Trigonometry DPP S-3 Solution

(1) $\sin\theta = \frac{12}{13}$

$\therefore 0<\theta<\frac{\pi}{2} \Rightarrow \cos\theta = +\sqrt{1-\frac{144}{169}} = \frac{5}{13}$

$\cos\phi = -3/5$

$\because \phi \in \left(\pi, \frac{3\pi}{2}\right)$ then $\sin\phi = -\sqrt{1-\cos^2\phi}$

$\phi \to$ 3rd Quad

$\sin\phi = -\sqrt{1-\frac{9}{25}}$

$\sin\phi = -4/5$

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$= \frac{12}{13}\times\frac{-3}{5} + \frac{5}{13}\times\frac{-4}{5}$

$= \frac{-36-20}{65} = -\frac{56}{65}$

(2) $\sin\alpha = \frac{15}{17}$

$\because \frac{\pi}{2}<\alpha<\pi$ then $\cos\alpha = -\sqrt{1-\sin^2\alpha}$

$= -\sqrt{1-\frac{225}{289}} = \frac{-8}{17}$

$\tan\beta = \frac{12}{5} = \frac{P}{B}$

$H = 13$ $\quad \sin\beta = -\frac{12}{13}$

$\because \pi<\beta<3\pi/2$ $\quad \cos\beta = -\frac{5}{13}$

$\beta \in$ 3rd Quadrant

$\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$

$= \frac{-12}{13}\times\frac{-8}{17} + \frac{5}{13}\times\frac{15}{17}$

$= +\frac{96+75}{221} = \frac{171}{221}$

(3) $\sin\theta = 3\sin(\theta+2\alpha)$

$\frac{\sin\theta}{\sin(\theta+2\alpha)} = \frac{3}{1}$

$\frac{\sin\theta+\sin(\theta+2\alpha)}{\sin\theta-\sin(\theta+2\alpha)} = \frac{3+1}{3-1}$

$\frac{2\sin\left(\frac{\theta+\theta+2\alpha}{2}\right)\cdot\cos\left(\frac{\theta-\theta-2\alpha}{2}\right)}{2\cos\frac{\theta+\theta+2\alpha}{2}\cdot\sin\frac{\theta-(\theta+2\alpha)}{2}} = \frac{4}{2}$

$\frac{\cancel{2}\sin(\theta+\alpha)\cos\alpha}{\cancel{2}\cos(\theta+\alpha)\sin(-\alpha)} = 2$

$\frac{\tan(\theta+\alpha)\cos\alpha}{-\sin\alpha} = 2$

$\frac{\tan(\theta+\alpha)}{-\tan\alpha} = 2 \Rightarrow \tan(\theta+\alpha) = -2\tan\alpha$

$\tan(\theta+\alpha) + 2\tan\alpha = 0$

(4) $\cos 15^\circ - \sin 15^\circ$

$\sqrt{2}\left(\frac{\cos 15^\circ}{\sqrt{2}} - \frac{\sin 15^\circ}{\sqrt{2}}\right)$

$\sqrt{2}(\sin 45^\circ\cos 15^\circ - \cos 45^\circ\sin 15^\circ)$

$\sqrt{2}(\sin(45^\circ-15^\circ))$

$\sqrt{2}\sin 30^\circ$

$\sqrt{2}\times\frac{1}{2} = \frac{1}{\sqrt{2}}$

(5) $\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$

$2\cos\left(\frac{\frac{\pi}{4}+\cancel{x}+\frac{\pi}{4}-\cancel{x}}{2}\right)\cdot\cos\left[\frac{\frac{\pi}{4}+x-\left(\frac{\pi}{4}-x\right)}{2}\right]$

$2\cos\left(\frac{\pi}{4}\right)\cdot\cos(2x)$

$2\times\frac{1}{\sqrt{2}}\cos 2x = \sqrt{2}\cos 2x$

(6) $\cos\left(\frac{\alpha-\beta}{2}\right) = 2\cos\left(\frac{\alpha+\beta}{2}\right)$

$\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{2}{1}$

$\frac{\cos\left(\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)-\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{2+1}{2-1}$

