Trigonometry DPPS-2 Solution

(17) $1-\sin 10^\circ \sin 50^\circ \sin 70^\circ$

$1-\sin 10^\circ \sin(60-10^\circ)\sin(60+10^\circ)$

$1-\frac{1}{4}\sin 10^\circ \times 3$ $\quad \left[\because \sin A \sin(60-A)\sin(60+A)=\frac{1}{4}\sin 3A\right]$

$1-\frac{1}{4}\sin 30^\circ$

$1-\frac{1}{4}\times\frac{1}{2}=1-\frac{1}{8}=\frac{7}{8}$

(18) $\operatorname{cosec} A+\cot A=\frac{2}{3}$ —(1)

then $\operatorname{cosec} A-\cot A=\frac{3}{2}$ —(2)

(1)+(2)

$2\operatorname{cosec} A=\frac{2}{3}+\frac{3}{2}=\frac{13}{6}$

$\operatorname{cosec} A=\frac{13}{12}=\frac{H}{P}$ $\quad B=5$

$\cos A=\frac{B}{H}=\frac{5}{13}$ (a is true DPP मे Answer galat है।)

(19) $5\sin x+4\cos x=3$

let $4\sin x-5\cos x=K$

$(5\sin x+4\cos x)^2+(4\sin x-5\cos x)^2=4^2+5^2$

$9+K^2=16+25$

$K^2=32$

$\boxed{K=4\sqrt{2}}$

(20) $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$\frac{\sin A/\cos A}{1-\frac{\cos A}{\sin A}}+\frac{\cos A/\sin A}{1-\sin A/\cos A}$

$\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$

$\frac{\sin^2 A}{\cos A(\sin A-\cos A)}-\frac{\cos^2 A}{\sin A(\sin A-\cos A)}$

$\frac{1}{\sin A-\cos A}\left[\frac{\sin^2 A}{\cos A}-\frac{\cos^2 A}{\sin A}\right]$

$\frac{1}{\sin A-\cos A}\left[\frac{\sin^3 A-\cos^3 A}{\sin A\cos A}\right]$

$\frac{\cancel{(\sin A-\cos A)}(\sin^2 A+\cos^2 A+\sin A\cos A)}{\cancel{\sin A-\cos A}\,(\sin A\cos A)}$

$=\frac{1+\sin A\cos A}{\sin A\cos A}$ $\quad \sec A\operatorname{cosec} A+1$

(1) $\frac{\sin^4\alpha}{2}+\frac{\cos^4\alpha}{3}=\frac{1}{5}$

$\frac{3\sin^4\alpha+2\cos^4\alpha}{6}=\frac{1}{5}$

$\frac{15\sin^4\alpha}{\cos^4\alpha}+\frac{10\cos^4\alpha}{\cos^4\alpha}=\frac{6}{\cos^4\alpha}$

$15\tan^4\alpha+10=6\sec^4\alpha$

$15\tan^4\alpha+10=6[1+\tan^2\alpha]^2$

$15\tan^4\alpha+10=6[1+\tan^4\alpha+2\tan^2\alpha]$

$15\tan^4\alpha+10=6+6\tan^4\alpha+12\tan^2\alpha$

$9\tan^4\alpha-12\tan^2\alpha+4=0$

$(3\tan^2\alpha-2)^2=0$

$\tan^2\alpha=2/3$ $\quad \cot^2\alpha=3/2$

$27\sec^6\alpha+8\operatorname{cosec}^6\alpha$

$=27[1+\tan^2\alpha]^3+8[1+\cot^2\alpha]^3$

$=27[1+\frac{2}{3}]^3+8[1+\frac{3}{2}]^3$

$=\cancel{27}\times\frac{125}{\cancel{27}}+\cancel{8}\times\frac{125}{\cancel{8}}$

$=250$

(11) $\cos A+\cos B+\cos C=\sin A+\sin B+\sin C=0$

$(\cos A+\cos B+\cos C)^2=0$ —(1)

$(\sin A+\sin B+\sin C)^2=0$ —(2)

