Chapter 04

Newton's Laws of Motion

CONTENT

1. INTRODUCTION

We have seen that part of physics is a study of motion, including accelerations (which is rate of change in velocity). Physics is also a study of what can cause an object to move. That cause is a force. Which is, loosely speaking, a push or pull on the object.

Sir Isaac Newton (1643-1727) developed theories of gravitation and later three laws of motion

1.1 Force & It's Classification

The concept of force gives us a quantitative description of the interaction between two bodies or between a body and its environment. As a result of everyday experiences, everybody has a basic understanding of the concept of forces.

Force: It is an frame independent parameter. It is the cause of change in motion i.e. cause of change in velocity.

- Produces or tries to produce motion in a body at rest.
- Stops or tries to stop a moving body.
- Produces a change in the shape of body.
- There are four fundamental forces in nature
  (i) Gravitational force
  (ii) Electromagnetic force
  (iii) Strong nuclear force
  (iv) Weak nuclear force

Interaction	Relative strength	Range(m)
Strong nuclear force	10^{38}	10^{-15}
Electromagnetic force	10^{36}	∞
Weak nuclear force	10^{25}	10^{-18}
Gravitational force	1	∞

1.2 Units for Measurement of Force :

Absolute units	Gravitational	Practical units
(i) N (M.K.S)	kg-wt or kg-f	(kg-force)
(ii) dyne (C.G.S)	g-wt or g-f	

- 1 kg-wt = 9.8 N
- 1 g-wt = 980 dyne
- 1 N = 10^5 dyne

1.3 Types of Forces

Based on the nature of the interaction between two bodies, forces may be broadly classified as under:

Non-contact forces (Field forces) : Forces that act between bodies separated by a distance without any actual contact.

Eg. Weight, electrostatic forces, etc.

Contact Forces : The force of interaction between two bodies generated because of their effective contact. Eg. normal reaction, frictional force, etc.

2. LAWS OF MOTION

2.1 Newton’s First Law of Motion :

- Every body continues its state of rest or uniform motion in a straight line unless compelled by an external force to change its state.
- This law defines the force and states that “force is a factor which can change the state of object.”
- Definition of force from Newton’s first law of motion:“Force is the push or pull which changes or tends tochange the state of rest or of uniform motion“.

2.2 Inertia

- It is the property of a body due to which it opposes any change in its state. Mass of a body is the measure of itsinertia of translational motion. It is difficult to change the state of rest or uniform motion of a body of heavier massand vice-versa.
- Mass of a body is quantitative or numerical measure of a body’s inertia.
- Larger the mass of a body, more will be its inertia.

2.2.1 Inertia of rest : It is the inability of a body to change its state of rest by itself.

Examples:

(i) When we shake a branch of a mango tree, the mangos fall down.

(ii) When a bus or train starts suddenly, the passengers sitting inside tends to fall backwards.

(iii) When a horse starts off suddenly, its rider falls backwards.

(iv) A coin is placed on cardboard and this cardboard is placed over a tumbler such that coin is above the mouth of tumbler. Now if the cardboard is removed with a sudden jerk, then the coin falls into the tumbler.

(v) The dust particles in a blanket fall off when it is beaten with a stick.

2.2.2 Inertia of motion: It is the inability of a body to change its state of uniform motion by itself.

Examples:

(i) When a bus or train stops suddenly, the passengers sitting inside lean forward.

(ii) A person who jumps out of a moving train may fall in the forward direction.

(iii) A bowler runs with the ball before throwing it, so that his speed of running gets added to the speed of the ball at the time of throw.

(iv) An athlete runs through a certain distance before taking a long jump because the velocity acquired during the running gets added to the velocity of athlete at the time of jump and hence he can jump over a longer distance.

(v) A ball is thrown in the upward direction by a passenger sitting inside a moving train. The ball will fall : back to the hands of the passenger, if the train is moving with constant velocity.

(vi) Ahead of the passenger, if the train is retarding (slowing down)

(vii) Behind of the passenger, if the train is accelerating (speeding up)

2.2.3 Inertia of direction: It is the inability of a body to change its direction of motion by itself.

Examples:

(i) When a straight running car turns sharply, the person sitting inside feels a force radially outwards.

(ii) Rotating wheels of vehicle throw out mud, mudguard fitted over the wheels prevent this mud for spreading.

(iii) When a knife is pressed against a grinding stone, the sparks produced move in the tangential direction.

2.3 Linear Momentum :

- The total quantity of motion possessed by a moving body is known as the momentum of the body. It is product of the mass and velocity of a body. It is a vector quantity whose direction is along the instantaneous velocity.

  momentum $= \vec{p} = m\vec{v}$

  SI Unit: kg-m/s

  Dimension: $[M\,L\,T^{-1}]$

2.4 Newton's Second Law of Motion

- The rate of change of momentum of any system is directly proportional to the net unbalanced force and this change in momentum takes place in the direction of the net unbalanced force.

  $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) \Rightarrow \vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$ (general form)

$$\boxed{\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}}$$

If m = constant then $\frac{dm}{dt} = 0$

$\Rightarrow \boxed{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}}$

If $\vec{v}$ = constant then $\frac{d\vec{v}}{dt} = 0$

$\Rightarrow \boxed{\vec{F} = \vec{v}\frac{dm}{dt}}$ $\Rightarrow$ Will be discussed later in chapter **"centre of mass"**

(eg. conveyor belt or rocket propulsion)

SPOT LIGHT

$\vec{F} = m\vec{a}$ is not an additional force acting on the body but it is the vector sum of all forces acting on the

2.5 Impulse

- When a large force is applied on a body for a very short interval of time, then the product of force and time interval is known as impulse.
- $d\vec{I} = \vec{F}dt = d\vec{p}$ $\quad \left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]$
- Units of impulse = N – s or kg-m/s
- Dimensions of impulse = [F] [t] = [m] [a] [t]= $[M^1L^1T^{-2}T^1] = [M^1L^1T^{-1}]$
- If this force is working from time t_1 to t_2, then integrating the above equation we get

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

$\Rightarrow$ Impulse = Change in momentum

- If a constant or average force acts on body, then

$$\Rightarrow \vec{I} = \int_{t_1}^{t_2} \vec{F}_{avg} dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} \Rightarrow \vec{I} = \vec{F}_{avg} \int_{t_1}^{t_2} dt = \int_{t_1}^{t_2} d\vec{p}$$

$$\Rightarrow \vec{I} = \vec{F}_{avg} [t]_{t_1}^{t_2} = [\vec{p}]_{\vec{p}_1}^{\vec{p}_2} \Rightarrow \vec{I} = \vec{F}_{avg}(t_2 - t_1) = (\vec{p}_2 - \vec{p}_1)$$

$$\boxed{\vec{I} = \vec{F}_{avg} \Delta t = \Delta \vec{p}}$$

DETECTIVE MIND

Impulse = change in linear momentum i.e. $\vec{I} = \vec{F}_{avg} \Delta t = \Delta \vec{p}$

Impulse - Momentum Theorem:

The impulse of force is equal to the change in momentum. This relation is known as impulse -momentum theorem.

SPOT LIGHT

- Force which can make a sudden change in momentum in very small interval of time is called impulsive force.

2.6 Law of Conservation of linear Momentum:

If net external force on a system is zero then the linear momentum of the system remains constant. According to Newton's II[nd] Law.

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

If $\vec{F}_{ext} = 0$

or Internal forces are large and impulsive then,

$$\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p}_{initial} = \vec{p}_{final}$$

$\Rightarrow$ $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \ldots\ldots + \vec{p}_n = \text{costant}$

(Conservation of linear momentum)

$\Rightarrow$ $\Delta(\vec{p}_1 + \vec{p}_2 + \ldots\ldots + \vec{p}_n) = 0$ $\Rightarrow$ $\Delta\vec{p}_1 + \Delta\vec{p}_2 + \ldots\ldots + \Delta\vec{p}_n = 0$

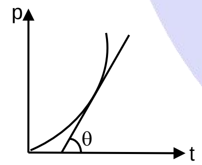
for two particles system $\vec{p}_1 + \vec{p}_2$ constant

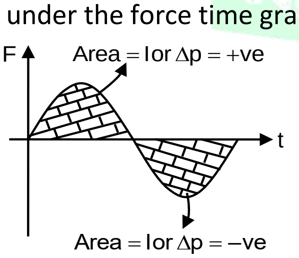
$\Delta\vec{p}_1 + \Delta\vec{p}_2$, (change in momentum of I[st] particle = -change in momentum of II[nd] particle)

SPOT LIGHT

- Linear momentum may be conserved even if there is present an external force on the system provided external force should be non-impulsive but this statement can be true only when there must be present an internal impulsive force on the body.

Important Points about Newton's Second Law of Motion:

- Newton's first law of motion defines force and second law of motion measures force. It gives the units, dimensions and magnitude of the force.
  Unit of force = (unit of mass) × (unit of acceleration) = 1kg × 1 m/s^2 = 1 N (newton)
  1N = 1 kg-m/s^2
  Dimension of force:- [m] [a] = $[m^1]\,[L^1T^{-2}] = [M^1L^1T^{-2}]$
- If a particle moves uniformly, means velocity = constant
  $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\text{cos tan t})}{dt} = \vec{0}$$
  so, $\vec{F} = m\vec{a} = m x \vec{0} = \vec{0}$
  It means that, in the absence of external force, a particle moves uniformly. This is Newton's first law of motion. It means that, we can derive mathematically Newton's first law of motion with the help of Newton's second law of motion.
- Accelerated motion is always due to an external force.
- Law of conservation of Linear Momentum (COLM) is applicable in the direction in which external force is zero, i.e.
  If $F_x = 0 \Rightarrow \frac{dp_x}{dt} = 0$, then p_x = constant
  If $F_y = 0 \Rightarrow \frac{dp_y}{dt} = 0$, then p_y = constant
  If $F_z = 0 \Rightarrow \frac{dp_z}{dt} = 0$, then p_z = constant
- The slope of momentum time graph is equal to the force on the particle
  e.g.
  

  Here, $F = \frac{dp}{dt} = \text{slope} = \tan\theta$
- Area under the force time graph represents impulse or change in momentum.
  

  e.g.
  Here, I or $\Delta p = \int F dt = \text{Area under F} - \text{t graph}$
- $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{\text{Area under the F-t graph}}{\text{Time interval}}$

- If the time interval is increased then the average force exerted will decrease and vice-versa.