$\frac{\cancel{2}\cos\alpha/2\cos\beta/2}{\cancel{2}\sin\frac{\alpha-\beta+\alpha+\beta}{2}/2\ \sin\frac{\alpha+\beta-\alpha+\beta}{2}/2} = 3$

$\boxed{\cot\alpha/2\cot\beta/2 = 3}$

$\tan\alpha/2\tan\beta/2 = \frac{1}{3}$

(7) $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$

$\frac{\cos\left(\frac{\pi}{4}+\theta\right)}{\sin\left(\frac{\pi}{4}+\theta\right)}\ \frac{\cos\left(\frac{\pi}{4}-\theta\right)}{\sin\left(\frac{\pi}{4}-\theta\right)} =$

$= \frac{\cos^2\frac{\pi}{4}-\sin^2\theta}{\sin^2\frac{\pi}{4}-\sin^2\theta} = \frac{\frac{1}{2}-\sin^2\theta}{\frac{1}{2}-\sin^2\theta} = 1$

(8) $\tan\theta_1 = K\cot\theta_2$

$$\frac{\sin\theta_1}{\cos\theta_1} = K\frac{\cos\theta_2}{\sin\theta_2}$$

$$\frac{1}{K} = \frac{\cos\theta_1\cos\theta_2}{\sin\theta_1\sin\theta_2}$$

$$\frac{1+K}{1-K} = \frac{\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2}{\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2}$$

$$\frac{1+K}{1-K} = \frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}$$

$$\boxed{\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)} = \frac{1-K}{1+K}}$$

(9) $2\sin\left(\theta+\frac{\pi}{3}\right) = \cos\left(\theta-\frac{\pi}{6}\right)$

$$2\left[\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}\right] = \cos\theta\cos\frac{\pi}{6}+\sin\theta\sin\frac{\pi}{6}$$

$$2\left[\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta\right] = \cos\theta\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\sin\theta$$

$$2\sin\theta+2\sqrt{3}\cos\theta = \sqrt{3}\cos\theta+\sin\theta$$

$$\sin\theta = -\sqrt{3}\cos\theta$$

$$\boxed{\tan\theta = -\sqrt{3}}$$

(10) $\cos^2 45^\circ - \sin^2 15^\circ$

$$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$$

$$\cos(45^\circ+15^\circ)\cos(45^\circ-15^\circ)$$

$$\cos 60^\circ \times \cos 30^\circ = \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

(11) $\cos^2\left(\frac{\pi}{6}+\theta\right) - \sin^2\left(\frac{\pi}{6}-\theta\right)$

$$\cos\left(\frac{\pi}{6}+\cancel{\theta}+\frac{\pi}{6}-\cancel{\theta}\right)\cdot\cos\left(\cancel{\frac{\pi}{6}}+\theta-\cancel{\frac{\pi}{6}}+\theta\right)$$

$$\cos(60^\circ)\cos 2\theta$$

$$\frac{1}{2}\cos 2\theta$$

(12) $\tan(A+B) = p$

$\tan(A-B) = q$

$$\tan(2A) = \tan(A+B+A-B)$$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}$$

$$= \frac{p+q}{1-pq}$$

(13) $\cos(\alpha+\beta) = \frac{3}{5}$ $\quad \sin(\alpha+\beta) = +\frac{4}{5}$

$0<\alpha<\frac{\pi}{4}$, $0<\beta<\frac{\pi}{4}$ $\Rightarrow$ $0<\alpha+\beta<\pi/2$

$\sin(\alpha-\beta) = \frac{5}{13}$ $\quad -\frac{\pi}{4}<\alpha-\beta<\frac{\pi}{4}$

then $\cos(\alpha-\beta) = \frac{12}{13}$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{4}{3}$$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{5/13}{12/13} = \frac{5}{12}$$

$$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \frac{\frac{48+15}{36}}{\frac{36-20}{36}}$$

$$\tan 2\alpha = \frac{63}{16}$$

(14) $\cos(\alpha+\beta) = \frac{4}{5}$, $0<\alpha<\frac{\pi}{4}$, $0<\beta<\pi/4$

