

Chapter 02

Sets and Relations



JEE-FLASHBACK



JEE MAINS QUESTION

Q.1 Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is -

[AIEEE-2004]

- (1) transitive (2) not symmetric
(3) reflexive (4) a function

Q.2 Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$, be relation on the set $A = \{3, 6, 9, 12\}$. The relation is - [AIEEE-2005]

- (1) reflexive and transitive only
(2) reflexive only
(3) an equivalence relation
(4) reflexive and symmetric only

Q.3 Let W denote the words in the English dictionary. Define the relation R by : $R = \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ have at least one letter in common}\}$. Then R is - [AIEEE 2006]

- (1) reflexive, symmetric and not transitive
(2) reflexive, symmetric and transitive
(3) reflexive, not symmetric and transitive
(4) not reflexive, symmetric and transitive

Q.4 Let R be the real line. Consider the following subsets of the plane $R \times R$: [AIEEE 2008]

$$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$$

$$T = \{(x, y) : x - y \text{ is an integer}\}.$$

Which one of the following is true?

- (1) Both S and T are equivalence relations on R
(2) S is an equivalence relation on R but T is not
(3) T is an equivalence relation on R but S is not
(4) Neither S nor T is an equivalence relation on R

Q.5 If A , B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then [AIEEE-2009]

- (1) $A = C$
(2) $B = C$
(3) $A \cap B = \phi$
(4) $A = B$

Q.6 Consider the following relations :

$R : \{(x, y) \mid x, y \text{ are real numbers and } x = wy \text{ for some rational number } w\}$

$$S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \mid m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn \right\}$$

Then [AIEEE-2010]

- (1) neither R nor S is an equivalence relation
(2) S is an equivalence relation but R is not an equivalence relation
(3) R and S both are equivalence relations
(4) R is an equivalence relation but S is not an equivalence relation

Q.7 Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty, is [AIEEE-2012]

- (1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

Q.8 If $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$ and $Y = \{9(n - 1) : n \in \mathbb{N}\}$, where \mathbb{N} is the set of natural numbers, then $X \cup Y$ is equal to [JEE(Main) 2014]

- (1) X (2) Y (3) \mathbb{N} (4) $Y - X$

Q.9 If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$, and $S = \{x \in \mathbb{R} : f(x) = f(-x)\}$; then S : [JEE(Main)2016]

- (1) contains exactly one element

- (2) contains exactly two elements.
 (3) contains more than two elements.
 (4) is an empty set.

Q.10 Two sets A and B are as under : $A = \{(a, b) \in \mathbb{R} \times \mathbb{R} : |a-5| < 1 \text{ and } |b-5| < 1\}$;
 $B = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$.
 Then;

[JEE(Main) 2018]

- (1) $A \cap B = \phi$ (an empty set)
 (2) Neither $A \subset B$ nor $B \subset A$
 (3) $B \subset A$
 (4) $A \subset B$

Q.11 Let $S = \{x \in \mathbb{R} : x \geq 0 \text{ and } 2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0\}$. Then S

[JEE(Main)-2018]

- (1) contains exactly four elements
 (2) is an empty set.
 (3) contains exactly one element.
 (4) contains exactly two elements.

Q.12 In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics, course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is:

[JEE Mains Online-2019]

- (1) 42 (2) 1 (3) 102 (4) 38

Q.13 Let $S = \{1, 2, 3, \dots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is :

[JEE Mains Online-2019]

- (1) $2^{50} + 1$ (2) $2^{50} (2^{50} - 1)$
 (3) $2^{50} - 1$ (4) $2^{100} - 1$

Q.14 Let Z be the set of integers. If $A = \{x \in \mathbb{Z} : 2^{(x+2)(x^2-5x+6)} = 1\}$ $B = \{x \in \mathbb{Z} : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is:

[JEE Mains Online-2019]

- (1) 2^{10} (2) 2^{15} (3) 2^{18} (4) 2^{12}

Q.15 If $A = \{x \in \mathbb{R} : |x| < 2\}$ and $B = \{x \in \mathbb{R} : |x-2| \geq 3\}$; then :

[JEE (Main) 2020]

- (1) $A - B = [-1, 2)$ (2) $A \cap B = (-2, -1)$
 (3) $A \cup B = \mathbb{R} - (2, 5)$ (4) $B - A = \mathbb{R} - (-2, 5)$

Q.16 If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

[JEE (Main) 2020]

- (1) $\{0, 1\}$
 (2) $\{-1, 0, 1\}$
 (3) $\{-2, -1, 0, 1, 2\}$
 (4) $\{-2, -1, 1, 2\}$

Q.17 Let R_1 and R_2 be two relations defined as follows:

[JEE (Main) 2020]

$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\}$ and

$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}$, where Q is the set of all rational numbers. Then :

- (1) R_1 is transitive but R_2 is not transitive.
 (2) R_1 and R_2 are both transitive.
 (3) R_2 is transitive but R_1 is not transitive.
 (4) Neither R_1 nor R_2 is transitive.

Q.18 A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

[JEE (Main) 2020]

- (1) 55 (2) 65 (3) 29 (4) 37

Q.19 Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$, where each X_i contains 10

elements and each Y_i contains 5 elements.