SPOT LIGHT

$\vec{F} = m\vec{a}$ is independently applicable along any direction with components net force long that direction and components of acceleration.

Examples:

(i) A cricketer lowers his hands while catching a ball so as to increase the time interval of momentum change consequently the average reaction force on his hands decreases. So he can save himself from getting hurt.

(ii) Shockers are provided in vehicles to avoid jerks.

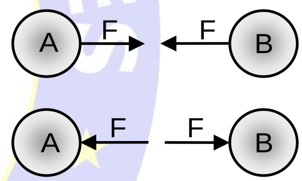
(iii) Buffers are provided in bogies of train to avoid jerks.

(iv) A person jumping on a hard cement floor receives more injuries than a person jumping on muddy or sandy road.

- In all above example the time interval is increased so that average force decrease.

2.7 Newton's Third Law of Motion :

- According to Newton's third law, to every action, there is always an equal (in magnitude) and opposite (in direction) reaction.
- Action and reaction are pair of forces
- Action reaction acts on different body & these forces are of same nature.
- Here $\vec{F}_{12}$ (force on first body due to second body) is equal in magnitude and opposite in direction $\vec{F}_{21}$ (force on second body due to first body).



The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive

$$\vec{F}_{12} = -\vec{F}_{21}$$

- We cannot produce a single isolated force in nature. Forces are always produced in action reaction pair.
- There is no time gap in between action and reaction. So we cannot say that action is the cause and reaction is its effect. Any one force can be action and the other reaction.
- Action- reaction law is applicable on both the states either at rest or in motion.
- Action and reaction is also applicable between bodies which are not in physical contact.
- Action - reaction law is applicable to all the interaction forces eg. gravitational force, electrostatic force, electromagnetic force, tension, friction, viscous force, etc.
- Action and reaction never cancel each other because they act on two different bodies.
  Eg. Walking, Swimming, Recoiling of gun when a bullet is fired from it, Rocket propulsion.
- Newton's III law can be derived from principle of conservation of linear momentum.
  if two particles of masses m_1 and m_2 are moving under the action of their mutually interacting forces with each other, such that no external force acts on the system, then momentum of system remains constant.

  i.e. $\Rightarrow \frac{\Delta\vec{p}_1}{\Delta t} = \frac{\Delta\vec{p}_2}{\Delta t} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$

  Force on 1^{st} due to 2^{nd} = – Force on 2^{nd} due to 1^{st}

SOLVED EXAMPLES

Example 1: A force $F = (6\hat{i} - 8\hat{j} + 10\hat{k})$ N produces an acceleration of $\sqrt{2}$ ms^{-2} in a body. Calculate the mass of the body.

(1) 5 kg (2) 8 kg (3) 10 kg (4) 15 kg

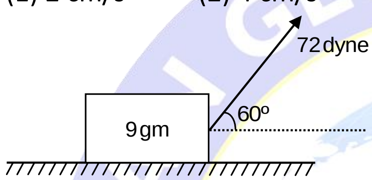
Solution: From Newton's IInd law $|\vec{F}| = |m\vec{a}| = ma$

$$\Rightarrow \text{Acceleration} = a = \frac{|\vec{F}|}{m} \Rightarrow m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{\sqrt{2}} = 10\text{kg}$$

Example 2: A force of 72 dynes is inclined at an angle 60° to the horizontal, find the acceleration in a mass of 9 g which moves under the effect of this force in the horizontal direction.

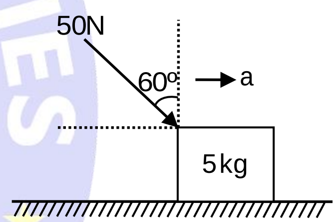
(1) 2 cm/s^2 (2) 4 cm/s^2 (3) 5 cm/s^2 (4) 6 cm/s^2

Solution:



F = ma $\Rightarrow$ 72 cos 60° = 9a $\Rightarrow$ a = 4 cm/sec^2

Example 3: A force of 50 N acts in the direction as shown in figure. The block is of mass 5 kg and resting on a smooth horizontal surface. Find out the acceleration of the block.



(1) $10\sqrt{3}$ m/s^2 (2) $15\sqrt{3}$ m/s^2 (3) $5\sqrt{3}$ m/s^2 (4) $8\sqrt{3}$ m/s^2

Solution: Horizontal component of the force = 50 sin 60° = $\frac{50\sqrt{3}}{2}$ acceleration of the block,

$$a = \frac{\text{component of force in the direction of acceleration}}{\text{mass}} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}\,\text{m/s}^2$$

Example 4: A machine gun fires a bullet of mass 50 gm with velocity 1000 ms^{-1}. If average force acting on gunis 200 N then find out the maximum number of bullets fired per minute.

(1) 230 (2) 235 (3) 240 (4) 245

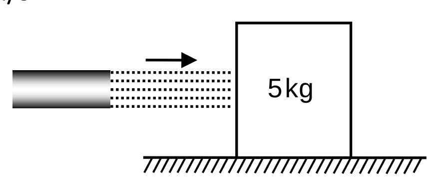
Solution: Average force acting on a gun = mnv

$200 = \frac{50}{1000} \times n \times 1000 = 4$ then no. of bullets fired per minute = 240

Example 5: A 5 kg block is resting on a frictionless plane. It is struck by a jet, releasing water at the rate of 3 kg/s emerging with a speed of 4 m/s. Calculate the initial acceleration of the block.

(1) 2.4 m/s^2 (2) 2.8 m/s^2 (3) 3.4 m/s^2 (4) 3.5 m/s^2

Solution: Force exerted on block F = $\left|\frac{\Delta P}{\Delta t}\right| = \left|\frac{mv_f - mv_i}{t}\right|$

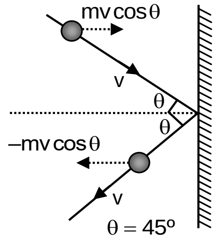


$\Rightarrow$ F = $\left|\frac{0 - 4m}{t}\right|$ = 4 × 3 = 12 N

So, acceleration of the block

$a = \frac{F}{m} = \frac{12}{5} = 2.4\,m/s^2$

Example 6: A ball of 0.20 kg hits a wall with a velocity of 25 m/s at an angle of 45°. If the ball rebounds at 90°to the direction of incidence, calculate the magnitude of change in momentum of the ball.

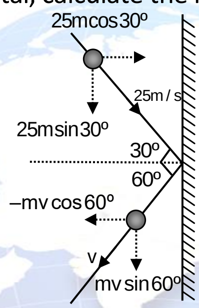


(1) $6\sqrt{3}$ N-s (2) $5\sqrt{3}$ N-s (3) $6\sqrt{2}$ N-s (4) $5\sqrt{2}$ N-s

Solution: Change in momentum = (–mvcos 45°) – (mv cos 45°) = – 2mv cos 45°

$|\Delta \vec{p}| = 2\,mv\cos 45° = 2 \times 0.2 \times 25 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}$ N-s

Example 7: A ball of 0.20 kg hits a wall with a velocity of 25 m/s at an angle of 30° with the horizontal. If the ball rebounds at 60° to the horizontal, calculate the magnitude of change in momentum of the ball.



(1) $\frac{10}{\sqrt{3}}$ N-s (2) $10\sqrt{3}$ N-s (3) $\frac{10}{\sqrt{2}}$ N-s (4) $10\sqrt{2}$ N-s

Solution. Change in momentum

= (–mv cos 60°) – (25m cos 30°) ....(i)

Change in momentum in the vertical direction must be equal to 0 as

$f_{ext.} = 0$ in vertical direction

25m sin 30° = mv sin 60°

$25\left(\frac{1}{2}\right) = V\left(\frac{\sqrt{3}}{2}\right) = v = \frac{25}{\sqrt{3}}$

$|D\vec{P}| = |(-mv\cos 60°) - (25m\cos 30°)|$

$= (0.2)\left(\frac{25}{\sqrt{3}}\right) \times \left(\frac{1}{2}\right) + 25\,(0.2)\left(\frac{\sqrt{3}}{2}\right) = \frac{10}{\sqrt{3}}$ N-s.

Example 8: A hammer of mass 1 kg moving with a speed of 6 m/s strikes a wall and comes to rest in 0.1s. Calculate:

(a) impulse of the force

(1) –6N-s (2) –8N-s (3) 10 N-s (4) 12 N-s

(b) average retarding force that stops the hammer.

(1) 60 N (2) 65 N (3) 70 N (4) 75 N

(c) average retardation of the hammer

(1) 50 m/s^2 (2) 54 m/s^2 (3) 56 m/s^2 (4) 60 m/s^2

Solution: (a) Impulse = $F \times \Delta t = m(v - u) = 1(0 - 6) = -6$Ns

(b) Average retarding force that stops the hammer

$\Rightarrow F = \frac{\text{Impulse}}{\text{time}} = \frac{6}{0.1} = 60N$

(c) Average retardation $a = \frac{F}{m} = \frac{60}{1} = 60\ m/s^2$

Example 9: A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that the ball gets turned back with a velocity of 20 m/s. If the duration of contact between the ball and bat is 0.01 s, find the impulse and the average force exerted on the ball by the bat.

(1) 5.8 N-s, 490 N (2) 4.8 N-s, 480 N (3) 5.4 N-s, 495 N (4) 2.8 N-s, 500 N

Solution: According to given problem, change in momentum of the ball

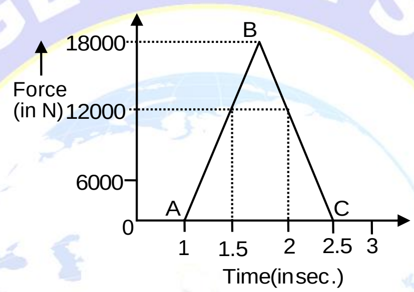
$\Delta p = p_f - p_i = m(v-u) = 150 \times 10^{-3}\,[20 - (-12)]$

So by impulse-momentum theorem, Impulse $I = \Delta p = 4.8$ N–s

And by the time average definition of force in case of impulse

$$F_{av} = \frac{I}{\Delta t} = \frac{\Delta p}{\Delta t} = \frac{4.80}{0.01} = 480\,N$$

Example 10: Figure shows an estimated force-time graph for a baseball struck by a bat. From this curve, determine



(a) Impulse delivered to the ball

(b) Average force exerted on the ball

Solution: (a) Impulse = Area under F-t curve = Area of $\Delta ABC = \frac{1}{2} \times 18000 \times (2.5 - 1) = 1.35 \times 10^4$ kg-m/s

(b) Average force $= \frac{\text{Impulse}}{\text{Time}} = \frac{1.35 \times 10^4}{(2.5 - 1)} = 9000$ N

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3. FREE BODY DIAGRAM:

➢ A diagram showing all external forces vectorially acting on an object is called "Free Body Diagram" (F.B.D.). A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces vectorially acting on it.