$0<\alpha+\beta<\pi/4$

$\sin(\alpha+\beta) = 3/5$

$-\frac{\pi}{4}<\alpha-\beta<\pi/4 \Rightarrow$

$\sin(\alpha-\beta) = \frac{5}{13}$ $\quad \cos(\alpha-\beta) = \frac{12}{13}$

Since $\sin(\alpha-\beta) = +ve$ then $0<\alpha-\beta<\pi/4$

$\tan(\alpha-\beta) = \frac{5}{12}$ $\quad \tan(\alpha+\beta) = 3/4$

$$\tan(\alpha+\beta+\alpha-\beta) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{5}{12}+3/4}{1-5/12\times\frac{3}{4}} = \frac{20+36/48}{33/48} = 56/33$$

(15) $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$

$= \dfrac{\cos 9^\circ\left[1+\dfrac{\sin 9^\circ}{\cos 9^\circ}\right]}{\cos 9^\circ\left[1-\dfrac{\sin 9^\circ}{\cos 9^\circ}\right]}$

$= \dfrac{\tan 45^\circ + \tan 9^\circ}{1-\tan 45^\circ \tan 9^\circ}$

$= \tan(45^\circ + 9^\circ) = \tan 54^\circ$

(16) if $A-B=\pi/4$

$(1+\tan A)(1-nB)$

$1-\tan B + nA - nA\, nB$ — (1)

$A-B=\pi/4$

$\tan(A-B)=\tan\frac{\pi}{4}$

$\dfrac{nA-nB}{1+nA\,nB}=1$

$nA-nB = 1+nA\,nB$ — (2)

put $nA-nB = 1+nA\,nB$ in (1)

$1+1+\cancel{nA\,nB}-\cancel{nA\,nB}=2$

(17) $\sin\left(\frac{31\pi}{3}\right)$

$\sin\left(10\pi+\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(18) $x=5\pi/6$

$2\sin 3\times\frac{5\pi}{6} + 3\cos 3\times\frac{5\pi}{6}$

$2\sin\left(\frac{5\pi}{2}\right)+3\cos\left(\frac{5\pi}{2}\right)$

$2\sin\left(2\pi+\frac{\pi}{2}\right)+3\cos\left(2\pi+\frac{\pi}{2}\right)$

$2\sin\pi/2 + 3\cos\pi/2$

$= 2\times1+3\times0 = 2$

(19) $\tan 75^\circ - \cot 75^\circ$

$\tan 75^\circ = \tan(90-15^\circ) = \cot 15^\circ$

$\cot(75^\circ) = \cot(90-15^\circ) = \tan 15^\circ$

$\cot 15^\circ - \tan 15^\circ$

$= 2+\sqrt{3} - (2-\sqrt{3})$

$= 2\sqrt{3}$

(20) $\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin 359^\circ$

if $A+B=360^\circ$

then $\sin A + \sin B = 0$

$(\sin 1^\circ + \sin 359^\circ) + (\sin 2^\circ + \sin 358^\circ)$

$+ (\sin 3^\circ + \sin 357^\circ) + \cdots$

$+ \sin 179^\circ + \sin 181^\circ + \sin 180^\circ$

$= 0+0 \cdots +0 = 0$

(21) $2\sec 2\alpha = \tan B + \cot B$

$\dfrac{2}{\cos 2\alpha} = \dfrac{\sin B}{\cos B} + \dfrac{\cos B}{\sin B}$

$\dfrac{2}{\cos 2\alpha} = \dfrac{\sin^2 B + \cos^2 B}{\sin B \cos B}$

$\dfrac{2}{\cos 2\alpha} = \dfrac{2}{2\sin B\cos B}$

$\dfrac{2}{\cos 2\alpha} = \dfrac{2}{\sin 2B}$

$\sin 2\alpha = \cos 2B = \sin\left(\frac{\pi}{2}-2B\right)$

$2\alpha = \frac{\pi}{2} - 2B$

$2\alpha + 2B = \pi/2$

$\boxed{\alpha + B = \frac{\pi}{4}}$

(22) $\tan 70^\circ = ?$

$\tan 70^\circ = \tan(50+20^\circ)$

$\tan 70^\circ = \dfrac{\tan 50^\circ + \tan 20^\circ}{1-\tan 50^\circ \tan 20^\circ}$

