Quadratic DPPS-2

① $x = \sqrt{1+\sqrt{1+\sqrt{1+\cdots\infty}}}$

$x = \sqrt{1+x}$

$x^2 = 1+x$

$x^2 - x - 1 = 0$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$x \neq \frac{-1-\sqrt{5}}{2}$ · $x = \frac{-1+\sqrt{5}}{2}$ ✓

② $5\cos A + 3 = 0$

$\cos A = -\frac{3}{5}$ $\quad 90 < A < 180^\circ$

$B = 3, H = 5, P = 4$

$\sin A = \frac{4}{5}$ $\quad \tan A = -\frac{4}{3}$

$\sin A, \tan A$

$x^2 - [\sin A + \tan A]x + \sin A \tan A = 0$

$x^2 - \left[\frac{4}{5} - \frac{4}{3}\right]x + \frac{4}{5} \times -\frac{4}{3} = 0$

$x^2 - \left(\frac{12-20}{15}\right)x - \frac{16}{15} = 0$

$\boxed{15x^2 + 8x - 16 = 0}$

③ $p, q, r \rightarrow AP$

$2q = p + r$

QE $\quad px^2 + qx + r = 0$

$q^2 - 4pr \geq 0$ (Roots are real)

$\left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$

$\frac{p^2 + r^2 + 2pr}{4} - 4pr \geq 0$

$p^2 + r^2 + 2pr - 16pr \geq 0$

$r^2 - 14pr + p^2 \geq 0$

$\frac{r^2}{p^2} - \frac{14pr}{p^2} + \frac{p^2}{p^2} \geq 0$

$\left(\frac{r}{p}\right)^2 - 2 \cdot 7 \times \frac{r}{p} + 49 - 49 + 1 \geq 0$

$\left(\frac{r}{p}\right)^2 - 2 \cdot 7 \cdot \frac{r}{p} + 49 \geq 48$

$\sqrt{\left(\frac{r}{p} - 7\right)^2} \geq \sqrt{48}$

$\boxed{\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}}$

⑤ $ax^2 + bx + c = 0$

$f(2) = 4a + 2b + c = 0$

then 2 is root of $ax^2 + bx + c = 0$

Since Imaginary roots are always occurs in pair

Hence 2nd root is also a real root

Hence Real roots

④ $\alpha, \beta \xrightarrow{\text{roots}} x^2 + px + 1 = 0$

$\alpha^2 + p\alpha + 1 = 0 \quad \alpha + \beta = -p$

$\beta^2 + p\beta + 1 = 0 \quad \alpha\beta = 1$

$\gamma, \delta \xrightarrow{\text{roots}} x^2 + qx + 1 = 0$

$\gamma^2 + q\gamma + 1 = 0 \quad \gamma + \delta = -q$

$\delta^2 + q\delta + 1 = 0 \quad \gamma\delta = 1$

$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$(\alpha\beta - \alpha\gamma - \beta\gamma + \gamma^2)(\alpha\beta + \alpha\delta + \beta\delta + \delta^2)$

$(1 - \gamma(-p) + \gamma^2)(1 + \delta \times (-p) + \delta^2)$

$(1 + p\gamma + \gamma^2)(1 - p\delta + \delta^2)$

$(-q\gamma + p\gamma)(-q\delta - p\delta)$

$\gamma(p - q) \times -\delta(p + q)$

$-\gamma\delta(p^2 - q^2)$

$-1(p^2 - q^2) = q^2 - p^2$

(6) $x = 2 + 2^{1/3} + 2^{2/3}$

$x - 2 = 2^{1/3} + 2^{2/3}$

$(x-2)^3 = (2^{1/3} + 2^{2/3})^3$

$x^3 - 8 - 6x(x-2) = 2 + 2^2 + 3 \times 2^{1/3} 2^{2/3} (2^{1/3} + 2^{2/3})$

$x^3 - 8 - 6x^2 + 12x = 2 + 4 + 6 \times (x-2)$

$x^3 - 8 - 6x^2 + 12x = 6 + 6x - 12$

$\boxed{x^3 - 6x^2 + 6x = 2}$

(7) $x^2 - 2x + \sin^2\alpha = 0$

$x^2 - 2x + 1 + \sin^2\alpha = 0 + 1$

$(x-1)^2 = 1 - \sin^2\alpha = \cos^2\alpha$

$(x-1)^2 = \cos^2\alpha$

$x - 1 = \pm \cos\alpha$

$x = 1 \pm \cos\alpha$

$-1 \le \cos\alpha \le 1$

$-1+1 \le 1 + \cos\alpha \le 1 + 1$

$0 \le 1 + \cos\alpha \le 2$

$\boxed{x \in [0, 2]}$

(8) $\alpha, \beta \xrightarrow{\text{roots}} x^2 + x + 1 = 0$, $\alpha + \beta = -1$, $\alpha\beta = 1$

