 357^\circ) + \cdots$

$+ \sin 179^\circ + \sin 181^\circ + \sin 180^\circ$

$= 0 + 0 \cdots + 0 = 0$

(21) $2\sec 2\alpha = \tan B + \cot B$

$\dfrac{2}{\cos 2\alpha} = \dfrac{\sin B}{\cos B} + \dfrac{\cos B}{\sin B}$

$\dfrac{2}{\cos 2\alpha} = \dfrac{\sin^2 B + \cos^2 B}{\sin B \cos B}$

$\dfrac{2}{\cos 2\alpha} = \dfrac{2}{2\sin B \cos B}$

$\dfrac{2}{\cos 2\alpha} = \dfrac{2}{\sin 2B}$

$\sin 2\alpha = \cos 2B = \sin\left(\frac{\pi}{2} - 2B\right)$

$2\alpha = \frac{\pi}{2} - 2B$

$2\alpha + 2B = \pi/2$

$\boxed{\alpha + B = \frac{\pi}{4}}$

(22) $\tan 70^\circ = ?$

$\tan 70^\circ = \tan(50 + 20^\circ)$

$\tan 70^\circ = \dfrac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \tan 20^\circ}$

$\tan 70^\circ - (\tan 70^\circ \tan 20^\circ)\tan 50^\circ = \tan 50^\circ + \tan 20^\circ$

$\tan 70^\circ - \tan 50^\circ = \tan 50^\circ + \tan 20^\circ$

$\boxed{\tan 70^\circ = 2\tan 50^\circ + \tan 20^\circ}$

(23) $\sin^2 17.5^\circ + \sin^2 72.5^\circ$

$\sin^2 17.5^\circ + \cos^2(17.5^\circ) = 1$

(24) $\sin^2 3^\circ + \sin^2 6^\circ + \sin^2 9^\circ + \cdots + \sin^2 84^\circ$
$+ \sin^2 87^\circ + \sin^2 90^\circ$

$(\sin^2 3^\circ + \sin^2 87^\circ) + (\sin^2 6^\circ + \sin^2 84^\circ) + \sin^2 90^\circ$
$+ (\sin^2 9^\circ + \sin^2 81^\circ) + \cdots$

$(1 + 1 + \cdots\cdots) + 1 + \sin^2 45^\circ$

$14 + 1 + \frac{1}{2} = \frac{31}{2}$

(25) $\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$

$$\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\sin 20^\circ \cos 20^\circ}$$

$$2\times 2\times \frac{(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{2\sin 20^\circ \cos 20^\circ}$$

$$4\,\frac{\sin(\cancel{60-20^\circ})}{\cancel{\sin 40^\circ}} = 4$$

(26) $\alpha \xrightarrow{\text{root}} 25\cos^2\theta + 5\cos\theta - 12 = 0$

$25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$25\cos^2\alpha + 20\cos\alpha - 15\cos\alpha - 12 = 0$

$5\cos\alpha\,[5\cos\alpha + 4] - 3\,[5\cos\alpha + 4] = 0$

$\boxed{\cos\alpha = -4/5}$ $\quad \cos\alpha \neq \frac{3}{5}$

$\sin\alpha = +3/5$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\times\frac{3}{5}\times -\frac{4}{5} = -\frac{24}{25}$

(27) $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2$
$+ 4(\sin 6x + \cos 6x)$

Put $x = 0$

$3(0-1)^4 + 6(0+1)^2 + 4(0+1)$

$3 + 6 + 4 = 13$

(28) $8\cos 2\theta + 8\sec 2\theta = 65$

$8\left[\cos 2\theta + \frac{1}{\cos 2\theta}\right] = 65$

$\cos 2\theta + \frac{1}{\cos 2\theta} = \frac{65}{8} = 8 + \frac{1}{8}$

$\Rightarrow \cos 2\theta = 8$ or $\frac{1}{8}$

But $\cos 2\theta \neq 8$

Hence $\cos 2\theta = \frac{1}{8}$

$4\cos 4\theta = 4[2\cos^2 2\theta - 1]$

$= 4\left[2\times\left(\frac{1}{8}\right)^2 - 1\right]$

$= 4\left[2\times\frac{1}{64} - 1\right]$

$= 4\left[\frac{1}{32} - 1\right] = 4\times\frac{-31}{32} = \frac{-31}{8}$

(29) $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$5(\sec^2 x - 1 - \cos^2 x) = 2[2\cos^2 x - 1] + 9$