Steps for F.B.D.

Step 1: Identify the object or system and isolate it from other objects.

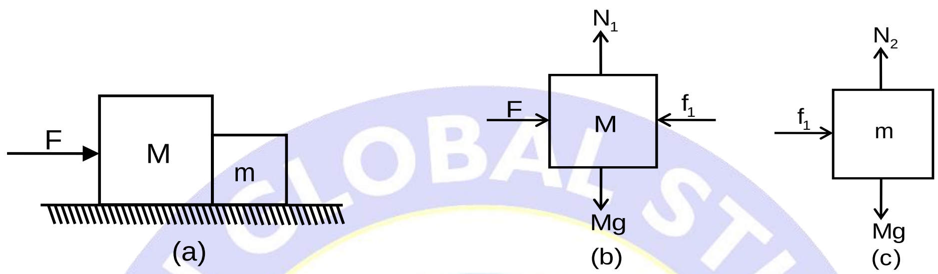
Step 2: First draw non-contact external force in the diagram. Generally it is weight.

Step 3: Draw contact forces which acts at the boundary (contact surface) of the object or system. Contact forces are normal, friction, tension and applied force.

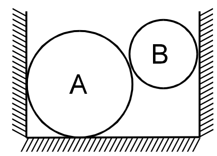
DETECTIVE MIND

In F.B.D, internal forces are not drawn, only external are drawn.

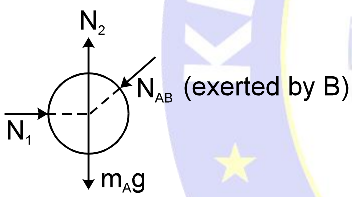
For example: If two bodies of masses m and M resting on a smooth floor are in contact and a force F is applied on M from the left as shown in figure (a), the free body diagram of M and m will be as shown in figures (b) and (c).



Example: Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.

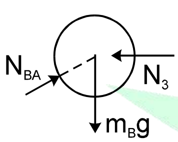


Solution: F.B.D. of sphere 'A' :



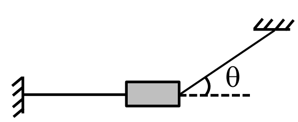
F.B.D. of sphere 'B' :

(exerted by A)

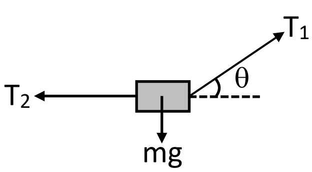


- Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

Example: A block of mass m is attached with two strings as shown in figure. Draw the free body diagram of the block.



Solution: The free body diagram of the block is as shown in Fig.

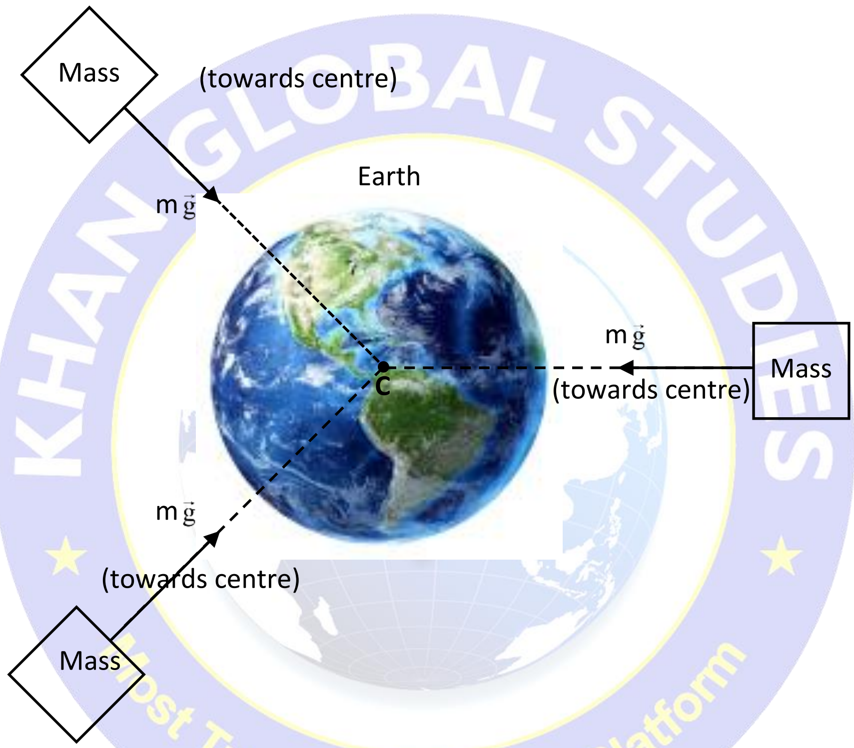


DETECTIVE MIND

Two forces in Newton's third law never occur in the same free body diagram. This is because a free body diagram shows forces acting on a single object, and the action-reaction pair in Newton's third law always act on different objects.

3.1 WEIGHT FORCE:

➢ Weight is the force with which earth pulls some body towards its center. If the body is near the earth's surface. This force equals $m\vec{g}$ where 'm' is the mass of the body & $\vec{g}$ is the acceleration due to gravity equals 9.8 m/sec.^{2} near the earth's surface. This force is vertically downwards (toward centre).



DETECTIVE MIND

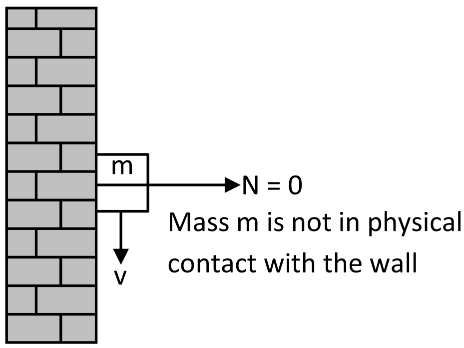
Weight force is the inherent property of earth but not the mass as weight force is applied by the earth on the body and weight of a body acts on the body itself.

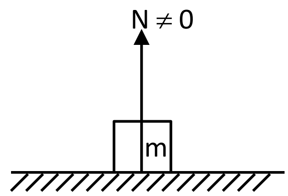
3.2 CONTACT FORCE:

➢ It is the force applied by a body over the other when they come in contact.

DETECTIVE MIND

The two bodies are said to be in physical contact if they apply contact force on each other

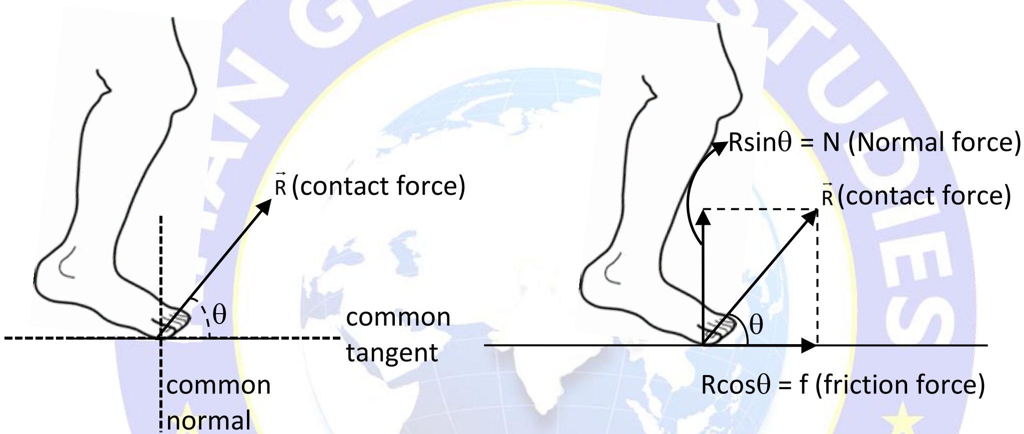




Mass m is applying normal force on the surface so it is in physical contact

3.3 NORMAL REACTION:

➢ Component of contact force along common normal is called normal force or normal reaction.



OR simply

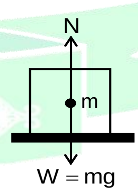
When a stationary flat body is placed on a surface then, that surface exerts a contact force on that flat body which is perpendicular to the surface and towards the flat body. This force is known as Normal Reaction.

e.g. A block of mass "m" is placed on a horizontal table. Then the forces exerted on it are :

(i) Downward gravitational force of attraction on body due to earth, means weight of the block (=mg).

(ii) Upward normal reaction exerted by the surface of table on the block.

Since the body is in equilibrium, therefore net force on it is zero.



$\therefore \vec{N} + \vec{W} = \vec{0}$

$\vec{N} = -\vec{W}$

$N = W \left[\because |\vec{N}| = |-\vec{W}|\right]$

$\therefore N = mg \quad [\because W = mg]$

DETECTIVE MIND

➢ Component of contact force along common tangent is called friction force.(will be discussed later).

➢ A body apply normal force on the surface in contact with but not the weight (is applied by earth).