If each elements of the set T is an element of exactly 20 of sets X_i 's, and exactly 6 of sets Y_i 's, then n is equal to :

[JEE (Main) 2020]

- (1) 15 (2) 50 (3) 30 (4) 45

Q.20 A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

[JEE (Main) 2020]

- (1) 36 (2) 38 (3) 63 (4) 54

MATHEMATICS

Q.21 Let $R = \{(P, Q) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$ be a relation, then the equivalence class of $(1, -1)$ is the set :

[JEE (Main) 2021]

- (1) $S = \{(x, y) \mid x^2 + y^2 = 4\}$
- (2) $S = \{(x, y) \mid x^2 + y^2 = 1\}$
- (3) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$
- (4) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Q.22 Let $A = \{2, 3, 4, 5, \dots, 30\}$ and ' \simeq ' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to :

[JEE (Main) 2021]

- (1) 5
- (2) 6
- (3) 8
- (4) 7

Q.23 Define a relation R over a class of $n \times n$ real matrices A and B as " ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$ ". Then which of the following is true?

[JEE (Main) 2021]

- (1) R is symmetric, transitive but not reflexive.
- (2) R is reflexive, symmetric but not transitive.
- (3) R is an equivalence relation
- (4) R is reflexive, transitive but not symmetric

Q.24 Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define

$B = \{T \subseteq A : \text{either } 1 \notin T \text{ or } 2 \in T\}$ and

$C = \{T \subseteq A : \text{the sum of all the elements of } T \text{ is a prime number}\}.$

Then the number of elements in the set $B \cup C$ is

[JEE (Main) 2022]

_____.

Q.25 Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$.

Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \emptyset\}$ is _____.

[JEE (Main) 2022]

Q.26 Let R_1 and R_2 be two relations defined on R by a $R_1b \Leftrightarrow ab \geq 0$ and $a R_2b \Leftrightarrow a \geq b$. Then,

[JEE (Main) 2022]

- (1) R_1 is an equivalence relation but not R_2
- (2) R_2 is an equivalence relation but not R_1
- (3) Both R_1 and R_2 are equivalence relations
- (4) Neither R_1 nor R_2 is an equivalence relation

Q.27 For $\alpha \in N$, consider a relation R on N given by $R = \{(x, y) : 3x + \alpha y \text{ is a multiple of } 7\}$. The relation R is an equivalence relation if and only if

[JEE (Main) 2022]

- (1) $\alpha = 14$
- (2) α is a multiple of 4
- (3) 4 is the remainder when α is divided by 10
- (4) 4 is the remainder when α is divided by 7

Q.28 Let $S = \left\{x \in [-6, 3] - \{-2, 2\} : \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and T

$= \{x \in Z : x^2 - 7|x| + 9 \leq 0\}.$

Then the number of elements in $S \cap T$ is

[JEE (Main) 2022]

- (1) 7
- (2) 5
- (3) 4
- (4) 3

Q.29 Let R be a relation from the set $\{1, 2, 3, \dots, 60\}$ to itself such that $R = \{(a, b) : b = pq, \text{ where } p, q \geq 3 \text{ are prime numbers}\}$. Then, the number of elements in R is :

[JEE (Main) 2022]

- (1) 600
- (2) 660
- (3) 540
- (4) 720

Q.30 The number of elements in the set

$$S = \left\{x \in R : 2 \cos \left(\frac{x^2 + x}{6} \right) = 4^x + 4^{-x} \right\}$$

[JEE (Main) 2022]

- (1) 1
- (2) 3
- (3) 0
- (4) infinite

Q.31 Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set

$$X = \{z \in C : \operatorname{Re}(az^2 + bz) = a \text{ and } \operatorname{Re}(bz^2 + az) = b\}$$

is equal to

[JEE (Main) 2023]

- (1) 0
- (2) 1
- (3) 3
- (4) 2

Q.32 Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is

[JEE (Main) 2023]

- (1) 752
- (2) 782
- (3) 792
- (4) 772

Q.33 Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that

$R = \{(x, y) \in A \times A : x - y \text{ is odd positive integer or } x - y = 2\}$. The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to _____.

[JEE (Main) 2023]

Q.34 Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is [JEE (Main) 2023]

- (1) an equivalence relation
- (2) symmetric but neither reflexive nor transitive
- (3) transitive but neither symmetric nor reflexive
- (4) reflexive but neither symmetric nor transitive

Q.35 The number of elements in the set $\{n \in \mathbb{Z} : |n^2 - 10n + 19| < 6\}$ is _____.

[JEE (Main) 2023]

Q.36 Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{(a_1, b_1), (a_2, b_2) \in (A \times B, A \times B) : a_1 \text{ divides } b_2 \text{ and } a_2 \text{ divides } b_1\}$ is [JEE (Main) 2023]

- (1) 36
- (2) 24
- (3) 18
- (4) 12

Q.37 An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events? [JEE (Main) 2023]

- (1) 15
- (2) 21
- (3) 10
- (4) 9

JEE ADVANCED QUESTION

Q.31 The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \leq 9x\}$ is [IIT-JEE 2009]

Q.32 Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to [JEE 2010]

- (1) 25
- (2) 34
- (3) 42
- (4) 41

Q.33 Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto function from Y to X , then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____.

[JEE(Advanced) 2018]

ANSWER KEY

JEE-FLASHBACK JEE-MAINS QUESTIONS

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|------|----|----|----|----|----|----|----|----|-----|-----|----|----|----|----|----|
| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Ans. | 2 | 1 | 1 | 3 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 2 | 2 | 4 |
| Que. | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Ans. | 2 | 4 | 1 | 3 | 1 | 4 | 4 | 3 | 107 | 112 | 4 | 4 | 4 | 2 | 1 |
| Que. | 31 | 32 | 33 | 34 | 35 | 36 | 37 | | | | | | | | |
| Ans. | 1 | 3 | 19 | 2 | 6 | 1 | 2 | | | | | | | | |

JEE-ADVANCED QUESTIONS

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|------|---|---|-----|
| Que. | 1 | 2 | 3 |
| Ans. | 7 | 4 | 119 |