$5\left[\frac{1}{\cos^2 x} - 1 - \cos^2 x\right] = 4\cos^2 x - 2 + 9$

$\frac{5}{\cos^2 x} - 5 - 5\cos^2 x = 4\cos^2 x + 7$

$9\cos^2 x + \frac{5}{\cos^2 x} + 12 = 0$

$9\cos^4 x + 12\cos^2 x - 5 = 0$

$9\cos^4 x + 15\cos^2 x - 3\cos^2 x - 5 = 0$

$3\cos^2 x(3\cos^2 x + 5) - 1(3\cos^2 x + 5) = 0$

$(3\cos^2 x + 5)(3\cos^2 x - 1) = 0$

$\cos^2 x = \frac{1}{3}$

$\cos 2x = 2\cos^2 x - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1$

$= 2\times\frac{1}{9} - 1 = -\frac{7}{9}$

(30) $x + \frac{1}{x} = 2\cos\alpha$

$x = e^{i\alpha} = \cos\alpha + i\sin\alpha$

$\frac{1}{x} = e^{-i\alpha} = \cos\alpha - i\sin\alpha$

$x^n + \frac{1}{x^n} = (e^{i\alpha})^n + (e^{-i\alpha})^n$

$= e^{in\alpha} + e^{-in\alpha}$

$= \cos n\alpha + \cancel{i\sin n\alpha}$
$+ \cos n\alpha - \cancel{i\sin n\alpha}$

$= 2\cos n\alpha$

(31)

$\sin 2\theta + \sin 2\phi = 1/2$ — ①

$\cos 2\theta + \cos 2\phi = 3/2$ — ②

$①^2 + ②^2$

$\sin^2 2\theta + \cos^2 2\theta + \sin^2 2\phi + \cos^2 2\phi$

$+ 2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = \frac{1}{4} + \frac{9}{4}$

$1 + 1 + 2[\cos 2(\theta - \phi)] = \frac{10}{4} = \frac{5}{2}$

$2[1 + \cos 2(\theta - \phi)] = \frac{5}{2}$

$\cancel{1} + 2\cos^2(\theta - \phi) - \cancel{1} = \frac{5}{4}$

$\boxed{\cos^2(\theta - \phi) = \frac{5}{8}}$

(32)

$\cos 2\alpha = \frac{3\cos 2B - 1}{3 - \cos 2B}$

$\frac{\cos 2\alpha + 1}{\cos 2\alpha - 1} = \frac{3\cos 2B - 1 + 3 - \cos 2B}{3\cos 2B - 1 - 3 + \cos 2B}$

$\frac{2\cos^2\alpha - \cancel{1} + \cancel{1}}{\cancel{1} - 2\sin^2\alpha - \cancel{1}} = \frac{2\cos 2B + 2}{4\cos 2B - 4}$

$-\cot^2\alpha = \frac{2(\cos 2B + 1)}{4(\cos 2B - 1)}$

$-\cot^2\alpha = \frac{1}{2}\left[\frac{2\cos^2 B - \cancel{1} + \cancel{1}}{\cancel{1} - 2\sin^2 B - \cancel{1}}\right]$

$\cancel{-}\cot^2\alpha = \cancel{-}\frac{1}{2}\cot^2 B$

$\cot^2 B = 2\cot^2\alpha$

$\cot B = \sqrt{2}\cot\alpha$

$\frac{1}{\cot\alpha} = \frac{\sqrt{2}}{\cot B}$

$\boxed{\tan\alpha = \sqrt{2}\tan B}$

(33) $2\sin A\cos 3A = 2\sin 3A\cos A$

$2\sin A\cos A(\cos^2 A - \sin^2 A)$

$\frac{1}{2}(2\sin 2A\cos 2A) = \frac{1}{2}\sin 4A$

(34)

$\frac{\cot x - \tan x}{\cot 2x}$

$\frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos 2x}{\sin 2x}} = \frac{\cos^2 x - \sin^2 x}{\sin x\cos x} \times \frac{\sin 2x}{\cos 2x}$