3.3.1 Effective or Apparent weight of a man in lift :

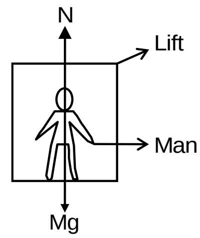
Case-I: If the lift is at rest or moving uniformly (a = 0), then

$N = mg$

$W_{app} = N$

So, $W_{app} = W_{actual}$

$W_{app} = mg_{effective} = W_{actual} = mg$



Case- II: If the lift is accelerating upwards, then -

Net upward force on man = ma

$N - mg = ma$

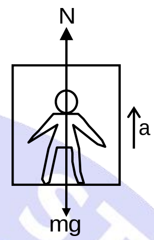
$N = mg + ma$

$N = m(g+a)$

W_{app} or $N = mg_{effective} = m(g + a)$

$g_{effective} = g + a$

So, $W_{app} > W_{actual}$



Case-III: If the lift is retarding upwards, then

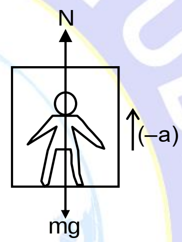
$N - mg = m(-a)$

$N = mg - ma$

W_{app} or $N = mg_{effective} = m(g-a)$

$g_{effective} = g - a$

So, $W_{app} < W_{actual}$



Case-IV: If the lift is accelerated downwards, then -

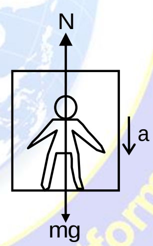
$mg - N = ma$

$N = mg - ma$

W_{app} or $N = mg_{effective} = m(g-a)$

$g_{effective} = g - a$

So, $W_{app} < W_{actual}$



Case-V: If the lift is retarding downwards, then -

$mg - N = m(-a)$

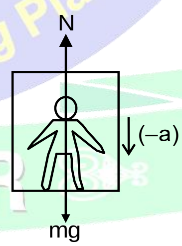
$mg - N = -ma$

$N = ma + mg$

W_{app} or $N = mg_{effective} = m(g + a)$

$g_{effective} = g + a$

So, $W_{app} > W_{actual}$



3.3.2 Two special cases of downward acceleration:

Special Case-I : If the lift is falling freely, it implies that its acceleration is equal to the acceleration due to gravity i.e. = g, then -

$W_{app.} = mg_{effective} = m(g-a) = m(g-g) = 0$

$W_{app.} = 0$

It means the man will feel weightless. This condition is known as condition of Weightlessness.

The apparent weight of any freely falling body is zero.

Special Case-II :

If the lift is accelerating downwards with an acceleration which is greater then 'g', then the man will move up with respect to the lift and stick to the ceiling.

3.4 MOTION OF BODIES IN CONTACT (CONTACT FORCE) :

Case-I: Two bodies in contact :

Free body diagram :

$F - N = m_1a$ ....(1)

$N = m_2a$ ....(2)

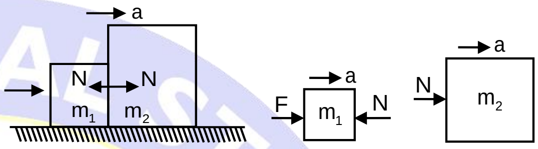
On adding the above equation

$F = m_1a + m_2a$ $\quad F = a(m_1+m_2)$ $\quad \Rightarrow a = \dfrac{F}{m_1+m_2}$ ......(3)

OR $\quad a = \dfrac{F_{net}}{m_{total}}$

Putting the value of 'a' from equation (3) in (2), we get

$N = \dfrac{m_2F}{m_1+m_2}$ (here N = contact force)



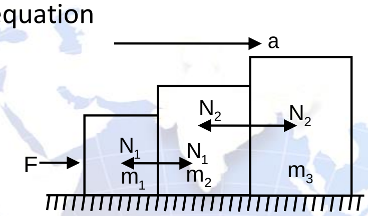
Case-II: Three bodies in contact :

$F - N_1 = m_1 a$ ....(1)

$N_1 - N_2 = m_2 a$ ....(2)

$N_2 = m_3 a$ ....(3)

On adding the above equation



$F = m_1a + m_2a + m_3a$

$F = a(m_1+m_2+m_3)$ $\quad \Rightarrow a = \dfrac{F}{m_1+m_2+m_3}$ ......(4)

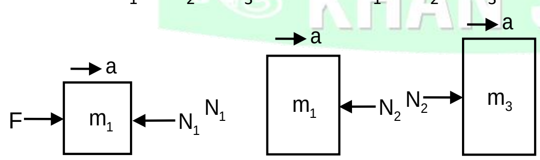
orPutting the value of 'a' from equation (4) in (3), we get

$N_2 = \dfrac{Fm_3}{m_1+m_2+m_3}$

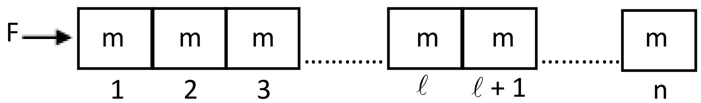
Putting the value of 'a' from (4) in (3), we get

$F - m_1a = N_1$

$N_1 = F - \dfrac{Fm_1}{m_1+m_2+m_3} \Rightarrow N_1 = \dfrac{(m_2+m_3)F}{m_1+m_2+m_3}$

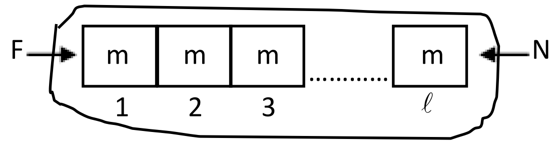


Case-III: n identical bodies in contact



Net acceleration of 'n' identical body system, $a = \dfrac{F_{applied}}{\text{total mass}} = \dfrac{F}{nm}$

F.B.D of body from 1^{st} to ℓ^{th} body assuming a single body



(normal forced applied by $(\ell + 1)^{th}$ block)

$$\Rightarrow F - N = (m\ell) \times \frac{F}{nm} \Rightarrow F - \frac{m\ell F}{nm} = N$$

$$\Rightarrow F\left[1 - \frac{\ell}{n}\right] = N \Rightarrow F\left[1 - \frac{\ell}{n}\right] = N$$

OR $\boxed{N = F\left[1 - \frac{\ell}{n}\right]}$ ⟶ Normal force between ℓ^{th} and $(\ell + 1)^{th}$ block

where n = total number of blocks

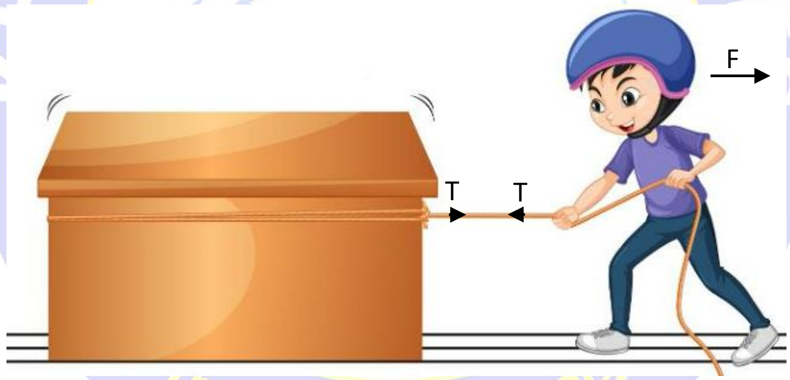
DETECTIVE MIND

- If there is no relative motion between the system of the bodies then all the bodies can be assumed to be a single body.(see above case-III)

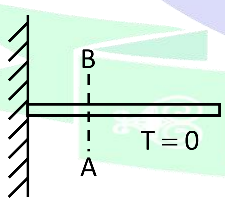
4. SYSTEM OF MASSES TIED BY STRINGS :

4.1 Tension in a String :

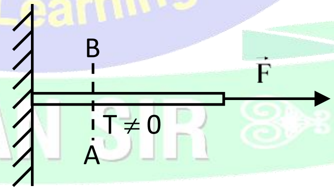
- It is intermolecular forces between the molecules of a string, which become active when the string is stretched.



- OR tension is the force with which any part of the body pulls the adjacent part towards itself.



At section AB tension = 0



At section AB tension ≠ 0

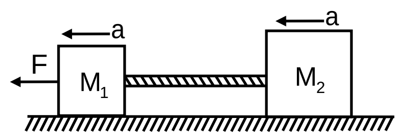
Here $\vec{F}$ is not tension force as it is applied by external agent but not applied by any part of the string

- Since here the system is at rest so coincidently magnitude of tension force = $|\vec{F}|$

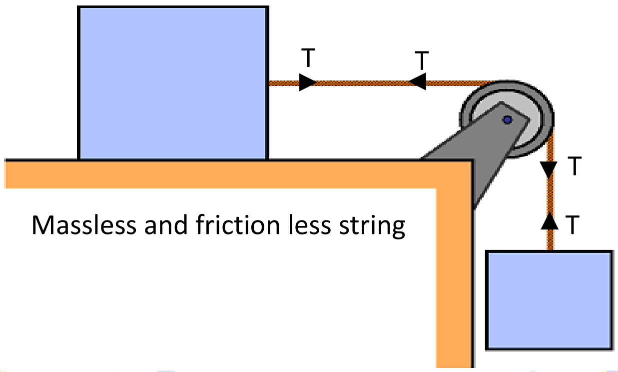
DETECTIVE MIND

- Tension force can change suddenly
- Direction of tension force is always away from the contact point.

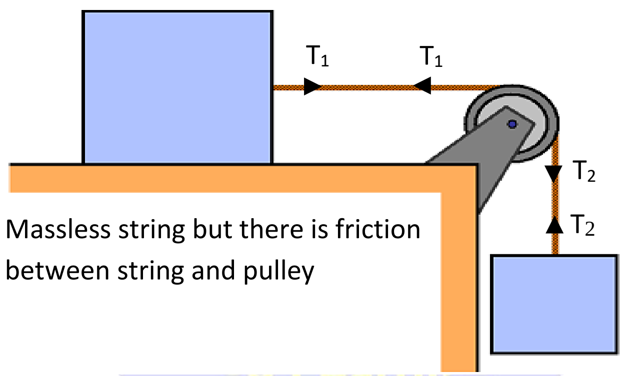
- Force of tension act on a body in the direction away from the point of contact or tied end of the string.
- String is assumed to be inextensible so that the magnitude of accelerations of the blocks tied to the strings are always same.



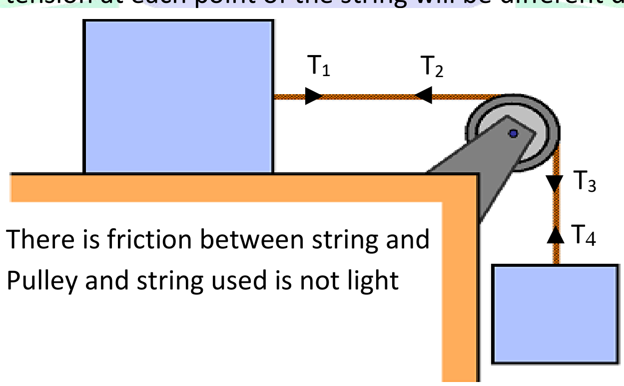
- If the string is massless and frictionless, tension throughout the string remains same.



