NDA

Logarithmic D.P.P.S-1 solution

① $\log_2 32 = ?$

$\log_{2^1} 2^5 = \frac{5}{1}(\log_2 2) = 5 \times 1 = 5$

② $\log_{\sqrt{8}} x = 3\frac{1}{3}$

$\log_{\sqrt{8}} x = \frac{10}{3}$

$x = (\sqrt{8})^{10/3}$

$x = (2^3)^{\frac{1}{2} \times \frac{10}{3}}$ 5

$x = 2^5 = 32$

③ $\log_{100}(0.1)$

$\log_{10^2} 10^{-1} = -\frac{1}{2}(\log_{10} 10) = -\frac{1}{2}$

④ $\log_2(\log_3 3^4)$

$\log_2(4\underbrace{\log_3 3})$

$= \log_2(4\times1) = \log_2 4 = \log_2 2^2$

$= 2(\log_2 2)$

$= \underline{\underline{2}}$

5 $(\log_3 5)(\log_{25} 27)$

$\log_3 5 \times \log_{5^2} 3^3$

$(\cancel{\log_3 5}) \times \frac{3}{2}(\cancel{\log_5 3})$

$\boxed{\frac{3}{2}}$

⑥ $\log_{10} 2 = 0.301$

$\log_{10} 25 = \log_{10} 5^2$

$= 2\log_{10} 5$

$= 2[\log_{10}(10/2)]$

$= 2[\log_{10} 10 - \log_{10} 2]$

$= 2[1 - 0.301]$

$= 2 \times 0.699 = 1.398$

⑦ $\log_3 9 + \log_5 25 + \log_2 8$

$\log_3 3^2 + \log_5 5^2 + \log_2 2^3$

$2(\log_3 3) + 2(\log_5 5) + 3(\log_2 2)$

$2 + 2 + 3 = 7$

⑧ $\log_2(\log_5 625)$

$\log_2(\log_5 5^4)$

$\log_2(4\underbrace{\log_5 5})$

$\log_2 2^2$

$2(\log_2 2)$

$= 2 \times 1 = 2$

⑨ $\log(\tan 1^\circ) + \log(\tan 2^\circ) + \dots + \log(\tan 89^\circ)$

$\log(\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ)$

$\log(\underbrace{\tan 1^\circ \tan 89^\circ}_{1}\ \underbrace{\tan 2^\circ \tan 88^\circ}_{1} \dots)$

$\log(1) = 0$ [$\because$ if $A+B = 90^\circ$, $\tan A \tan B = 1$]

⑩ $\log_9 x - \log_9\left(\frac{x}{10} + \frac{1}{9}\right) = 1$

$\log_9 x - \log_9\left(\frac{9x+10}{90}\right) = 1$

$\log_9\left(\frac{x}{\frac{9x+10}{90}}\right) = 1$

$\frac{\overset{10}{\cancel{90}}x}{9x+10} = \cancel{9}$

$10x = 9x + 10$

$\boxed{x = 10}$

⑪ $\log_a mn = \log_a m + \log_a n$

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(12) $\log_{10} 10 + \log_{10} 10^2 + \log_{10} 10^3 + \ldots + \log_{10} 10^n$

$\log_{10} 10 + 2\log_{10} 10 + 3\log_{10} 10 + \ldots + n\log_{10} 10$

$(\log_{10} 10)\,[1+2+3+ \ldots + n]$

$\frac{n(n+1)}{2}$

(13) $\log_{10} 40000 - \log_{10} 4$

$\log_{10} \frac{40000}{4}$ $\left[\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)\right]$

$\log_{10} 10000 = \log_{10} 10^4 = \underline{4}$

(14) $\log_{10} 2x = 1$

$2x = 10^1$

$2x = 10$

$\boxed{x = 5}$

(15) $\log_8 x = \frac{2}{3}$ $\left(\because \log_a x = y \quad x = a^y\right)$

$x = 8^{2/3}$

$x = (2^{\cancel{3}})^{2/\cancel{3}}$

$\boxed{x = 4}$

(16) $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$

$\frac{9}{16} = x^{-1/2}$

$\left(\frac{9}{16}\right)^2 = \left(x^{-1/2}\right)^2$

$\frac{81}{256} = x^{-1} = \frac{1}{x}$

$= \boxed{x = \frac{256}{81}}$

(17) $\log_{10^4} x = -\frac{1}{4}$

$\frac{1}{4} \log_{10} x = -\frac{1}{4}$

$\log_{10} x = -1$

$x = 10^{-1} = \frac{1}{10} = 0.1$

(18) $\log_{32} x = 0.8$

$x = (32)^{0.8}$

$x = (2^5)^{\frac{8}{10}}$

$x = 2^{\cancel{5} \times \cancel{8}^{4} / \cancel{10}_{2}}$

$\boxed{x = 16}$

(19) $\log_x (0.1) = -\frac{1}{3}$

$(0.1) = x^{-1/3}$

$\left(\frac{1}{10}\right)^3 = \left(x^{-1/3}\right)^3$

$\frac{1}{1000} = x^{-1} = \frac{1}{x}$

$\boxed{x = 1000}$

(20) $\log_{10} (x-15) + \log_{10} 10 = 4$

$\log_{10} (x-15) + 1 = 4$

$\log_{10} (x-15) = 3$

$x-15 = 10^3 = 1000$

$\boxed{x = 1015}$

(21) $\log_4 (x^2+x) - \log_4 (x+1) = 2$

$\log_4 \frac{x^2+x}{x+1} = 2$ $(x+1>0)$

$\log_4 \frac{\cancel{(x+1)}x}{\cancel{(x+1)}} = 2$

$\log_4 x = 2$

$\boxed{x = 16}$

(22) $\log_e x + \log_e (1+x) = 0$

$\log_e x(1+x) = 0$ $\quad (\log_a m + \log_a n = \log_a mn)$

$\log_e (x^2+x) = 0$

$x^2 + x = e^0 = 1$

$\boxed{x^2 + x - 1 = 0}$

(23) $\log_8 x + \log_8 \left(\frac{1}{6}\right) = \frac{1}{3}$

$\log_8 x \times \frac{1}{6} = \frac{1}{3}$

$\frac{x}{6} = 8^{1/3}$

$\frac{x}{6} = (2^{\cancel{3}})^{1/\cancel{3}}$

$\frac{x}{6} = 2$ $\quad \boxed{x = 12}$

(24) $\log_5 3 \times \log_3 625$

$\log_5 3 \times \log_3 5^4$

$\log_5 3 \times 4 \times \log_3 5$

$4 \times \underbrace{(\log_5 3 \times \log_3 5)}_{1}$

$4 \times 1 = 4$

(25) $\log_{60} 3 + \log_{60} 4 + \log_{60} 5$

$\log_{60} 3 \times 4 \times 5 = \log_{60} (60) = 1$

(26) $\log_y x = \frac{\log_e x}{\log_e y}$

(27) $\log 360$

$= \log 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= \log 2^3 \times 3^2 \times 5$

$= \log 2^3 + \log 3^2 + \log 5$

$= 3 \log 2 + 2 \log 3 + \log 5$

(28) $\log_b a^2 \; \log_c b^2 \; \log_a c^2$

$2 \log_b a \times 2 \log_c b \times 2 \log_a c$

$8 \frac{\cancel{\log a}}{\cancel{\log b}} \times \frac{\cancel{\log b}}{\cancel{\log c}} \times \frac{\cancel{\log c}}{\cancel{\log a}}$

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