$= \frac{2\sin x\cos x}{\sin x\cos x} = 2$

(35) $\tan x = \frac{3}{4} = \frac{P}{B}$ $H = 5$

$\pi < x < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$x \in 2^{nd}$ Quadrant

$\cos x = -\frac{4}{5}$

$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}}$

$= -\sqrt{\frac{1+(-4/5)}{2}}$

$= -\sqrt{\frac{1}{10}}$

(36)

$\operatorname{cosec}\theta = \frac{p+q}{p-q}$

$\frac{1}{\sin\theta} = \frac{p+q}{p-q}$

$\frac{1+\sin\theta}{1-\sin\theta} = \frac{p+q+p-q}{p+q-(p-q)}$

$\frac{\sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}{\sin^2\theta/2 + \cos^2\theta/2 - 2\sin\theta/2\cos\theta/2} = \frac{p}{q}$

$\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)^2 = \frac{p}{q}$

$\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2} = \sqrt{\frac{p}{q}}$

$\frac{1 + \tan\theta/2}{1 - \tan\theta/2} = \sqrt{\frac{p}{q}}$

$\tan\left(\frac{\pi}{4} + \theta/2\right) = \sqrt{\frac{p}{q}}$

$\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sqrt{\frac{q}{p}}$

(37) $90 < A < 180^\circ$ $\quad 45 < \frac{A}{2} < 90^\circ$

$\sin A = \frac{4}{5}$

$\cos A = -3/5 \quad \tan A = -\frac{4}{3}$

$\tan A = \frac{2\tan A/2}{1-\tan^2 A/2}$

$-\frac{4}{3} = \frac{2t}{1-t^2}$ Let $\tan A/2 = t$

$-4 + 4t^2 = 6t$

$4t^2 - 6t - 4 = 0$

$2t^2 - 3t - 2 = 0$

$2t^2 - 4t + t - 2 = 0$

$2t(t-2) + 1(t-2) = 0$

$t = 2 \quad t = -1/2$

$\boxed{\tan A/2 = 2}$ $\quad \tan A/2 \neq -\frac{1}{2}$ $\because 45 < \frac{A}{2} < 90^\circ$

(38) $0 < \theta < 90^\circ$

$0 < \theta/2 < 45^\circ$

$\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}} = \frac{\sqrt{x-1}}{\sqrt{2x}} = \frac{P}{H}$

$B = \sqrt{H^2 - P^2} = \sqrt{2x-(x-1)}$

$B = \sqrt{x+1}$

$\tan\frac{\theta}{2} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$

$\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2} = \frac{2 \times \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 - \frac{x-1}{x+1}}$

$= \frac{\cancel{2}\sqrt{x-1}}{\sqrt{x+1}} \times \frac{x+1}{(\cancel{x+1-x+1})}$

$= \sqrt{x-1}\,\sqrt{x+1}$

$\boxed{\tan\theta = \sqrt{x^2-1}}$

(39) $\tan(\frac{\pi}{4} + \theta/2) + \tan(\frac{\pi}{4} - \theta/2)$

$\frac{\sin(\frac{\pi}{4}+\theta/2)}{\cos(\frac{\pi}{4}+\theta/2)} + \frac{\sin(\pi/4-\theta/2)}{\cos(\frac{\pi}{4}-\theta/2)}$

$\frac{\sin(\frac{\pi}{4}+\theta/2)\cos(\frac{\pi}{4}-\theta/2) + \sin(\frac{\pi}{4}-\theta/2)\cos(\frac{\pi}{4}+\frac{\theta}{2})}{\cos(\frac{\pi}{4}+\theta/2)\cos(\frac{\pi}{4}-\theta/2)}$

$\frac{\sin(\frac{\pi}{4}+\theta/2+\frac{\pi}{4}-\theta/2)}{\cos^2\frac{\pi}{4} - \sin^2\theta/2}$

$\frac{\sin\pi/2}{(\frac{1}{\sqrt{2}})^2 - \sin^2\theta/2} = \frac{2}{1-2\sin^2\theta/2}$