$(1)^2+(2)^2$

$\cos^2 A+\cos^2 B+\cos^2 C+2[\cos A\cos B+\cos B\cos C+\cos C\cos A]$

$+\sin^2 A+\sin^2 B+\sin^2 C$

$+2\sin A\sin B+2\sin B\sin C+2\sin C\sin A=0$

$=(\cos^2 A+\sin^2 A)+(\cos^2 B+\sin^2 B)+(\cos^2 C+\sin^2 C)$

$+2[\cos A\cos B+\sin A\sin B]+2[\cos B\cos C+\sin B\sin C]$

$+2[\cos C\cos A+\sin C\sin A]$

$=1+1+1+2[\cos(A-B)+\cos(B-C)+\cos(C-A)]=0$

$\cos(A-B)+\cos(B-C)+\cos(C-A)=-3/2$

(3) $\cos\theta = -\frac{3}{5}$, $\pi < \theta < 3\pi/2$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$

$\cos\theta/2 = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1-3/5}{2}}$

$\cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$ $B=1$ $H=\sqrt{5}$ $P=2$

$\sin\theta/2 = +\frac{2}{\sqrt{5}}$

$\cos\theta/2 = -\frac{1}{\sqrt{5}}$

$\tan\theta/2 = -\frac{2}{1} = -2$

$\tan\theta/2 + \sin\theta/2 + 2\cos\theta/2$

$= -2 + \cancel{\frac{2}{\sqrt{5}}} - \cancel{\frac{2}{\sqrt{5}}} = -2$

(4) $\sin 6^\circ + \sin 54^\circ + \sin 126^\circ + \cos 156^\circ$

$\sin 6^\circ + \sin(90^\circ - 36^\circ) + \sin(90^\circ + 36^\circ)$
$+ \cos(90^\circ + 66^\circ)$

$= \cos 36^\circ + \cos 36^\circ + (\sin 6^\circ - \sin 66^\circ)$

$= 2\cos 36^\circ + 2\cos\frac{6+66^\circ}{2} \cdot \sin\frac{(6-66^\circ)}{2}$

$= 2\cos 36^\circ + 2\cos 36^\circ \sin(-30^\circ)$

$= 2\cos 36^\circ [1 - \sin 30^\circ]$

$= 2\cos 36^\circ \times [1 - \frac{1}{2}]$

$= \cos 36^\circ = \frac{\sqrt{5}+1}{4}$

(5) $\cot\theta = -\frac{2}{3}$, $\theta \notin 4$th Quad.

Hence $\theta \in$ 2nd Quadrant $B=2$

$P=3$

$H=\sqrt{13}$

$\sin\theta = +\frac{3}{\sqrt{13}}$

$\cos\theta = -\frac{2}{\sqrt{13}}$ $\tan\theta = -\frac{3}{2}$

$\cot\theta = -2/3$

$\frac{\left(5\times\frac{3}{\sqrt{13}} - \frac{2}{\sqrt{13}}\right)^2}{-\frac{3}{2} - \frac{2}{3}} = \frac{(\sqrt{13})^2}{\frac{-13}{6}} = -6$

(6) $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

$81^{\sin^2 x} + 81^{1-\sin^2 x} = 30$

$81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$

Let $81^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

$t^2 - 30t + 81 = 0$

$t = 27$ $t = 3$

$81^{\sin^2 x} = 27$ $81^{\sin^2 x} = 3$

$3^{4\sin^2 x} = 3^3$ $3^{4\sin^2 x} = 3^1$

$4\sin^2 x = 3$ $4\sin^2 x = 1$

$\sin x = \frac{\sqrt{3}}{2}$ $\sin x = \frac{1}{2}$

$x = 60^\circ$ $x = 30^\circ$

(7) $\frac{1}{\sin 250^\circ} + \frac{\sqrt{3}}{\cos 290^\circ}$

$\frac{1}{\sin(270^\circ - 20^\circ)} + \frac{\sqrt{3}}{\cos(270^\circ + 20^\circ)}$

$\frac{1}{-\cos 20^\circ} + \frac{\sqrt{3}}{\sin 20^\circ}$

$\frac{-\sin 20^\circ + \sqrt{3}\cos 20^\circ}{\sin 20^\circ \cos 20^\circ}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\sin 20^\circ \cos 20^\circ}$

$= 2\times 2\frac{(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{2\sin 20^\circ \cos 20^\circ}$