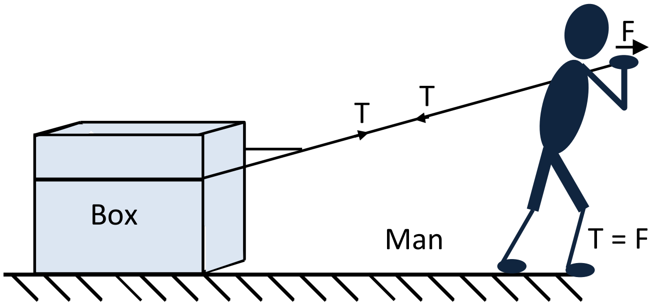
- If the string is massless but not frictionless, at every contact tension changes.



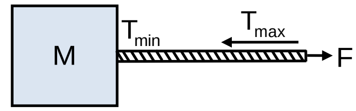
- If the string is not light, tension at each point of the string will be different depending on the acceleration.



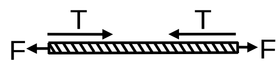
- If a force is directly applied to a string, say a man is pulling a string from the other end with some force, then tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box moves or not, man moves or not.



- String is assumed to be massless unless stated, hence tension in it remains the same everywhere and equal to the applied force. However, if a string has a mass, tension at different points will be different being maximum (=applied force) at the end through which force is applied and minimum at the other end connected to a body.



- In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e. T = F) and is directed inwards opposite to F.



- Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break beyond a certain limit. The maximum tension which a string can bear without breaking is called its "**breaking strength**". It is finite for a string and depend on its material and dimensions.

DETECTIVE MIND

A thread or string is said to be ideal if it is in massless, inextensible and frictionless. In ideal thread magnitude of tension is same everywhere.

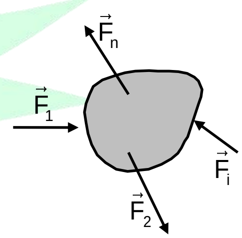
4.2 TRANSLATIONAL EQUILIBRIUM:

- A body in a state of rest or moving with constant velocity is said to be in translational equilibrium.
- If a body is in translational equilibrium in a certain inertial frame of reference, it must have no linear acceleration.
- When a body is at rest, it is in **static equilibrium**.
- If a body is moving with constant velocity, it is in **dynamic equilibrium**.

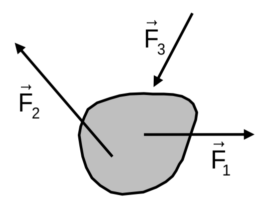
4.2.1 Conditions for translational equilibrium

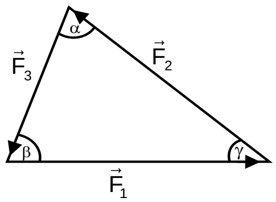
- For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero. If several external forces $\vec{F}_1, \vec{F}_2$ ..... $\vec{F}_i$ ..... and $\vec{F}_n$ act simultaneously on a body and the body is in translational equilibrium, the resultant of these must be zero. Cartesian components, we have :

$\sum F_{ix} = 0$ $\sum F_{iy} = 0$ $\sum F_{iz} = 0$



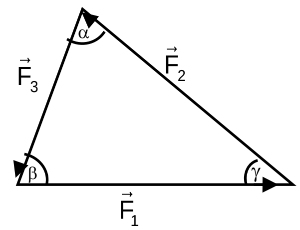
- If a body is acted upon by a single external force, it cannot be in equilibrium.
- If a body is in equilibrium under the action of only two external forces must be equal and opposite.
- If a body is in equilibrium under the action of three forces, their resultant must be zero. Therefore, according to the triangle law of vector addition they must be coplanar and should form a closed triangle.





$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \Rightarrow$$

The situation can be analyzed by either graphical or analytical method.



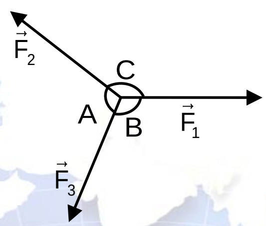
Lami's Theorem

Graphical method makes use of sine rule or Lami's theorem.

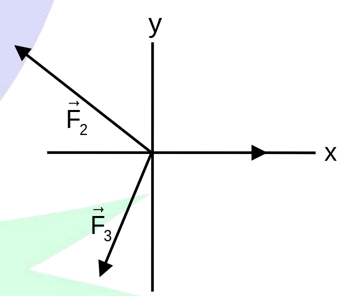
Sine rule : $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$

DETECTIVE MIND

If an object is in equilibrium under the effect of three forces then solving the question using Lami's theorem is more fruitful.



Lami's theorem : $\frac{F_1}{\sin A} = \frac{F_1}{\sin B} = \frac{F_1}{\sin C}$



- Analytical method makes use of Cartesian components. Since the forces involved form a closed triangle, they lie in a plane and a two-dimensional Cartesian frame can be used to resolve the forces. As far as possible orientation of the x-y frame is selected in such a manner that angles made by difference forces with the axes should have convenient values.

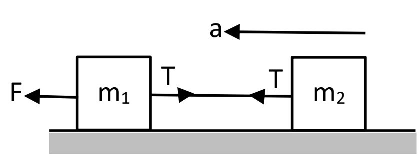
$$\sum \vec{F}_x = \vec{0} \Rightarrow \vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} = \vec{0}$$

$$\sum \vec{F}_y = \vec{0} \Rightarrow \vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} = \vec{0}$$

- Problems involving more than three forces should be analyzed by analytical method. However, in some situations there may be some parallel or antiparallel forces and they should be combined first to minimize the number of forces. This may sometimes reduce a system involving more than three forces to a three-force system.

5. MOTION OF CONNECTED BODIES

5.1 Two Connected Bodies :



$F - T = m_1a$ ...(1)

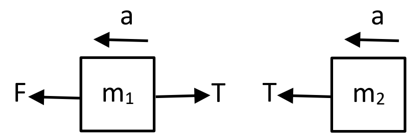
$T = m_2a$ ...(2)

On adding the above equations

$F = m_1a + m_2a$

$F = a(m_1 + m_2)$

$$a = \frac{F}{m_1 + m_2} \quad ...(3)$$



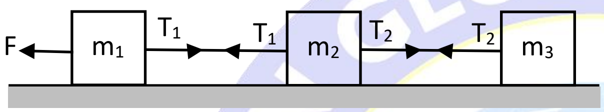
or $a = \dfrac{F_{net}}{m_{total}}$

Putting the value of 'a' from equation (3) in (2), we get -

$$T = \frac{m_2F}{m_1 + m_2}$$

5.2 Three Connected Bodies :

Free body diagrams : -



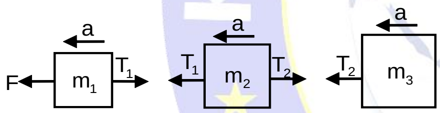
$F - T_1 = m_1a$ ....(1)

$T_1 - T_2 = m_2a$ ....(2)

$T_2 = m_3a$ ....(3)

On adding above equations

$F = (m_1 + m_2 + m_3)\, a$



$$\therefore a = \frac{F}{m_1 + m_2 + m_3} \quad \text{or} \quad a = \frac{F_{net}}{m_{total}}$$

Put the value of 'a' in equation (1)

$T_1 = F - m_1a$

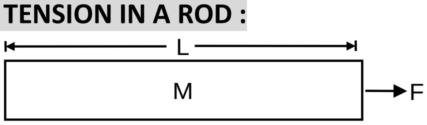
$$\Rightarrow T_1 = F - \frac{m_1F}{m_1 + m_2 + m_3} = \frac{m_1F + m_2F + m_3F - m_1F}{m_1 + m_2 + m_3}$$

$$T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

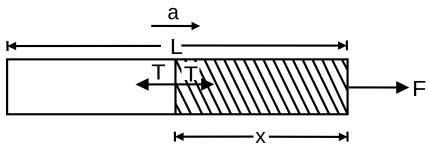
Similarly, putting the value of 'a' in equation (3)

$$T_2 = \frac{m_3F}{m_1 + m_2 + m_3}$$

5.3 TENSION IN A ROD :



Given Mass of Rod = M



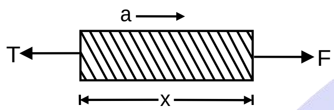
Length of Rod = L

F - T = ma $\Rightarrow$ T = F – ma

$\Rightarrow \quad T = F - m\dfrac{F}{M}$ ...(1) $\left(\because a = \dfrac{F}{M}\right)$

Mass of length 'L' = M

Mass of unit length = $\dfrac{M}{L}$



$\therefore$ Mass(m) of length 'x' = $\dfrac{M}{L}x$

Put the value of 'm' in equation (1)

$T = F - \left(\dfrac{M}{L}x\right)\dfrac{F}{M} \Rightarrow \boxed{T = F\left(1 - \dfrac{x}{L}\right)}$

5.4 BODIES HANGING VERTICALLY

Since all the bodies are in equilibrium, therefore net force on each body is zero

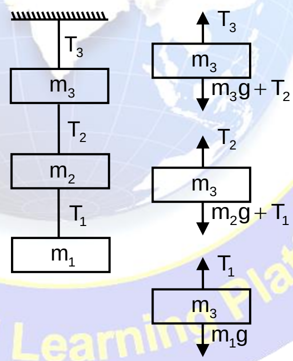
$\boxed{T_1 = m_1 g}$ ...(1)

$T_2 = T_1 + m_2 g$

$\Rightarrow \quad \boxed{T_2 = (m_1 + m_2)g}$ ...(2)

$T_3 = T_2 + m_3 g$

$\Rightarrow \quad \boxed{T_3 = (m_1 + m_2 + m_3)g}$



5.4.1 Accelerating Upwards

For m_1, $T_1 - m_1 g = m_1 a$

$\Rightarrow \boxed{T_1 = m_1(g + a)}$ ...(1)

For m_2, $T_2 - m_2 g - T_1 = m_2 a$

$\Rightarrow T_2 = m_2 a + m_2 g + T_1$

$\Rightarrow \boxed{T_2 = (m_1 + m_2)(g + a)}$ (from equation 1)

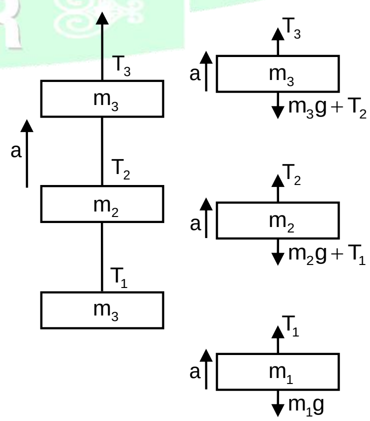
$\Rightarrow T_2 = (m_1 + m_2)(g + a)$ ...(2)