$= \frac{2}{\cos\theta} = 2\sec\theta$

(40) $\cos\theta = \frac{1}{2}(a + \frac{1}{a})$

$a + \frac{1}{a} = 2\cos\theta$

$(a+\frac{1}{a})^3 = (2\cos\theta)^3$

$a^3 + \frac{1}{a^3} + 3 \times \cancel{a} \times \frac{1}{\cancel{a}}(a + \frac{1}{a}) = 8\cos^3\theta$

$a^3 + \frac{1}{a^3} + 3 \times 2\cos\theta = 8\cos^3\theta$

$a^3 + \frac{1}{a^3} = 8\cos^3\theta - 6\cos\theta$

$= 2[4\cos^3\theta - 3\cos\theta]$

$a^3 + \frac{1}{a^3} = 2 \times \cos 3\theta$

$\frac{1}{2}(a^3 + \frac{1}{a^3}) = \cos 3\theta$

(41) $\cos^3 110^\circ + \cos^3 10^\circ + \cos^3 130^\circ$

Let $a = \cos 110^\circ$ $b = \cos 10^\circ$ $c = \cos 130^\circ$

$a+b+c = \cos 110^\circ + \cos 10^\circ + \cos 130^\circ$

$= 2\cos 60^\circ \cos 50^\circ + \cos(180^\circ - 50^\circ)$

$= 2\cos 60^\circ \cos 50^\circ - \cos 50^\circ$

$= 2 \times \frac{1}{2} \times \cos 50^\circ - \cos 50^\circ = 0$

If $a+b+c = 0$ then $a^3+b^3+c^3 = 3abc$

$\cos^3 110^\circ + \cos^3 10^\circ + \cos^3 130^\circ$

$= 3\cos 110^\circ \cos 10^\circ \cos^3 130^\circ$

$3\cos(180^\circ-70^\circ)\cos 10^\circ \cos(180^\circ-50^\circ)$

$3\cos 10^\circ \cos 50^\circ \cos 70^\circ$

$3\cos 10^\circ \cos(60-10^\circ)\cos(60+10^\circ)$

$3\times\frac{1}{4}\cos 3\times 10^\circ = \frac{3}{4}\cos 30^\circ$

$= \frac{3}{4}\times\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$

(42) $f(\theta)=(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$

$f\left(\frac{\pi}{4}\right)=\left(1+\cot\frac{\pi}{4}-\operatorname{cosec}\frac{\pi}{4}\right)\left(1+\tan\frac{\pi}{4}+\sec\frac{\pi}{4}\right)$

$=(1+1-\sqrt{2})(1+1+\sqrt{2})$

$f\left(\frac{\pi}{4}\right)=(2-\sqrt{2})(2+\sqrt{2}) = 2$

(43) $(\tan\alpha+\tan\beta)(1-\cot\alpha\cot\beta)+(\cot\alpha+\cot\beta)(1-\tan\alpha\tan\beta)$

$(\tan\alpha+\tan\beta)\left(1-\frac{1}{\tan\alpha\tan\beta}\right)+\left[\frac{1}{\tan\alpha}+\frac{1}{\tan\beta}\right](1-\tan\alpha\tan\beta)$

$\frac{(\tan\alpha+\tan\beta)(\tan\alpha\tan\beta-1)}{\tan\alpha\tan\beta}+\frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}[1-\tan\alpha\tan\beta]$

$\left(\frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}\right)(\tan\alpha\tan\beta-1)-\frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}[\tan\alpha\tan\beta-1]$

$=0$

Hence indep of α and β both

(44) $\cot A = \tan((n-1)A)$

$\tan(n-1)A = \tan\left(\frac{\pi}{2}-A\right)$

$(n-1)A = \frac{\pi}{2}-A$

$(n-1+1)A = \pi/2$

$\boxed{A = \frac{\pi}{2n}}$

(45) $\tan A-\tan B = \frac{1}{2}$

$\cot A-\cot B=\frac{1}{3}$

$\cot(A-B) = \frac{\cot A\cot B+1}{\cot B-\cot A}$

$= \frac{\cot A\cot B+1}{(-1/3)}$

$\cot A-\cot B = \frac{1}{3}$

$\frac{1}{\tan A}-\frac{1}{\tan B}=\frac{1}{3}$

$\frac{\tan B-\tan A}{\tan A\tan B}=\frac{1}{3}$

$\frac{-\frac{1}{2}}{\tan A\tan B}=\frac{1}{3}$

$\boxed{\cot A\cot B = -\frac{2}{3}}$

$\cot(A-B) = \frac{-\frac{2}{3}+1}{-1/3} = \frac{1/3}{-1/3} = -1$

(46) $2\cos\theta+\sin\theta=1$ —— (1)