$\tan 70^\circ - (\tan 70^\circ \tan 20^\circ)\tan 50^\circ = \tan 50^\circ + \tan 20^\circ$

$\tan 70^\circ - \tan 50^\circ = \tan 50^\circ + \tan 20^\circ$

$\boxed{\tan 70^\circ = 2\tan 50^\circ + \tan 20^\circ}$

(23) $\sin^2 17.5^\circ + \sin^2 72.5^\circ$

$\sin^2 17.5^\circ + \cos^2(17.5^\circ) = 1$

(24)

$\sin^2 3^\circ + \sin^2 6^\circ + \sin^2 9^\circ + \dots + \sin^2 84^\circ + \sin^2 87^\circ + \sin^2 90^\circ$

$(\sin^2 3^\circ + \sin^2 87^\circ) + (\sin^2 6^\circ + \sin^2 84^\circ) + \sin^2 90^\circ + (\sin^2 9^\circ + \sin^2 81^\circ) + \dots$

$(\underbrace{1+1+ \dots}) + 1 + \sin^2 45^\circ$

$14 + 1 + \frac{1}{2} = \frac{31}{2}$

(25) $\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$

$\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\sin 20^\circ \cos 20^\circ}$

$\frac{2 \times 2 \times (\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{2\sin 20^\circ \cos 20^\circ}$

$4 \frac{\sin(60 - 20^\circ)}{\sin 40^\circ} = 4$

(26) $\alpha \xrightarrow{\text{root}} 25\cos^2\theta + 5\cos\theta - 12 = 0$

$25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$25\cos^2\alpha + 20\cos\alpha - 15\cos\alpha - 12 = 0$

$5\cos\alpha[5\cos\alpha + 4) - 3[5\cos\alpha + 4] = 0$

$\boxed{\cos\alpha = -4/5}$ $\quad \cos\alpha \neq \frac{3}{5}$

$\sin\alpha = +3/5$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \times \frac{3}{5} \times \frac{-4}{5} = \frac{-24}{25}$

(27) $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

Put $x = 0$

$3(0-1)^4 + 6(0+1)^2 + 4(0+1)$

$3 + 6 + 4 = 13$

(28) $8\cos 2\theta + 8\sec 2\theta = 65$

$8\left[\cos 2\theta + \frac{1}{\cos 2\theta}\right] = 65$

$\cos 2\theta + \frac{1}{\cos 2\theta} = \frac{65}{8} = 8 + \frac{1}{8}$

$\Rightarrow \cos 2\theta = 8$ or $\frac{1}{8}$

But $\cos 2\theta \neq 8$

Hence $\cos 2\theta = \frac{1}{8}$

$4\cos 4\theta = 4[2\cos^2 2\theta - 1]$

$= 4\left[2 \times \left(\frac{1}{8}\right)^2 - 1\right]$

$= 4\left[2 \times \frac{1}{64} - 1\right]$

$= 4\left[\frac{1}{32} - 1\right] = 4 \times \frac{-31}{32} = \frac{-31}{8}$

(29) $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$5(\sec^2 x - 1 - \cos^2 x) = 2[2\cos^2 x - 1] + 9$

$5\left[\frac{1}{\cos^2 x} - 1 - \cos^2 x\right] = 4\cos^2 x - 2 + 9$

$\frac{5}{\cos^2 x} - 5 - 5\cos^2 x = 4\cos^2 x + 7$

$9\cos^2 x + \frac{5}{\cos^2 x} + 12 = 0$

$9\cos^4 x + 12\cos^2 x - 5 = 0$

$9\cos^4 x + 15\cos^2 x - 3\cos^2 x - 5 = 0$

$3\cos^2 x(3\cos^2 x + 5) - 1(3\cos^2 x + 5) = 0$

$3\cos^2 x(3\cos^2 x + 5)(3\cos^2 x - 1) = 0$

$(3\cos^2 x + 5)$

$\cos^2 x = \frac{1}{3}$

$\cos 2x = 2\cos^2 x - 1 = \frac{2}{3} - 1 = \frac{-1}{3}$

$\cos 4x = 2\cos^2 2x - 1$

$= 2 \times \frac{1}{9} - 1 = \frac{-7}{9}$

(30) $x + \frac{1}{x} = 2\cos\alpha$

$x = e^{i\alpha} = \cos\alpha + i\sin\alpha$

$\frac{1}{x} = e^{-i\alpha} = \cos\alpha - i\sin\alpha$

$x^n + \frac{1}{x^n} = (e^{i\alpha})^n + (e^{-i\alpha})^n$

$= e^{in\alpha} + e^{-in\alpha}$

$= \cos n\alpha + \cancel{i\sin n\alpha} + \cos n\alpha - \cancel{i\sin n\alpha}$