$x^2 - [\alpha^2 + \beta^2 + \alpha^{-2} + \beta^{-2}]x + (\alpha^2 + \beta^2)(\alpha^{-2} + \beta^{-2}) = 0$

$x^2 - \left[\alpha^2 + \beta^2 + \frac{1}{\alpha^2} + \frac{1}{\beta^2}\right]x + (\alpha^2 + \beta^2)\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right) = 0$

$x^2 - \left[(\alpha+\beta)^2 - 2\alpha\beta + \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}\right]x + (\alpha^2 + \beta^2)\frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = 0$

$x^2 - \left[(-1)^2 - 2 \times (1) + \frac{(-1)^2 - 2}{(1)}\right] + \frac{((-1)^2 - 2)((-1)^2 - 2)}{(1)} = 0$

$x^2 - [-1 - 1]x + (-1) \times (-1) = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

(9) $b^2 = 4(ac + 5d^2) \Rightarrow b^2 - 4ac = 20d^2$

$ax^2 + bx + c = 0$

$D = b^2 - 4ac = 20d^2$

$D = 20d^2 > 0$ But $20d^2 = D \to$ is not a perfect square

Hence irrational roots

(10) $\sin c, \cos c \xrightarrow{\text{roots}} 2x^2 - px + 1 = 0$

$\sin c + \cos c = \frac{p}{2}$ $\sin c \cos c = \frac{1}{2}$

$(\sin c + \cos c)^2 = \frac{p^2}{4}$

$1 + 2 \sin c \cos c = \frac{p^2}{4}$

$1 + \cancel{2} \times \frac{1}{\cancel{2}} = \frac{p^2}{4}$

$p^2 = 8$

$p = \pm 2\sqrt{2}$ $\boxed{p \neq -2\sqrt{2}}$

$\boxed{p = 2\sqrt{2}}$ one value of p

(11) $4x^2 + 6px + 1 = 0$

roots are equal

$D = 0$

$b^2 = 4ac$

$(6p)^2 = 4 \times 4 \times (1)$

$36p^2 = 16$

$p^2 = \frac{16}{36} = \frac{4}{9}$

$\boxed{p = \frac{2}{3}}$

(12) $\alpha = 1 + \sqrt{3}$ coeff are rational

then $\beta = 1 - \sqrt{3}$

QE $x^2 - [\alpha + \beta]x + \alpha\beta = 0$

$x^2 - [1 + \sqrt{3} + 1 - \sqrt{3}]x + (1 + \sqrt{3})(1 - \sqrt{3}) = 0$

$x^2 - 2x + (1 - 3) = 0$

$\boxed{x^2 - 2x - 2 = 0}$

(13) $\alpha = \frac{1}{2+\sqrt5} \times \frac{2-\sqrt5}{2-\sqrt5}$

$\alpha = \frac{2-\sqrt5}{(2)^2-5} = \frac{2-\sqrt5}{-1} = \sqrt5 - 2$

then $\beta = -2-\sqrt5$

QE $x^2 - [\alpha+\beta]x + \alpha\beta = 0$

$x^2 - [-2+\cancel{\sqrt5} + (-2-\cancel{\sqrt5})]x + (-2+\sqrt5)(-2-\sqrt5) = 0$

$x^2 - (-4)x + (-2)^2 - (5) = 0$

$\boxed{x^2 + 4x - 1 = 0}$

(14) $2x^2 + (k+1)x + 8 = 0$

$D = 0$ (Equal roots)

$b^2 = 4ac$

$(k+1)^2 = 4\times2\times8$

$k^2+1+2k = 64$

$k^2+2k-63=0$

$k^2+9k-7k-63=0$

$k(k+9)-7(k+9)=0$

$\boxed{k=-9}$ $\boxed{k=7}$

(15) $x^2+5x+k=0 \rightarrow$ Imaginary roots

$\boxed{D<0}$

$(5)^2 - 4k < 0$

$25 < 4k$

$k > \frac{25}{4}$

$k > 6.25$

$\boxed{k=7}$ (min)

(16) let $\alpha = 2+i\sqrt3$ $\beta = 2-i\sqrt3$

$2+i\cancel{\sqrt3} + 2 - i\cancel{\sqrt3} = -p$

$\boxed{p=-4}$

$(2+i\sqrt3)(2-i\sqrt3) = \frac{q}{1}$

$2^2 + (\sqrt3)^2 = \frac{q}{1}$

$\boxed{q=7}$

$(p,q) = (-4,7)$

(17) $ix^2 - 4x - 4i = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4\times i \times -4i}}{2\times i}$