$(2\cos\theta)^2=(1-\sin\theta)^2$

$4\cos^2\theta = 1+\sin^2\theta-2\sin\theta$

$4-4\sin^2\theta = 1+\sin^2\theta-2\sin\theta$

$5\sin^2\theta-2\sin\theta-3=0$

$5\sin^2\theta-5\sin\theta+3\sin\theta-3=0$

$5\sin\theta(\sin\theta-1)+3(\sin\theta-1)=0$

$\boxed{\sin\theta=1}$ $\sin\theta=-3/5$

Put $\sin\theta=-3/5$ in (1)

$2\cos\theta = 1+3/5$ $\cos\theta=4/5$

$\sin\theta = -3/5 \quad \cos\theta = 4/5$

$7\cos\theta + 6\sin\theta = 7\times\frac{4}{5} + 6\times\frac{-3}{5}$

$= \frac{28-18}{5} = 2$

(47) $\tan x + \sec x = 2\cos x$

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$1+\sin x = 2\cos^2 x$

$1+\sin x = 2(1-\sin^2 x)$

$1+\sin x = 2-2\sin^2 x$

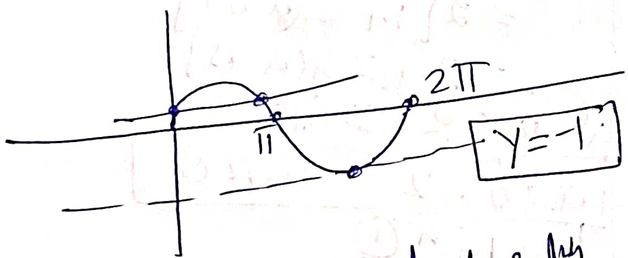
$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1 = 0$

$2\sin x(\sin x+1) - 1(\sin x+1) = 0$

$(\sin x+1)(2\sin x-1) = 0$

$\sin x = -1 \quad \sin x = 1/2$



3 sol^n → But 1 sol^n neglected $\sin x = -1$ ie $x = 3\pi/2$

for $x = 3\pi/2$ eq^n is not defined

so final Ans: 2 sol^n

(48) $\sin x + \cos x = \frac{1}{3}$

$\sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{9}$

$1+\sin 2x = \frac{1}{9}$

$\sin 2x = \frac{1}{9} - 1 = -\frac{8}{9}$

$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

$-\frac{8}{9} = \frac{2t}{1+t^2}$

$-4-4t^2 = 9t$

$4t^2+9t+4 = 0$

$t = \frac{-9 \pm \sqrt{81-64}}{8}$

$t = \frac{-9 \pm \sqrt{17}}{8}$

$\boxed{t = \frac{-9+\sqrt{17}}{8}}$

(49) $f(\theta) = (1+\cos\theta)(1+\cos 3\theta)(1+\cos 5\theta)(1+\cos 7\theta)$

$f(\frac{\pi}{8}) = (1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$

$\boxed{\because \cos\theta = \cos(-\theta)}$

$= (1+\cos\frac{\pi}{8})(1+\cos\frac{7\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})$

$= (1+\cos\frac{\pi}{8})(1+\cos(\pi-\frac{\pi}{8}))[1+\cos(\frac{\pi}{2}-\frac{\pi}{8})][1+\cos(\frac{\pi}{2}+\frac{\pi}{8})]$

$= (1+\cos\frac{\pi}{8})(1-\cos\frac{\pi}{8})(1+\sin\frac{\pi}{8})(1-\sin\frac{\pi}{8})$

