Chapter 04

Trigonometric Equation

JEE-FLASHBACK

JEE MAINS QUESTION

Q.1 The number of values of α in $[0,2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$ is: **[JEE(Main) 2014]**

(1) 6 (2) 4 (3) 3 (4) 1

Q.2 The number of $x \in [0,2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is

[JEE(Main) 2016]

(1) 2 (2) 6 (3) 4 (4) 8

Q.3 If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is :-

[JEE(Main) 2016]

(1) 9 (2) 3 (3) 5 (4) 7

Q.4 If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{x}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to - **[JEE (Main)-2018]**

(1) $\frac{20}{9}$ (2) $\frac{2}{3}$ (3) $\frac{13}{9}$ (4) $\frac{8}{9}$

Q.5 The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2},\pi\right)$ is : **[JEE(Main) 2018]**

(1) 3 (2) 4 (3) 2 (4) 1

Q.6 If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is:

[JEE(Main) 2013 and 2019]

(1) 3 (2) 1 (3) 4 (4) 2

Q.7 The sum of all values of $\theta \in \left(0,\frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is : **[JEE(Main) 2019]**

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

Q.8 The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\ldots.\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is

[JEE(Main) 2019]

(1) $\frac{1}{1024}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{256}$

Q.9 Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

[JEE(Main) 2019]

(1) $\frac{13\pi}{6}$ (2) $\frac{5\pi}{3}$

(3) 2π (4) π

Q.10 If [x] denotes the greatest integer $\le$ x, then the system of linear equations

$[\sin\theta]\, x + [-\cos\theta]\, y = 0$

$[\cot\theta]\, x + y = 0$ **[JEE(Main) 2019]**

(1) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi,\frac{7\pi}{6}\right)$

(2) Has a unique solution if

$\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(3) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(4) Have infinitely many solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Q.11 Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to : **[JEE (Main)-2019]**

(1) R (2) [1, 4]

(3) [3, 7] (4) [2, 6]

Q.12 The number of solutions of the equation

$1 + \sin^4 x = \cos^2 3x, x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is **[JEE (Main)-2019]**

(1) 3 (2) 5 (3) 7 (4) 4

Q.13 The number of distinct solution of the equation, $\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$ in the interval, $[0, 2\pi]$ is _____. **[JEE (Main)-2020]**

Q.14 If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$, has real solutions for θ, then λ lies in the interval : **[JEE (Main)-2020]**

(1) $\left(-\frac{5}{4}, -1\right)$ (2) $\left[-1, \frac{-1}{2}\right]$

(3) $\left(-\frac{1}{2}, -\frac{1}{4}\right)$ (4) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

Q.15 If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta, y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ then : **[JEE (Main)-2021]**

(1) xyz = 4 (2) xy – z = (x + y)z

(3) xy + yz + zx = z (4) xy + z = (x + y)z

Q.16 All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in : **[JEE (Main)-2021]**

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Q.17 If $\sqrt{3}\left(\cos^2 x\right) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ are _____. **[JEE (Main)-2021]**

Q.18 The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is _____. **[JEE (Main)-2021]**

Q.19 The number of solutions of the equation

$|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is __. **[JEE (Main)-2021]**

Q.20 If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for $\alpha \in R$, then the value of $27\sec^6\alpha + 8\,\text{cosec}^6\alpha$ is equal to : **[JEE (Main)-2021]**

(1) 350 (2) 250 (3) 400 (4) 500

Q.21 The number of solution of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to **[JEE (Main)-2021]**

(1) 11 (2) 7 (3) 5 (4) 9

Q.22 The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$ is equal to : **[JEE (Main)-2021]**

(1) 8π (2) 11π (3) 12π (4) 9π

Q.23 The sum of solutions of the equation

$\frac{\cos x}{1 + \sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is : **[JEE (Main)-2021]**

(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$ (3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

Q.24 Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to ____.

[JEE (Main)-2021]

Q.25 If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is : **[JEE (Main)-2021]**

(1) $\left(3,\frac{13\pi}{9}\right)$ (2) $\left(2,\frac{2\pi}{3}\right)$

(3) $\left(2,\frac{8\pi}{9}\right)$ (4) $\left(3,\frac{5\pi}{3}\right)$

Q.26 The number of solutions of $|\cos x| = \sin x$ such that $-4\pi \le x \le 4\pi$ is- **[JEE (Main)-2022]**

(1) 4 (2) 6 (3) 8 (4) 12

Q.27 $2\sin^2\theta - \cos 2\theta = 0$, $2\cos^2\theta + 3\sin\theta = 0$. If sum of all solutions of θ in $[0, 2\pi]$ is $k\pi$, then find k_____. **[JEE (Main)-2022]**

Q.28 Let $S = \left\{\theta\in[0,2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$.