$= 4\frac{\sin(60-20^\circ)}{\sin 40^\circ} = 4\frac{\cancel{\sin 40^\circ}}{\cancel{\sin 40^\circ}} = 4$

(8) $\cos^2\left(\frac{\pi}{6}+\theta\right) - \sin^2\left(\frac{\pi}{6}-\theta\right)$

$\cos^2 A - \sin^2 B = \cos(A+B)\cdot\cos(A-B)$

$\cos\left(\frac{\pi}{6} + \cancel{\theta} + \frac{\pi}{6} - \cancel{\theta}\right)\cdot\cos\left(\cancel{\frac{\pi}{6}} + \theta - \cancel{\frac{\pi}{6}} + \theta\right)$

$\cos 60^\circ \cos(2\theta)$

$\frac{1}{2}\cos 2\theta$

(9) $\cos 15^\circ - \sin 15^\circ$

$$\sqrt{2}\left(\frac{\cos 15^\circ}{\sqrt{2}} - \frac{\sin 15^\circ}{\sqrt{2}}\right)$$

$$\sqrt{2}\left(\sin 45^\circ \cos 15^\circ - \cos 45^\circ \sin 15^\circ\right)$$

$$\sqrt{2}\,\sin(45^\circ - 15^\circ)$$

$$\sqrt{2}\,\sin 30^\circ$$

$$\sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

(10) $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)\times(1-\sin\theta)}{1+\sin\theta \quad 1-\sin\theta}} \quad (\because 1-\sin\theta > 0)$$

$$= \frac{1-\sin\theta}{\sqrt{\cos^2\theta}} = \frac{1-\sin\theta}{|\cos\theta|}$$

$\frac{\pi}{2} < \theta < 3\pi/2$ then $\cos\theta < 0$

$$|\cos\theta| = -\cos\theta$$

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1-\sin\theta}{-\cos\theta} = \tan\theta - \sec\theta$$

(11) $\alpha \in [\pi \ \pi]$

$$\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha} \times \frac{1-\sin\alpha}{1-\sin\alpha}} = \sqrt{\frac{(1-\sin\alpha)^2}{\cos^2\alpha}}$$

$$= \frac{1-\sin\alpha}{|\cos\alpha|}$$

if $\cos\alpha > 0$ $\quad \frac{1-\sin\alpha}{\cos\alpha} = \sec\alpha - \tan\alpha$

then $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(12) let $\sin 2 - \sin 1 = 2\cos\frac{2+1}{2} \cdot \sin\frac{2-1}{2}$

$$= 2\cos\frac{3}{2}\,\sin\frac{1}{2} > 0$$

$1^c = 57\ldots^\circ$
$2^c = 114\ldots^\circ$
$3^c = 171\ldots^\circ$

$\sin 2 - \sin > 0$

$\sin 2 > \sin 1$ — (1)

$$\sin 3 - \sin 1 = 2\cos\frac{3+1}{2} \cdot \sin\frac{3-1}{2}$$

$$= 2\cos 2\,\sin 1 < 0$$

$\sin 3 < \sin 1$ — (2)

from (1) and (2) $\sin 3 < \sin 1 < \sin 2$

$$\frac{10\sin^4\alpha}{\cos^4\alpha} + \frac{15\cos^4\alpha}{\cos^4\alpha} = \frac{6}{\cos^4\alpha}$$