For m_3, F = T_3 and $T_3 - m_3 g - T_2 = m_3 a$

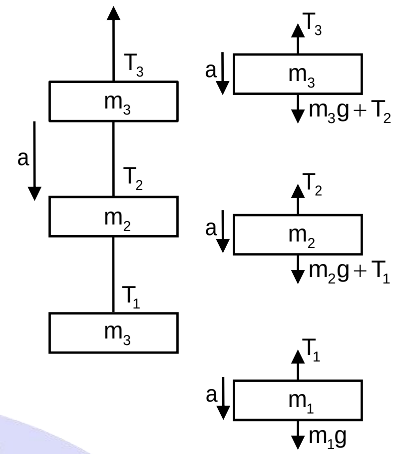
$\Rightarrow T_3 = T_2 + m_3 g + m_3 a$

$\Rightarrow T_3 = (m_1 + m_2)(g + a) + m_3(g + a)$ (from equation 2)

$\Rightarrow \boxed{T_3 = (m_1 + m_2 + m_3)(g + a)}$ ...(3)



5.4.2 Bodies Accelerating Vertically Downwards



For m_1, $m_1g - T_1 = m_1a$

$\Rightarrow \boxed{T_1 = m_1(g-a)}$ ...(1)

for m_2, $m_2g + T_1 - T_2 = m_2a$

$\Rightarrow T_2 = m_2g - m_2a + T_1$

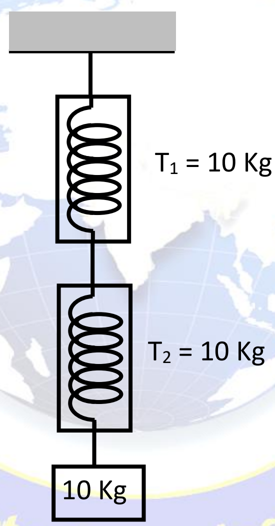
$\Rightarrow T_2 = m_2(g-a) + m_1(g-a)$ (from equation 1)

$\boxed{T_2 = (m_1 + m_2)(g-a)}$ ...(2)

Similarly $\Rightarrow \boxed{T_3 = (m_1 + m_2 + m_3)(g-a)}$ ...(3)

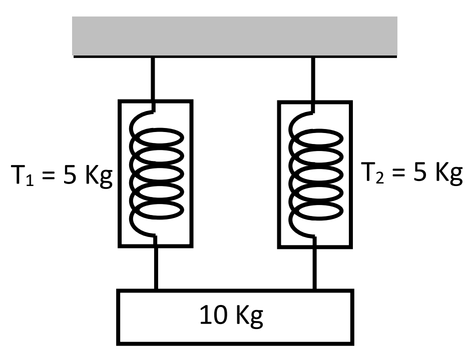
- If several spring balance are connected in series, then the reading of each balance is the same and is equal to the applied load (Spring balances have negligible mass so they are assumed massless)

  e.g. $T_1 = T_2 = 10$ kg-wt



- If several spring balances are connected in parallel and symmetrically to the load, then the applied load is equally divided in all the balances; so the reading of each balance will be $= \frac{\text{applied load}}{\text{no. of balance}}$

  e.g. $T_1 = T_2 = 10 / 2 = 5$ kg-wt



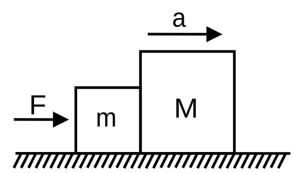
SOLVED EXAMPLES

Example 11: Consider a box of mass 10 kg resting on a horizontal table and acceleration due to gravity to be 10 m/s^2.
(a) Draw the free body diagram of the box.
(b) Find the value of the force exerted by the table on the box.
(c) Find value of the force exerted by the box on the table.
(d) Does force exerted by table on the box and weight of the box form third law action-reaction pair ?

Solution: (a)N_1 : Force exerted by table on the box.

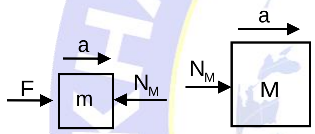
(b)The block is in equilibrium.
$\sum \vec{F} = \vec{0} \Rightarrow mg - N_1 = 0 \Rightarrow N_1 = 100$ N
(c)N_1 = 100 N : Because force by table on the box and force by box on table Make action-reaction pair.
(d) No

Example 12: Two blocks of masses m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (=35N) is applied to m. Find the force of contact between the blocks. Will the force of contact remain same if F is applied to M?

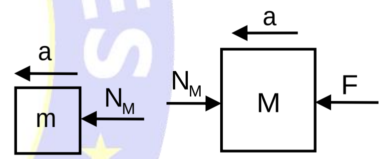


Solution: As the blocks are rigid both will move with same acceleration under the action of a force F

$$a = \frac{F}{m+M} = \frac{35}{2+5} = 5\,m/s^2$$

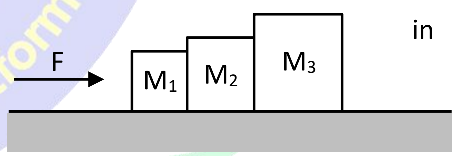


Force of contact $N_M = Ma = 5 \times 5 = 25$ N
If the force is applied to M then its action on m will be
$N_m = ma = 2 \times 5 = 10$ N.

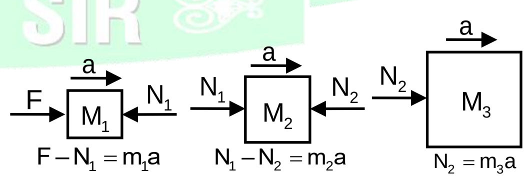


Note :- From this problem it is clear that acceleration does not depend on the fact that whether the force is applied to m or M, but force of contact does.

Example 13: Three blocks of masses m_1 = 1 kg m_2=1.5 kg and m_3=2 kg are in contact with each other on a frictionless surface as shown in fig. Find the
(a) Horizontal force F needed to push the blocks as a single unit with an acceleration of 4 m/s^2
(b) Resultant force on each block and
(c) Magnitude of contact forces between the blocks.



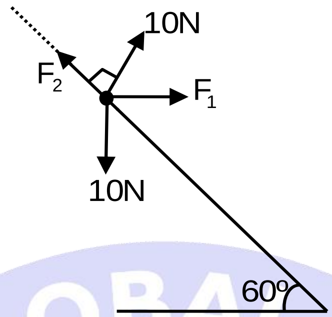
Solution: (a) $F = (m_1 + m_2 + m_3)\,a$
$= (1 + 1.5 + 2) \times 4$
$= 4.5 \times 4 = 18$ N



(b) For m_1
$F - N_1 = m_1 a = 1 \times 4 \Rightarrow F - N_1 = 4$ N
for m_2,
$N_1 - N_2 = m_2\,a = 1.5 \times 4 = 6$
$\Rightarrow N_1 - N_2 = 6$ N
for m_3,
$N_2 = m_3 a = 2 \times 4$
$\Rightarrow N_2 = 8$ N

(c)Contact force between m_2 and m_3 is $N_2 = 8$ N and contact force between m_1 and m_2 is $N_1 = N_2 + 6 = 8 + 6 = 14N$.

Example 14: In the given figure if all the forces are in equilibrium then calculate F_1 and F_2.



Solution: Resolve the forces along x-direction & y-direction

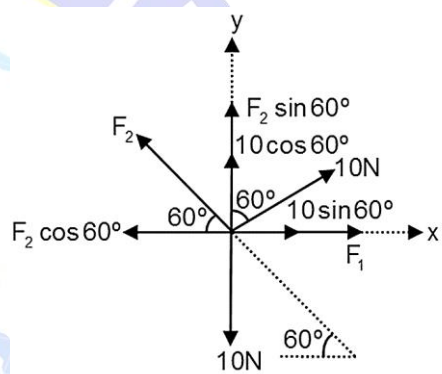
In x-direction

$\sum \vec{F}_x = \vec{0}$ (for equilibrium)

$F_1 + 10 \sin 60° = F_2 \cos 60°$

$\Rightarrow F_1 + 10\left(\frac{\sqrt{3}}{2}\right) = F_2\left(\frac{1}{2}\right)$

$\Rightarrow 2F_1 - F_2 = -10\sqrt{3}$ ....(i)



In y-direction

$\sum \vec{F}_y = \vec{0}$ $F_2 \sin 60° = 10 \cos 60° = 10$

$\Rightarrow F_2\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) = 10 \Rightarrow F_2\left(\sqrt{3}\right) + 10 = 20$

$\Rightarrow F_2 = \left(\frac{10}{\sqrt{3}} N\right)$

Now from equation (i)

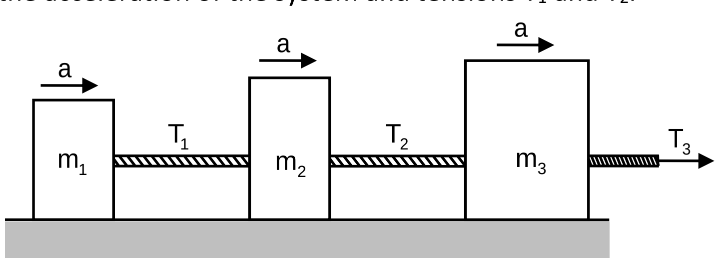
$\Rightarrow 2F_1 - F_2 = -10\sqrt{3}$ $\Rightarrow 2F_1 - \frac{10}{\sqrt{3}} = -10\sqrt{3}$

$\Rightarrow 2\sqrt{3}\, F_1 - 10 = -30 \Rightarrow 2\sqrt{3}\, F_1 = -20 \Rightarrow F_1 = \frac{-10}{\sqrt{3}}$ N

Example 15: Three blocks are connected by strings as shown in the figure below and are pulled by a force $T_3 = 120$ N. If $m_1 = 5$ kg,

$m_2 = 10$ kg and $m_3 = 15$ kg.

Calculate the acceleration of the system and tensions T_1 and T_2.