$= 2\cos n\alpha$

(31)

$\sin 2\theta + \sin 2\phi = 1/2$ —①

$\cos 2\theta + \cos 2\phi = 3/2$ —②

$①^2 + ②^2$

$\sin^2 2\theta + \cos^2 2\theta + \sin^2 2\phi + \cos^2 2\phi$

$+ 2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = \frac{1}{4} + \frac{9}{4}$

$1 + 1 + 2[\cos 2(\theta - \phi)] = \frac{10}{4} = \frac{5}{2}$

$2[1 + \cos 2(\theta - \phi)] = \frac{5}{2}$

$\cancel{1} + 2\cos^2(\theta - \phi) - \cancel{1} = \frac{5}{4}$

$\boxed{\cos^2(\theta - \phi) = \frac{5}{8}}$

(32) $\cos 2\alpha = \frac{3\cos 2B - 1}{3 - \cos 2B}$

$\frac{\cos 2\alpha + 1}{\cos 2\alpha - 1} = \frac{3\cos 2B - 1 + 3 - \cos 2B}{3\cos 2B - 1 - 3 + \cos 2B}$

$\frac{2\cos^2\alpha - \cancel{1} + \cancel{1}}{\cancel{1} - 2\sin^2\alpha - \cancel{1}} = \frac{2\cos 2B + 2}{4\cos 2B - 4}$

$-\cot^2\alpha = \frac{2(\cos 2B + 1)}{4(\cos 2B - 1)}$

$-\cot^2\alpha = \frac{1}{2}\left[\frac{2\cos^2 B - \cancel{1} + \cancel{1}}{\cancel{1} - 2\sin^2 B - \cancel{1}}\right]$

$\cancel{-}\cot^2\alpha = \cancel{-}\frac{1}{2}\cot^2 B$

$\cot^2 B = 2\cot^2\alpha$

$\cot B = \sqrt{2}\cot\alpha$

$\frac{1}{\cot\alpha} = \frac{\sqrt{2}}{\cot B}$

$\boxed{\tan\alpha = \sqrt{2}\tan B}$

(33) $2\sin A\cos^3 A - 2\sin^3 A\cos A$

$2\sin A\cos A(\cos^2 A - \sin^2 A)$

$\frac{1}{2}(2\sin 2A\cos 2A) = \frac{1}{2}\sin 4A$

(34)

$\frac{\cot x - \tan x}{\cot 2x}$

$\frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos 2x}{\sin 2x}} = \frac{\cancel{\cos^2 x - \sin^2 x}}{\sin x\cos x} \times \frac{\sin 2x}{\cancel{\cos 2x}}$

$= \frac{2\cancel{\sin x\cos x}}{\cancel{\sin x\cos x}} = 2$

(35) $\tan x = \frac{3}{4} = \frac{P}{B}$ $H = 5$ $\pi < x < \frac{3\pi}{2}$

$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}}$ $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$= -\sqrt{\frac{1+(-4/5)}{2}}$ $x \in$ 2nd Quadrant

$\cos x = -\frac{4}{5}$

$= -\sqrt{\frac{1}{10}}$

(36) $\operatorname{cosec}\theta = \frac{p+q}{p-q}$

$\frac{1}{\sin\theta} = \frac{p+q}{p-q}$

$\frac{1+\sin\theta}{1-\sin\theta} = \frac{p+q+p-q}{p+q-(p-q)}$

$\frac{\sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}{\sin^2\theta/2 + \cos^2\theta/2 - 2\sin\theta/2\cos\theta/2} = \frac{p}{q}$