$= (1-\cos^2\frac{\pi}{8})(1-\sin^2\frac{\pi}{8})$

$= \frac{1}{4}(4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8})$

$= \frac{1}{4}(2\sin\frac{\pi}{8}\cos\frac{\pi}{8})^2$

$= \frac{1}{4}\times(\sin\frac{\pi}{4})^2$

$= \frac{1}{4}\times(\frac{1}{\sqrt{2}})^2 = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

(50)

$\sin^2\alpha + \cos^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$

$\sin^2\alpha + \cos^2(\alpha+\beta) + [\cos(\alpha-\beta) - \cos(\alpha+\beta)]\cos(\alpha+\beta)$

$\sin^2\alpha + \cancel{\cos^2(\alpha+\beta)} + \cos(\alpha-\beta)\cos(\alpha+\beta) - \cancel{\cos^2(\alpha+\beta)}$

$\sin^2\alpha + (\cos^2\alpha - \sin^2\beta)$

$= 1 - \sin^2\beta = \cos^2\beta$

Hence expression is indep of α.

(51) $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$

$(\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ)$

$2\cos\frac{12+132^\circ}{2}\cos\frac{12^\circ-132^\circ}{2} + 2\cos\frac{84+156^\circ}{2}\cos\left(\frac{84-156^\circ}{2}\right)$

$2\cos 72^\circ \cos 60^\circ + 2\cos 120^\circ \cos 36^\circ$

$\cancel{2}\sin 18^\circ \times \frac{1}{\cancel{2}} + 2\cos(90+30^\circ)\cos 36^\circ$

$\sin 18^\circ + 2\times(-\sin 30^\circ)\times\cos 36^\circ$

$\sin 18^\circ - \cancel{2}\times\frac{1}{\cancel{2}}\times\cos 36^\circ$

$\frac{\sqrt{5}-1}{4} - \left[\frac{\sqrt{5}+1}{4}\right] = -\frac{1}{2}$

(52) $\cos\theta = \cos\alpha\cos\beta$

$\frac{\cos\theta}{\cos\alpha} = \frac{\cos\beta}{1}$

$\frac{\cos\theta+\cos\alpha}{\cos\theta-\cos\alpha} = \frac{\cos\beta+1}{\cos\beta-1}$

$\frac{\cancel{2}\cos\frac{\theta+\alpha}{2}\cos\left(\frac{\theta-\alpha}{2}\right)}{-\cancel{2}\sin\frac{\theta+\alpha}{2}\sin\left(\frac{\theta-\alpha}{2}\right)} = \frac{\cancel{2}\cos^2\beta/2 \cancel{-1+1}}{\cancel{1}-\cancel{2}\sin^2\beta/2 - 1}$

$\cancel{-}\cot\left(\frac{\theta+\alpha}{2}\right)\cot\left(\frac{\theta-\alpha}{2}\right) = \cancel{-}\cot^2\beta/2$

$\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right) = \tan^2\beta/2$

(53) $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$

$\alpha - \beta = 2\theta$

$\frac{\cos 3\theta}{\cos\theta} = \frac{4\cos^3\theta - 3\cos\theta}{\cos\theta}$

$= 4\cos^2\theta - 3$

$= 2(2\cos^2\theta) - 3$

$= 2(1+\cos 2\theta) - 3$

$\frac{\cos 3\theta}{\cos\theta} = 2\cos 2\theta - 1$ — (1)

$a^2+b^2 = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$= \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$

$= 2 + 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

$= 2 + 2[\cos\alpha\cos\beta + \sin\alpha\sin\beta]$

$= 2[1 + \cos(\alpha-\beta)]$

$= 2 + 2\cos(\alpha-\beta)$

$a^2+b^2 = 2 + 2\cos 2\theta$

$\boxed{a^2+b^2-2 = 2\cos 2\theta}$

↳ put in (1)