Solution: (i) Acceleration of the system

$$a = \frac{F}{m_1 + m_2 + m_3} = \frac{120}{5+10+15} = 4 \text{ m/s}^2$$

(ii) $T_1 = m_1 a = 5 \times 4 = 20$ N

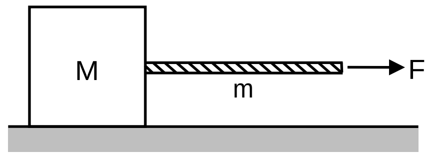
$T_2 = (m_1 + m_2)\, a = (5 + 10)\, 4 = 60$ N

Example 16: A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find the

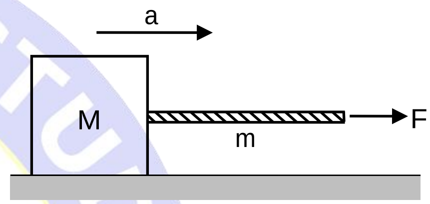
(i) Acceleration of the rope and the block

(ii) Force that the rope exerts on the block

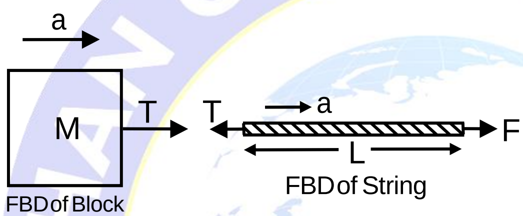
(iii) Tension in the rope at its mid point.



Solution: (i) Acceleration $a = \frac{F}{(m+M)}$

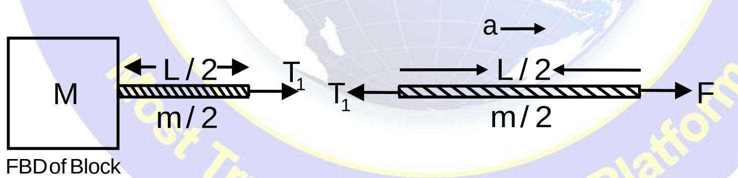


(ii) Force exerted by the rope on the block is



$$T = Ma = \frac{M.F}{(m+M)}$$

(iii) $T_1 = \left(\frac{m}{2} + M\right) a = \left(\frac{m+2M}{2}\right)\left(\frac{F}{m+M}\right)$



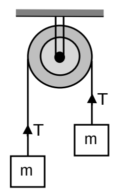
Tension in rope at midpoint is $T_1 = \frac{(m+2M)F}{2(m+M)}$

6. PULLEY SYSTEMS

- Ideal pulley is considered weightless and frictionless.
- Ideal string is massless and inextensible.
- A pulley may change the direction of force in the string but not the tension.

Some Cases of Pulley

Case I: $m_1 = m_2 = m$



Tension in the string

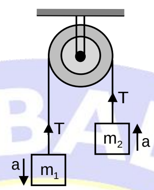
$T = mg$

Acceleration 'a' = zero

Reaction at the point of suspension of the pulley or thrust on pulley.

$R = 2T = 2\,mg$.

Case II: $m_1 > m_2$



now for mass m_1,

$m_1g - T = m_1a$ ...(i)

$T - m_2g = m_2a$ ...(ii)

adding (i) and (ii)

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} \text{ and } T = \frac{2m_1m_2}{(m_1 + m_2)}g$$

$$= \frac{2W_1W_2}{W_1 + W_2}$$

$$\text{acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}} \times \text{acceleration due to gravity}$$

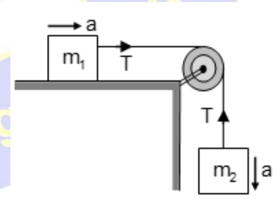
$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{total mass to be pulled}} \times \text{acceleration due to gravity}$$

Reaction at the suspension point of pulley (or thrust on pulley)

$$R = 2T = \frac{4m_1m_2g}{(m_1 + m_2)} = \frac{4W_1W_2}{W_1 + W_2}$$

Case III: For mass m_1 : $T = m_1a$

For mass m_2 : $m_2g - T = m_2a$



$$\text{acceleration } a = \frac{m_2g}{(m_1 + m_2)} \text{ and } T = \frac{m_1m_2}{(m_1 + m_2)}g$$

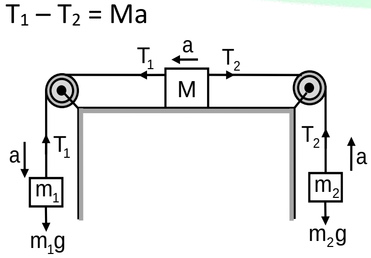
Reaction at suspension point of pulley $R = \sqrt{2}T$

Case IV: $(m_1 > m_2)$

$m_1g - T_1 = m_1a$ (i)

$T_2 - m_2g = m_2a$ (ii)

$T_1 - T_2 = Ma$ (iii)



adding (i), (ii) and (iii)

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)}g$$

Case V: Mass suspended over a pulley along with another on an inclined plane.

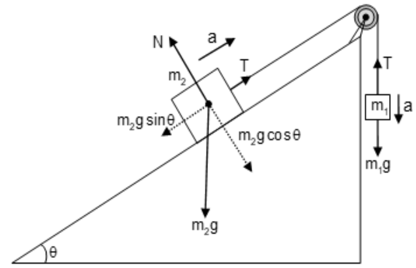
For mass m_1 : $m_1g - T = m_1a$ ....(1)

For mass m_2 : $T - m_2g \sin\theta = m_2a$ ....(2)

From equ. (1) and (2)

acceleration $a = \dfrac{(m_1 - m_2 \sin\theta)}{(m_1 + m_2)} g$

$T = \dfrac{m_1 m_2 (1 + \sin\theta)}{(m_1 + m_2)} g$



Case VI: Masses m_1 and m_2 are connected by a string passing over a pulley $m_1\sin\alpha > m_2\sin\beta$

$m_1g\sin\alpha - T = m_1a$ ...(i)

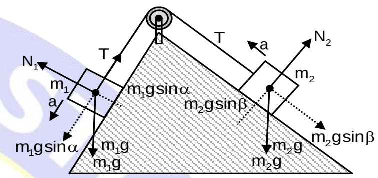
$T - m_2g \sin\beta = m_2a$ ...(ii)

After solving equation (i) and (ii)

Acceleration

$a = \dfrac{(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)} g$

Tension $T = \dfrac{m_1 m_2 (\sin\alpha + \sin\beta)}{(m_1 + m_2)} g$



Case VII: For mass m_1 : $T_1 - m_1g = m_1a$ ....(1)

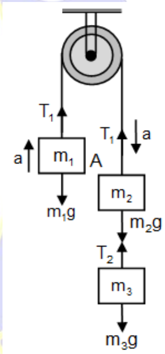
For mass m_2 : $m_2g + T_2 - T_1 = m_2a$ ....(2)

For mass m_3 : $m_3g - T_2 = m_3a$ ...(3)

From equ. (1), (2) & (3)

$\Rightarrow a = \dfrac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} g$

we can calculate tensions T_1 and T_2 from above questions.



DETECTIVE MIND

A pulley is said to be ideal if it is frictionless. If pulley is not ideal then tension in the thread hanging on the both of the pulley is different.

6.1 MECHANICAL ADVANTAGE

The ratio of load to effort is called mechanical advantage (M.A.)

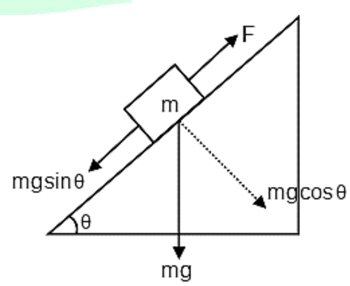
Thus, mathematically, M.A. = $\dfrac{\text{Load}}{\text{effort}}$

e.g. : Mechanical advantage of an inclined plane.

$F = mg \sin\theta$

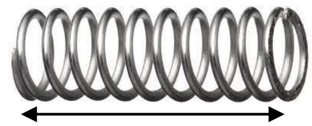
M.A. of inclined plane

$= \dfrac{mg}{F} = \dfrac{mg}{mg\sin\theta} = \dfrac{1}{\sin\theta} = \text{cosec}\theta$



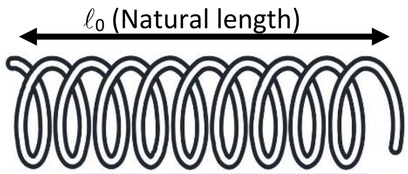
7. SPRING FORCE

- Natural length (ℓ_0): Length of spring when it is in relaxed state (condition) or length of spring at which it is manufactured.

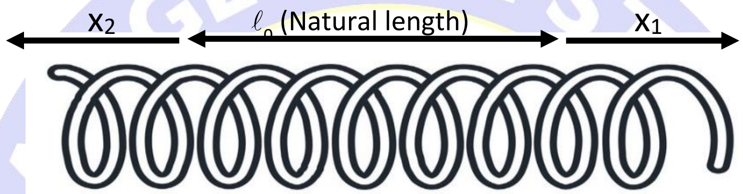


ℓ_0 (Natural length)

- Elongation: increment in length from natural length.
- Contraction / compression: Decrement in length from natural length.

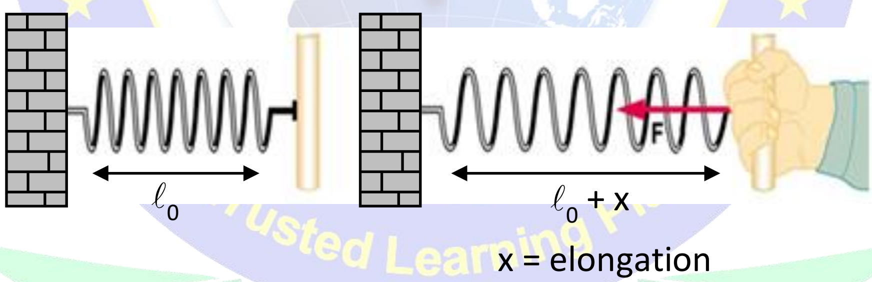


If one end of spring is displaced by x_1 rightward and other end displaced by x_2 leftward then total elongation of the spring = $x_1 + x_2 = x$ (say)



