KHAN GLOBAL STUDIES

K.G.S. campus, Near Sai Mandir, Mussalahpur Hatt, Patna – 6
Mob. : 8877918018, 8757354880

Defence (Mathematics)

By : Prashant Sir

Quadratic Equation short Note

Polynomials

An expression of the form

$a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ $n \in$ whole No is called a polynomial of degree n where $a_n \neq 0$, $a_0, a_1, a_2 \ldots a_n \in R$

Quadratic equation

$ax^2 + bx + c = 0,\ a \neq 0$

Roots of an equation

Values of the variable for which an equation is satisfied are called the roots of the equation

Identity

If $f(x) = 0$ is a satisfied by every value of x in the domain of $f(x)$ then it is called an identity.

Ex $(a + b)^2 = a^2 + b^2 + 2ab$

$ax^2 + bx + c = 0$, $a \neq 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$D = b^2 - 4ac$

$D \rightarrow$Discriminant

Nature of roots

(1) $D > 0 \rightarrow$ roots are real and distinct

(2) $D = 0$, real and Equal

(3) $D < 0 \Rightarrow$ Imaginary roots

(4) If $a, b, c \in$ Real No, If one root is $p + iq$ then other root must be $p - iq$

(5) If $a, b, c \in$ rational No, if one root is $p + \sqrt{q}$ then other root must be $p - \sqrt{q}$

Note

1. If α and β are roots of $ax^2 + bx + c = 0$ then
$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

2. If α, β are the roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

3. Equation whose roots are α and β is given by $x^2 - [\alpha + \beta]x + \alpha\beta = 0$

4. For real roots $D \geq 0$

5. If $a, b, c \in Q$ and D is a perfect square then roots are rational.

6. If $a = 1$ and $b, c \in I$ and roots are rational numbers then roots must be integers.

7. If $a + b + c = 0$ and one root of $ax^2 + bx + c = 0$ is 1 and other root $\frac{c}{a}$

8. $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

9. $ax^2 + bx + c = 0$, $a \neq 0$, $\alpha, \beta \xrightarrow{roots} ax^2 + bx + c = 0$. $a \neq 0$, $|\alpha - \beta| =$ difference of roots $= \frac{\sqrt{D}}{|a|}$

10. Let α, β, γ are the roots of cubic equation $ax^3 + bx^2 + cx + d = 0$

$\alpha + \beta + \gamma = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\alpha\beta\gamma = -\frac{d}{a}$

11. Let $\alpha, \beta, \gamma, \delta$ are the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$

$\alpha + \beta + \gamma + \delta = \frac{-b}{a}$

$\alpha\beta\gamma\delta = \frac{e}{a}$

$\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta = \frac{-d}{a}$

$\alpha\beta + \beta\gamma + \gamma\delta + \delta\alpha + \alpha\gamma + \beta\delta = \frac{c}{a}$

12. If the degree of a polynomial equation is odd then numbers of real roots will also be odd. If follows that atleast one of the roots will be real.

13. If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$ then $ax^3 + bx^2 + cx + d = a(x - \alpha)(x - \beta)(x - \gamma)$

14. If $\alpha, \beta, \gamma, \delta$ one the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$

then $ax^4 + bx^3 + cx^2 + dx + e = a(x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$

15. If one root is k times the other root, QE $ax^2 + bx + c = 0$
Condition $\frac{(k+1)^2}{k} = \frac{b^2}{ac}$

16. If ratio of roots of the equation $ax^2 + bx + c = 0$ be $p : q$ then
$(p+q)^2 ac = pqb^2$

17. If $a > 0$ and $x = \sqrt{a + \sqrt{a + \sqrt{a + \ldots\infty}}}$
then $x = \frac{1+\sqrt{4a+1}}{2}$

18. Remainder theorem :
If $f(x)$ is divided by $x - k$ then remainder is $f(k)$

19. If D_1 is Discriminant of $ax^2 + bx + c = 0$ and D_2 is Discriminant of $a_1x^2 + b_1x + c_1 = 0$

(a) If $D_1 + D_2 \geq 0 \Rightarrow$ atleast one of D_1 and $D_2 \geq 0$

$\Rightarrow$ atleast one of the equation has real roots.