$\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)^2 = \frac{p}{q}$

$\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2} = \sqrt{\frac{p}{q}}$

$\frac{1+\tan\theta/2}{1-\tan\theta/2} = \sqrt{\frac{p}{q}}$

$\tan\left(\frac{\pi}{4} + \theta/2\right) = \sqrt{\frac{p}{q}}$

$\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sqrt{\frac{q}{p}}$

(37) $90 < A < 180^\circ$ $\quad 45 < \frac{A}{2} < 90^\circ$

$\sin A = \frac{4}{5}$

$\cos A = -3/5 \quad \tan A = -\frac{4}{3}$

$\tan A = \frac{2\tan A/2}{1-\tan^2 A/2}$

$-\frac{4}{3} = \frac{2t}{1-t^2}$ let $\tan A/2 = t$

$-4 + 4t^2 = 6t$

$4t^2 - 6t - 4 = 0$

$2t^2 - 3t - 2 = 0$

$2t^2 - 4t + t - 2 = 0$

$2t(t-2) + 1(t-2) = 0$

$t = 2 \quad t = -1/2$

$\boxed{\tan A/2 = 2}$ $\quad \tan A/2 \neq -\frac{1}{2}$

$\because 45 < \frac{A}{2} < 90^\circ$

(38) $0 < \theta < 90^\circ$

$0 < \theta/2 < 45^\circ$

$\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}} = \frac{\sqrt{x-1}}{\sqrt{2x}} = \frac{P}{H}$

$B = \sqrt{H^2 - P^2} = \sqrt{2x - (x-1)}$

$B = \sqrt{x+1}$

$\tan\frac{\theta}{2} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$

$\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2} = \frac{2 \times \frac{\sqrt{x-1}}{\sqrt{x+1}}}{1 - \frac{x-1}{x+1}}$

$= \frac{\cancel{2}\sqrt{x-1}}{\sqrt{x+1}} \times \frac{x+1}{\cancel{(x+1-x+1)}}$

$= \sqrt{x-1}\,\sqrt{x+1}$

$\boxed{\tan\theta = \sqrt{x^2-1}}$

(39) $\tan(\frac{\pi}{4} + \theta/2) + \tan(\frac{\pi}{4} - \theta/2)$

$\frac{\sin(\frac{\pi}{4}+\theta/2)}{\cos(\frac{\pi}{4}+\theta/2)} + \frac{\sin(\pi/4-\theta/2)}{\cos(\frac{\pi}{4}-\theta/2)}$

$\frac{\sin(\frac{\pi}{4}+\theta/2)\cos(\frac{\pi}{4}-\theta/2) + \sin(\frac{\pi}{4}-\theta/2)\cos(\frac{\pi}{4}+\frac{\theta}{2})}{\cos(\frac{\pi}{4}+\theta/2)\cos(\frac{\pi}{4}-\theta/2)}$

$\frac{\sin(\frac{\pi}{4}+\theta/2+\frac{\pi}{4}-\theta/2)}{\cos^2\frac{\pi}{4} - \sin^2\theta/2}$

$\frac{\sin\pi/2}{(\frac{1}{\sqrt{2}})^2 - \sin^2\theta/2} = \frac{2}{1-2\sin^2\theta/2}$

$= \frac{2}{\cos\theta} = 2\sec\theta$

(40) $\cos\theta = \frac{1}{2}(a + \frac{1}{a})$

$a + \frac{1}{a} = 2\cos\theta$

$(a+\frac{1}{a})^3 = (2\cos\theta)^3$

$a^3 + \frac{1}{a^3} + 3 \times \cancel{a} \times \frac{1}{\cancel{a}}(a + \frac{1}{a}) = 8\cos^3\theta$

$a^3 + \frac{1}{a^3} + 3 \times 2\cos\theta = 8\cos^3\theta$

$a^3 + \frac{1}{a^3} = 8\cos^3\theta - 6\cos\theta$

$= 2[4\cos^3\theta - 3\cos\theta]$

$a^3 + \frac{1}{a^3} = 2 \times \cos 3\theta$

$\frac{1}{2}(a^3 + \frac{1}{a^3}) = \cos 3\theta$

(41)