then $\frac{\cos 3\theta}{\cos\theta} = a^2+b^2-2-1$

$= a^2+b^2-3$

(54) $\tan(\pi\tan x) = \cot(\pi\tan x)$

$\tan(\pi\tan x) = \tan\left(\frac{\pi}{2} - \pi\tan x\right)$

$\pi\tan x = \frac{\pi}{2} - \pi\tan x$

$\pi\tan x + \pi\cot x = \pi/2$

$\boxed{\tan x + \cot x = 1/2}$

$\frac{1}{\sin x\cos x} = \frac{1}{2}$

$\boxed{\sin x\cos x = 2}$

↳ which is not possible

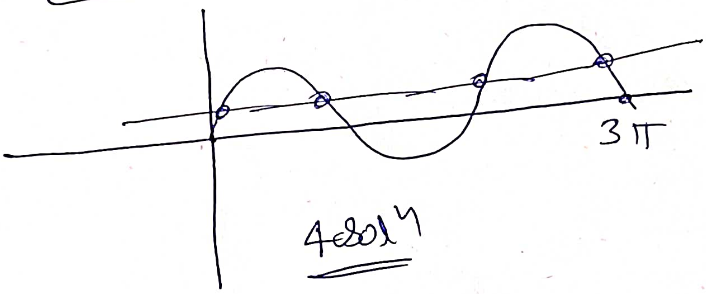
(55) $2\sin^2 x + 5\sin x - 3 = 0$

$2\sin^2 x + 6\sin x - \sin x - 3 = 0$

$2\sin x(\sin x + 3) - 1(\sin x + 3) = 0$

$(\sin x + 3)(2\sin x - 1) = 0$

$\boxed{\sin x = -3}$ $\boxed{\sin x = 1/2}$



4 sol^n

(56) $\cos A = \frac{3}{4}$ $32 \sin A/2 \sin 5A/2$

$= 16(2\sin A/2 \sin 5A/2)$

$= 16\left[\cos\left(\frac{A}{2} - \frac{5A}{2}\right) - \cos\left(A/2 + \frac{5A}{2}\right)\right]$

$= 16[\cos(2A) - \cos(3A)]$

$= 16[2\cos^2 A - 1 - [4\cos^3 A - 3\cos A]]$

$= 16\left[2 \times \frac{9}{16} - 1 - \left[4 \times \frac{27}{64} - 3 \times \frac{3}{4}\right]\right]$

$= 16\left[\frac{9}{8} - 1 - \left[\frac{27}{16} - \frac{9}{4}\right]\right]$

$= 16\left[\frac{1}{8} - \left(\frac{27-36}{16}\right)\right]$

$= 16\left[\frac{2+9}{16}\right] = 11$

(57) $\sin x + \cos y = a$, $\cos x + \sin y = b$

$\frac{\sin x + \cos y}{\cos x + \sin y} = \frac{a}{b}$

$\frac{\sin x + \cos y + \cos x + \sin y}{\sin x + \cos y - \cos x - \sin y} = \frac{a+b}{a-b}$

$\frac{(\sin x + \sin y) + (\cos y + \cos x)}{(\sin x - \sin y) + (\cos y - \cos x)} = \frac{a+b}{a-b}$

$\frac{2\sin\frac{x+y}{2} \cdot \cos\frac{x-y}{2} + 2\cos\frac{x+y}{2} \cos\frac{x-y}{2}}{2\cos\frac{x+y}{2} \sin\frac{x-y}{2} + 2\sin\frac{x+y}{2} \sin\frac{x-y}{2}} = \frac{a+b}{a-b}$

$\frac{2\cos\frac{x-y}{2}\left[\sin\frac{x+y}{2} + \cos\frac{x+y}{2}\right]}{2\sin\frac{x-y}{2}\left[\cos\frac{x+y}{2} + \sin\frac{x+y}{2}\right]} = \frac{a+b}{a-b}$

$\cot\left(\frac{x-y}{2}\right) = \frac{a+b}{a-b}$

$\tan\left(\frac{x-y}{2}\right) = \frac{a-b}{a+b}$

(58) $2\tan\alpha + \cot\beta = \tan\beta$

$\tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta \tan\alpha}$

$= \frac{2\tan\alpha + \cot\beta - \tan\alpha}{1 + \tan\alpha \tan\beta}$

$= \frac{\tan\alpha + \cot\beta}{1 + \tan\alpha \tan\beta}$

$= \frac{\tan\alpha + \frac{1}{\tan\beta}}{1 + \tan\alpha \tan\beta}$

$= \frac{\cancel{(1 + \tan\alpha \tan\beta)}}{\tan\beta \cancel{(1 + \tan\alpha \tan\beta)}}$

$= \cot\beta$.