(b) If $D_1 + D_2 < 0$ then atleast one of the equation has imaginary roots

<u>Transformation of Quadratic Equations</u>

Let α, β be the roots of equation $ax^2 + bx + c = 0$, $a \neq 0$, then the equation whose roots are

(1) $\frac{1}{\alpha}, \frac{1}{\beta}$

Let $x = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{x}$

Replace x by $\frac{1}{x}$ we get

$a + bx + cx^2 = 0$

(2) $\alpha + k, \beta + k$

Replace x by $x - k$

(3) $\alpha - k, \beta - k$

Replace x by $x + k$

(4) roots are $\alpha k, \beta k$

Replace x by $\frac{x}{k}$

(5) $\frac{\alpha}{k}, \frac{\beta}{k}$

Replace x by xk

Range of Quadratic Expression
$Y = ax^2 + bx + c$
If $a > 0$ Range , $\left[\frac{-D}{4a}, \infty\right)$
min value $= \frac{-D}{4a}$
If Range , $\left(-\infty, \frac{-D}{4a}\right]$
max value $= \frac{-D}{4a}$

Range in restricted Domain
$y = ax^2 + bx + c$ if $x \epsilon [x_1, x_2]$
$= f(x)$

(i) If $\frac{-b}{2a} \notin [x_1, x_2]$ then
Range
$= [\min\{f(x_1), f(x_2)\}, \max\{f(x_1), f(x_2)\}]$

(ii) If $\frac{-b}{2a} \epsilon [x_1, x_2]$ then
Range =
$[\min \{f(x_1), f(x_2), f\left(\frac{-b}{2a}\right)\}, \max \{f(x_1), f(x_2), f\left(\frac{-b}{2a}\right)\}]$

Note

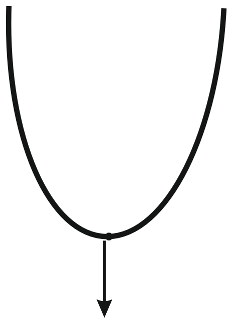
- If $D < 0$, then
if $a > 0 \Rightarrow ax^2 + bx + c > 0\ \forall x$
if $a < 0 \Rightarrow ax^2 + bx + c < 0\ \forall x$
- If $f(x) \geq 0\ \forall\ x \epsilon R$ then $a > 0$ and $D \leq 0$
- If $f(x) \leq 0\ \forall\ x \epsilon R$ then $a < 0$ and $D \leq 0$

Condition for common roots
$a_1x^2 + b_1x + c_1 = 0 \rightarrow QE(1)$
$a_2x^2 + b_2x + c_2 = 0 \rightarrow QE(2)$

- <u>Rules for finding common root</u>

Making coefficient of x^2 same and then subtracting the two equations we get common root.

- If both roots are common then
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If D is $-ve$ of any one of the QE then both roots are common.
- If $p + \sqrt{q}$ is one of the common root then both roots are common.
-



Vertex$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

- An equation of the form
  $(x-)a(x-b)(x-c)(x-d) = Ax^2$
  Where $ab = cd$
  Divided by x^2 in both sides
  then put $y = x + \frac{ab}{x}$
- An equation of th form
  $(x-a)^4 + (x-b)^4 = A$
  Put $y = \frac{x-a+x-b}{2}$
  then solve
- $|f(x) + g(x)| = |f(x)| + |g(x)|$
  Passible if $f(x)g(x) \geq 0$
- $y = ax^3 + bx^2 + cx + d$

a<0

a<0

- $y = ax^4 + bx^3 + cx^2 + dx + e$

If a>0

If a<0

Note

1. If both roots are $+ve$

   (a) $D \geq 0$ (b) Sum > 0

   (c) $p > 0$

2. If both roots are $-ve$

   (a) $D \geq 0$ (b) Sum < 0

   (c) $p > 0$

3. Roots are of opposite signs

   (a) $D > 0$ (b) $p < 0$

4. Quadratic equation ax^2+bx+c=0 roots are reciprocal to each other than $a = c$
5. Roots are equal in magnitude but opposite in sign

   (a) $D > 0$ (b) $S = 0$

   (c) $p < 0$

6. Attleast one root of equation is positive then either one root is positive or both roots are positive
7. If atleast one roots of equation is negative then either one root is negative or both roots are negative.

<u>Location of roots</u>

1. If k lies between the roots of f(x)=0 then

   (a) $D > 0$ (b) $af(k) < 0$

2. If both roots are less than k

   (a) $D \geq 0$ (b) $af(k) > 0$

   (c) $\frac{-b}{2a} < k$

3. If both the roots are greater than k

   (a) $D \geq 0$ (b) $af(k) > 0$

   (c) $k < \frac{-b}{2a}$

4. If exactly one root lies in the interval (k_1, k_2)

   (a) $D > 0$ (b) $f(k_1)f(k_2) < 0$

5. Both the roots lying in the interval (k_1, k_2)

   (a) $D \geq 0$

   (b) $af(k_1) > 0$ and $af(k_2) > 0$

   (c) $k_1 < \frac{-b}{2a} < k_2$

6. If one root is less than k_1 and other root is more than k_2
   $\Rightarrow k_1$ and k_2 lies b/w the roots $af(k_1) < 0$ and $af(k_2) < 0$
7. If k_1 and k_2 lies b/w the roots

   (a) $D > 0$ (b) $af(k_1) < 0$

   (c) $af(k_2) < 0$