$x = \frac{4 \pm \sqrt{16+16i^2}}{2i}$

$x = \frac{4\pm 0}{2i} = \frac{4}{2i} = \frac{2\times i}{i\times i}$

$x = \frac{2i}{i^2} = \frac{2i}{-1} = -2i$

(18) if $a+b+c=0$,

$4ax^2+3bx+2c=0$

$D = (3b)^2 - 4\times4a\times2c$

$= 9b^2 - 32ac$

$= 9\times(a+c)^2 - 32ac$

$= 9(a^2+c^2+2ac) - 32ac$

$= 9a^2+9c^2+18ac-32ac$

$= 9a^2+9c^2-14ac$

$= 7a^2+7c^2-14ac+2a^2+2c^2$

$= 7(a^2+c^2-2ac)+2(a^2+c^2)$

$= 7(a-c)^2+2(a^2+c^2) \geq 0$

real roots

(19) $ax^2+b=0$

roots are real and distinct

$D>0$

$(0)^2-4\times a\times b>0 \Rightarrow -4ab>0$

$\boxed{ab<0}$

(20) $a(b-c)x^2+b(c-a)x+c(a-b)=0$

Roots are equal then $\boxed{D=0}$

$a(b-c)+b(c-a)+c(a-b)=0$

then roots are $1, \frac{c(a-b)}{a(b-c)}$

given roots are equal.

$1=\frac{c(a-b)}{a(b-c)}$

$ab-ac=ac-bc$

$ab+bc=2ac$

$\boxed{b=\frac{2ac}{a+c}}$

$a,b,c \rightarrow HP$

(21) $4x^2+px+9=0$

roots are equal then $D=0$

$(p)^2-4\times4\times9=0$

$p^2-144=0$

$\sqrt{p^2}=\sqrt{144}$

$\boxed{|p|=12}$

(22) $x^2+px+q=0$

$\because 1, p, q \in R$

if $\alpha=3+4i$ then $\beta=3-4i$

$3+\cancel{4i}+3-\cancel{4i}=-p$

$\boxed{p=-6}$

$(3+4i)(3-4i)=q$ (p,q)

$3^2+4^2=q=25$ $(-6,25)$

(23) $(a+b-2c)x^2-(2a-b-c)x+(a-2b+c)=0$

$a+b-2c-(2a-b-c)+a-2b+c$

$=\cancel{a+b-2c-2a+b+c+a-2b+c}$

$=0$

then roots are $1, \frac{a-2b+c}{a+b-2c}$

(24) $2ax^2+(2a+b)x+b=0, a\neq0$

$D=(2a+b)^2-4\times2a\times b$

$D=4a^2+b^2+4ab-8ab$

$=4a^2+b^2-4ab$

$D=(2a-b)^2 \rightarrow$ ~~should be~~ perfect square

perfect square (पूर्ण वर्ग)

then roots are rational

(मूल परिमेय)

(25) $kx^2+1=kx+3x-11x^2$

$(k+11)x^2-(k+3)x+1=0$

roots are real and Equal

(मूल बराबर और वास्तविक)

$D=0$

$(k+3)^2-4\times(k+11)=0$

$k^2+9+6k-4k-44=0$

$k^2+2k-35=0$

$k^2+7k-5k-35=0$

$k(k+7)-5(k+7)=0$

$(k+7)(k-5)=0$

$k=-7$ or $k=5$

(26) $x^2-2(1+3K)x+7(3+2K)=0$

$\boxed{D=0}$

$(2(1+3K))^2-4\times7\times(3+2K)=0$

$4(1+9K^2+6K)-4\times7\times(3+2K)=0$

$4(1+9K^2+6K-21-14K)=0$

$9K^2-8K-20=0$

$9K^2-18K+10K-20=0$

$9K[K-2]+10[K-2]=0$

$(K-2)(9K+10)=0$

$K=2 \quad K=-\frac{10}{9}$

(27) $ix^2-2(1+i)x+2-i$

Let $\alpha=2-i$

$\alpha+\beta=-\frac{(-2(1+i))}{i}$

$2-i+\beta=\frac{2(1+i)}{i}$

$2-i+\beta=\frac{2(1+i)}{i\times i}\times i$

$2-i+\beta=\frac{2(i+i^2)}{i^2}$

$2-i+\beta=\frac{2(-1+i)}{(-1)}$

$2-i+\beta=-2(-1+i)$

$\cancel{2}-i+\beta=\cancel{2}-2i$

$\beta=-2i+i=-i$