$$10\tan^4\alpha + 15 = 6\sec^4\alpha$$

$$10\tan^4\alpha + 15 = 6[1+\tan^2\alpha]^2$$

$$10\tan^4\alpha + 15 = 6[1+\tan^4\alpha+2\tan^2\alpha]$$

$$10\tan^4\alpha + 15 = 6+6\tan^4\alpha+12\tan^2\alpha$$

$$4\tan^4\alpha - 12\tan^2\alpha + 9 = 0$$

$\tan^2\alpha = t$

$$4t^2 - 12t + 9 = 0$$

$$(2t-3)^2 = 0 \quad \boxed{t = 3/2}$$

$$\tan^2\alpha = 3/2 \quad \cot^2\alpha = 2/3$$

$$27\operatorname{cosec}^6\alpha + 8\sec^6\alpha$$

$$27[1+\cot^2\alpha]^3 + 8[1+\tan^2\alpha]^3$$

$$27\left[1+\frac{2}{3}\right]^3 + 8\left[1+\frac{3}{2}\right]^3$$

$$27\times\left(\frac{5}{3}\right)^3 + 8\times\left(\frac{5}{2}\right)^3$$

$$27\times\frac{125}{27} + 8\times\frac{125}{8} = 250$$

(14) $a = \sin x\cos^3 x \quad b = \cos x\sin^3 x$

$$a-b = \sin x\cos^3 x - \cos x\sin^3 x$$

$$= \sin x\cos x(\cos^2 x - \sin^2 x)$$

$$= \frac{1}{2}(2\sin x\cos x\cos 2x)$$

$$= \frac{1}{2}\times\frac{2\times\sin 2x\cos 2x}{2}$$

$$= \frac{1}{4}(2\sin 2x\cos 2x)$$

$$= \frac{1}{4}\sin 4x$$

$\therefore \sin 4x > 0$ if $0 < 4x < \pi$

$a-b = \frac{1}{4}\sin 4x > 0$ if $0 < x < \frac{\pi}{4}$

(15) $\frac{3\pi}{4} < \theta < \pi$

$\sqrt{2\cot\theta + \frac{1}{\sin^2\theta}}$

$= \sqrt{2\cot\theta + \operatorname{cosec}^2\theta}$

$= \sqrt{2\cot\theta + 1 + \cot^2\theta} = \sqrt{(1+\cot\theta)^2}$

$= |1+\cot\theta|$

$\because 1+\cot\theta < 0$ in $\theta \in \left(\frac{3\pi}{4}, \pi\right)$

Hence $|1+\cot\theta| = -1 - \cot\theta = k - \cot\theta$

$\boxed{k = -1}$

(16) $\sin x + \sin^2 x + \sin^3 x = 1$

$\sin x + \sin^3 x = 1 - \sin^2 x = \cos^2 x$

squaring both sides

$(\sin x + \sin^3 x)^2 = (\cos^2 x)^2$

$\sin^2 x + \sin^6 x + 2\sin^4 x = \cos^4 x$

$1 - \cos^2 x + (1-\cos^2 x)^3 + 2(1-\cos^2 x)^2 = \cos^4 x$

$1 - \cos^2 x + 1 - \cos^6 x - 3\cos^2 x(1-\cos^2 x)$
$+ 2[1 + \cos^4 x - 2\cos^2 x] = \cos^4 x.$

$1 - \cos^2 x + 1 - \cos^6 x - 3\cos^2 x + 3\cos^4 x$
$+ 2 + 2\cos^4 x - 4\cos^2 x = \cos^4 x$

$-\cos^6 x + 4\cos^4 x - 8\cos^2 x + 4 = 0$

$\cos^6 x - 4\cos^4 x + 8\cos^2 x = 4$

(17) $\sin^3\alpha + \sin^3\left(\frac{2\pi}{3}+\alpha\right) + \sin^3\left(\frac{4\pi}{3}+\alpha\right)$

Let $a = \sin\alpha$ $b = \sin\left(\frac{2\pi}{3}+\alpha\right)$ $c = \sin\left(\frac{4\pi}{3}+\alpha\right)$

$a+b+c$

$= \sin\alpha + \sin(120+\alpha) + \sin(240+\alpha)$

$= \sin\alpha + 2\sin\left(\frac{120+\alpha+240+\alpha}{2}\right)\cdot\cos\left(\frac{120+\alpha-240-\alpha}{2}\right)$

$= \sin\alpha + 2\sin(180+\alpha)\cos(60^\circ)$

$= \sin\alpha - 2\sin\alpha \cdot \frac{1}{2} = 0$

$a+b+c = 0$

then $a^3+b^3+c^3 = 3abc$

$\sin^3\alpha + (\sin(120+\alpha))^3 + \sin^3(240+\alpha)$

$= 3\sin\alpha\sin(120+\alpha)\sin(240+\alpha)$

$= 3\sin\alpha\sin(180^\circ-(60-\alpha))\sin(180^\circ+(60+\alpha))$

$= 3\sin\alpha\sin(60-\alpha) \times -\sin(60+\alpha)$

$= -3\sin\alpha\sin(60-\alpha)\sin(60+\alpha)$

$= -3 \times \frac{1}{4}\sin 3\alpha$

$= -\frac{3}{4}\sin 3\alpha$ $\left[\because \sin A\sin(60-A)\sin(60+A) = \frac{1}{4}\sin 3A\right]$