Then n (S) + $\sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$

is equal to : **[JEE MAIN 2022]**

(1) 0 (2) –2 (3) –4 (4) 12

Q.29 Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left[-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right]$.

Then the number of elements in the set

$A=\left\{\theta\in S : \tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$

is..... **[JEE MAIN 2022]**

Q.30 Let

$S = \left\{\theta\in(0,2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\right\}$.

Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$, $\theta \in S$, is-

[JEE MAIN 2022]

Q.31 The number of elements in the set

$$S=\left\{x\in R : 2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}\right\}$$

[JEE MAIN 2022]

(1) 1 (2) 3 (3) 0 (4) infinite

Q.32 The number of elements in the set

$S=\left\{\theta\in[0,2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\right\}$

is **[JEE (Main)-2023]**

(1) 10 (2) 8 (3) 12 (4) 9

JEE ADVANCED QUESTION

Q.1 The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is- **[IIT-2002]**

(1) 4 (2) 8 (3) 10 (4) 12

Q.2 For which interval for θ, the inequation $(2\sin^2\theta - 5\sin\theta + 2) > 0$. When $0 < \theta < 2\pi$ **[IIT-2006]**

(1) $\left(\frac{13\pi}{48}, 2\pi\right)$ (2) $\left(0,\frac{\pi}{8}\right)\cup\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$

(3) $\left(\frac{\pi}{8},\frac{5\pi}{6}\right)$ (4) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},2\pi\right)$

Q.3 The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is- **[IIT-2007]**

(1) zero (2) one

(3) two (4) four

Q.4 For $0 < \theta < \frac{\pi}{2}$, the solution(s) of

$$\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$$ is(are) **[IIT-JEE - 2009]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{12}$ (4) $\frac{5\pi}{12}$

Q.5 The number of values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $\theta \ne \frac{n\pi}{5}$ for n = 0, ±1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **[IIT-JEE-2010]**

Q.6 Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy **[IIT-JEE 2012]**

(1) $0 < \phi < \frac{\pi}{2}$ (2) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$

(3) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (4) $\frac{3\pi}{2} < \phi < 2\pi$

Q.7 For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has **[JEE (Advanced) 2014]**

(1) infinitely many solutions

(2) three solutions

(3) one solution

(4) no solution

Q.8 The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is **[JEE (Advanced) 2015]**

Q.9 Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to **[JEE (Advanced) 2016]**

(1) $-\frac{7\pi}{9}$ (2) $-\frac{2\pi}{9}$ (3) 0 (4) $\frac{5\pi}{9}$

Q.10 Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}\,a\cos x + 2b\sin x = c$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____. **[JEE(Advanced) 2018]**

Answer Q.11 and Q.12 by appropriately matching the lists based on the information given in the paragraph.

Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi\sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order:

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$, $Z = \{x : g(x) = 0\}$

$W = \{x : g'(x) = 0\}$

List-I contains sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I		List-II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Q.11 Which of the following is the only correct combination? **[JEE(Advanced) 2019]**

(1) IV-(Q), (T)

(2) III-(R), (U)

(3) III-(P), (Q), (U)

(4) IV-(P), (R), (S)

Q.12 Which of the following is the only correct combination? **[JEE(Advanced) 2019]**

(1) I-(Q), (U)

(2) I-(P), (R)

(3) II-(Q), (T)

(4) II-(R), (S)

Q.13 Consider the following lists

List-I		List-II	
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos 2x = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

The correct option is: **[JEE(Advanced) 2022]**

(1) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)

(2) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)

(3) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)

(4) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

ANSWER KEY

JEE-FLASHBACK

JEE MAINS QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	4	4	3	4	4	2	3	3	1	4	2	8	2	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	4	1	11	1	2	3	4	1	56	1	3	3	3	5	16
Que.	31	32													
Ans.	1	4													

JEE ADVANCED QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	2	4	3	3,4	3	1,3,4	4	8	3	0.5	4	3	2