(18) $\sin 2\theta = \frac{3}{4}$

$\sin^3\theta + \cos^3\theta$

$= (\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$

$(\sin\theta + \cos\theta) = \sqrt{1+\sin 2\theta}$

$= \sqrt{1+\frac{3}{4}} = \frac{\sqrt{7}}{2}$

$\frac{\sqrt{7}}{2} \times \left[1 - \frac{1}{2}2\sin\theta\cos\theta\right]$

$\frac{\sqrt{7}}{2}\left(1 - \frac{1}{2}\sin 2\theta\right)$

$\frac{\sqrt{7}}{2}\left(1 - \frac{1}{2}\times\frac{3}{4}\right)$

$\frac{\sqrt{7}}{2} \times \left(1 - \frac{3}{8}\right) = \frac{5\sqrt{7}}{16}$

(19) $\cos A + \cos B = 1$

$2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2} = 1$

$2\cos\frac{60^\circ}{2}\cos\left(\frac{A-B}{2}\right) = 1$

$2\cos 30^\circ\cos\frac{A-B}{2} = 1$

$2 \times \frac{\sqrt{3}}{2}\cos\left(\frac{A-B}{2}\right) = 1$

$\cos\left(\frac{A-B}{2}\right) = \frac{1}{\sqrt{3}}$

$\sin\frac{A-B}{2} = \sqrt{1-\cos^2\frac{A-B}{2}}$

$= \sqrt{1-\frac{1}{3}} = \sqrt{2/3}$

$$|\cos A - \cos B| = \left|2\sin\frac{A+B}{2}\cdot\sin\left(\frac{A-B}{2}\right)\right| \quad \left(\because A+B = \frac{\pi}{3} = 60^\circ\right)$$

$$= 2\sin\frac{60^\circ}{2} \times \sqrt{\frac{2}{3}}$$

$$= 2\times\sin 30^\circ \times \sqrt{2/3}$$

$$= 2\times\frac{1}{2}\times\sqrt{2/3}$$

$$= \sqrt{2/3}.$$

(20) $\sin\frac{\pi}{18}\ \sin\frac{5\pi}{18}\ \sin\frac{7\pi}{18}$

$\sin 10^\circ \sin 50^\circ \sin 70^\circ$

$\sin 10^\circ \sin(60-10^\circ)\sin(60+10^\circ)$

$\frac{1}{4}\sin 3\times 10^\circ = \frac{1}{4}\sin 30^\circ = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

(21) $\tan\theta + \tan\left(\frac{\pi}{3}+\theta\right) + \tan\left(\theta - \frac{\pi}{3}\right) = K\tan 3\theta$

$$\tan\theta + \frac{\tan 60^\circ + \tan\theta}{1-\tan 60^\circ\tan\theta} + \frac{\tan\theta - \tan 60^\circ}{1+\tan\theta\tan 60^\circ} = K\tan 3\theta$$

$$\frac{\tan\theta}{1} + \frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta} + \frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}$$

$$\frac{\tan\theta(1-3\tan^2\theta) + (\sqrt{3}+\tan\theta)(1+\sqrt{3}\tan\theta) + (\tan\theta-\sqrt{3})(1-\sqrt{3}\tan\theta)}{(1-3\tan^2\theta)}$$

$$\frac{\tan\theta - 3\tan^3\theta + \cancel{\sqrt{3}} + 3\tan\theta + \tan\theta + \cancel{\sqrt{3}\tan^2\theta} + \tan\theta - \cancel{\sqrt{3}\tan^2\theta} - \cancel{\sqrt{3}} + 3\tan^2\theta}{1-3\tan^2\theta}$$

$$= \frac{9\tan\theta - 3\tan^3\theta}{1-3\tan^2\theta} = 3\left[\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right]$$

$$= 3\tan 3\theta = K\tan 3\theta$$

$K=3$