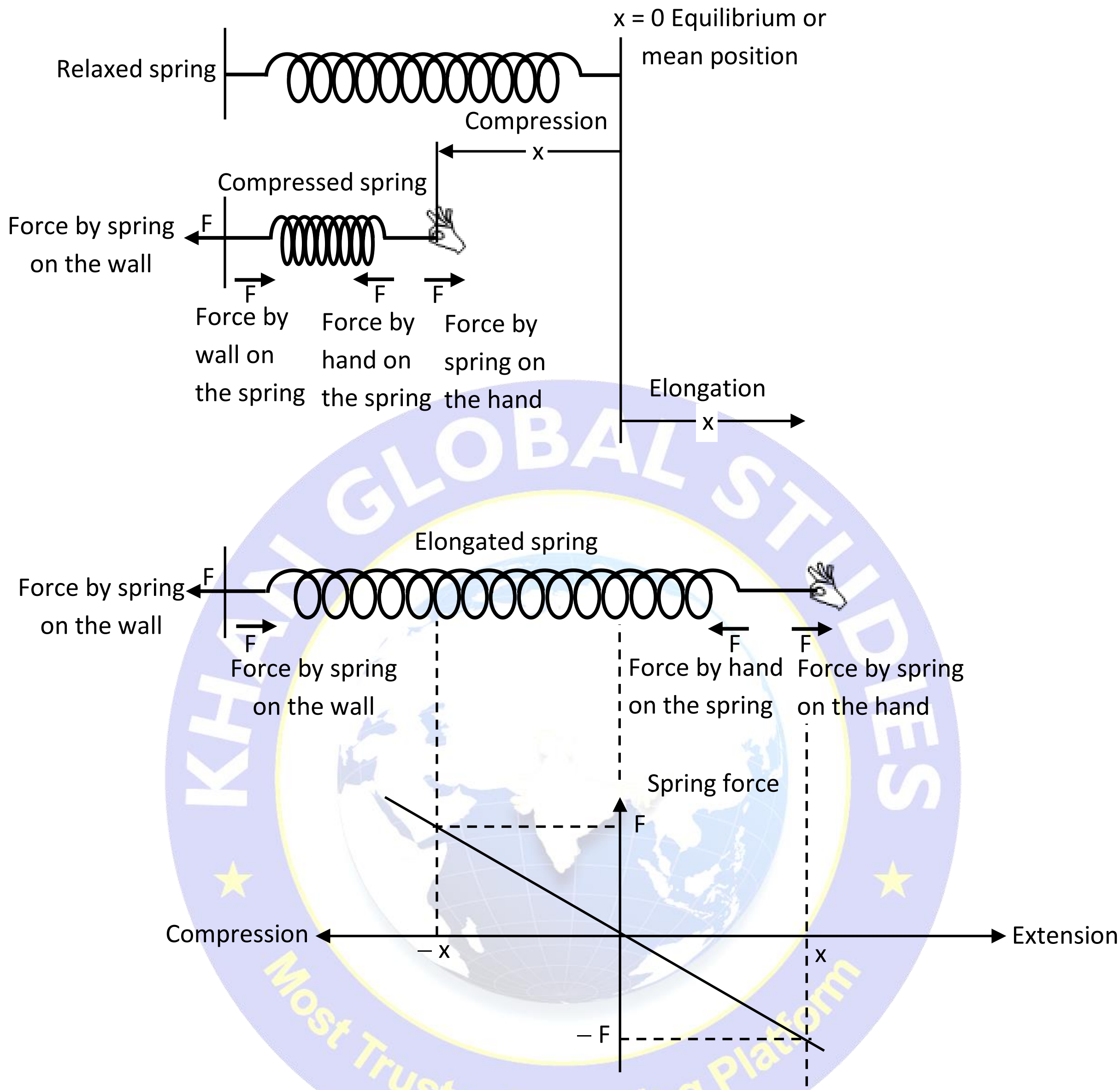
Elongation in spring = $x_1 + x_2 = x$

- If one end of spring is fixed to the wall and other free end of a spring is pulled by hand. The force applied by the spring on the wall and the force applied by the wall on the spring form a Newton's third law action-reaction pair. Similarly, force by hand on the spring and force by the spring on the hand form another Newton's third law action-reaction pair.



7.1 Hooke's Law :

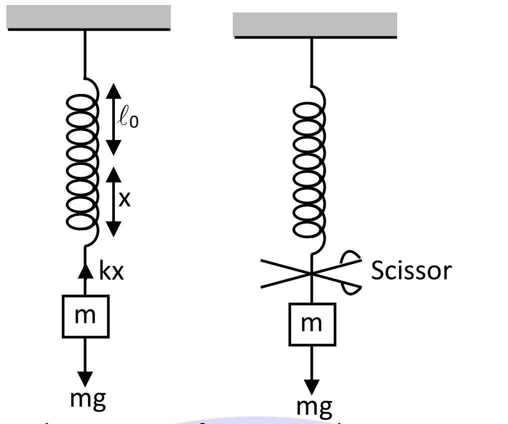
How spring force varies with deformation in length x of the spring is also shown in the following figure.



- The force F varies linearly with x and acts in a direction opposite to x. Therefore, it is expressed by the equation $= -kx$
- The negative sign represents the fact that force F is always opposite to x.
- The constant of proportionality k is known as force constant of the spring or simply as spring constant.
- The modulus of slope of the graph equals the spring constant.
- S.I. unit of k: N/m
- Dimensions of spring constant are $[MT^{-2}]$

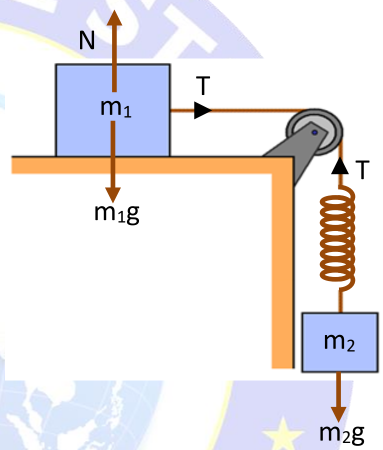
SPOT LIGHT

- Spring force cannot change suddenly.
- Spring force can change suddenly and becomes spring force zero, if it is cut.
- Spring force is non-impulsive force as it cannot make a large change in momentum in small interval of time.



kx = mg   After cutting kx = 0, as x = 0

7.2 For a pulley-spring system (at steady state)



tension $T = \frac{m_1 m_2 g}{(m_1 + m_2)}$

If x is the extension in the spring then T = kx

$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1 + m_2)}$

In this case if acceleration comes in picture then

$a = \frac{m_2 g}{m_1 + m_2}$

DETECTIVE MIND

A spring is said to be ideal if it is massless. Tension or compression force in the spring is called spring force.

8. FRAME OF REFERENCE :

- A system with respect to which the position or motion of a particle is described, is known as a frame of reference. or it is the place from where observation is taken we can classify frames of reference into two categories :

8.1 Inertial Frame of Reference :

- The frame for which is applicable, is known as inertial frame of reference.
- All the frames which are at rest or moving uniformly w.r.t. ground are inertial frame.

8.2 Non–Inertial Frame of Reference :

- The frame for which law of inertial is not applicable is known as non-inertial frame of reference.
- All the frames which are accelerating or rotating w.r.t. ground will be non-inertial frames.

SPOT LIGHT

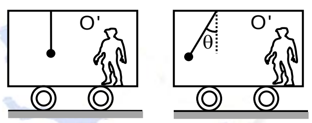
- Although earth is an accelerating frame, it is taken to be an inertial frame of reference.
- Other than earth, or accelerating frames are taken to be non-inertial frame of reference.

8.3 PSEUDO FORCE

- Newton's second law is not applicable from non-inertial frame but it some observer in a non-inertial frame wishes to apply $F_{netext.} = m\vec{a}$ it has to consider hypothetical force (false force) acting on the object in addition to other forces acting on the object.

  OR simply it is fictitious force or an apparent force or a correction force which is used to explain the motion of objects in non-inertial reference frames.
- This force always acts in the direction opposite to that of acceleration of frame and its magnitude is equal to the product of mass of the body and the acceleration of the non-inertial reference frame.

  $\boxed{\vec{F} = -m\vec{a}}$
- Pseudo force does not follow action-reaction law as it is not a real force acting on the body.

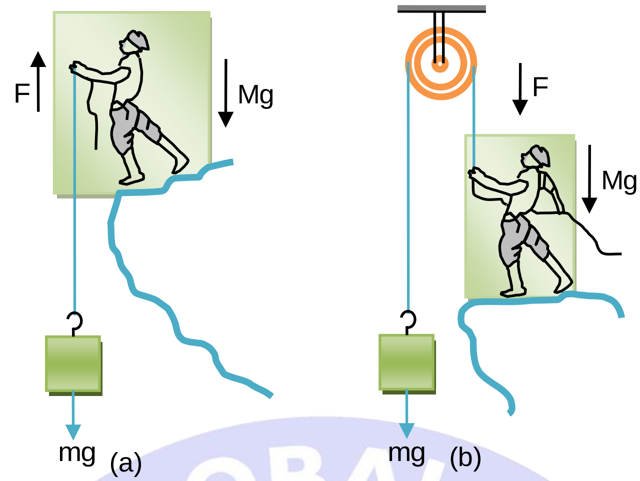


SPOT LIGHT

- Pseudo force is not the real force so it is not included in free body diagram (F.B.D).
- Showing pseudo force vectorially in F.B.D is wrong method.

SOLVED EXAMPLES

Example 17: A block of mass 25 kg is raised in two different ways by a 50 kg man as shown in fig. What is the action in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the yielding of the floor?

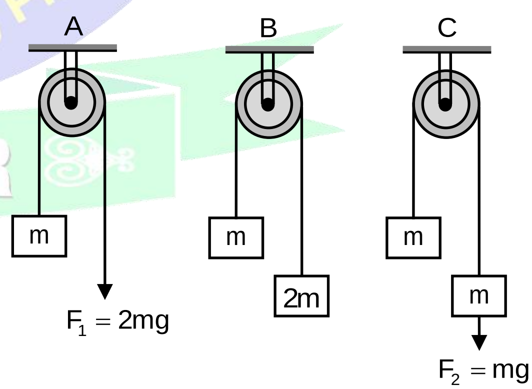


Solution: Mass of the block, m = 25 kg ;
mass of the man, M = 50 kg
Force applied to lift the block
F = mg = 25 × 9.8 = 245 N
Weight of the man,
Mg = 50 × 9.8 = 490 N

(a) When the block is raised by the man by applying a force F in the upward direction, reaction being equal and opposite to F will act on the floor in addition to the weight of the man.
$\therefore$ Action on the floor Mg + F = 490 + 245 =735 N

(b) When the block is raised by the man applying force F over the rope (passing over the pulley) in the downward direction, reaction being equal and opposite to F will act on the floor against the weight of the man.
$\therefore$ Action on the floor Mg – F = 490 – 245 = 245N
Since floor yields to a normal force of 700N, mode (b) should be adopted by the man to lift the block.

Example 18: In the figure blocks A, B and C have accelerations a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes 2mg and mg respectively. Find the value of a_1, a_2 and a_3.



Solution:

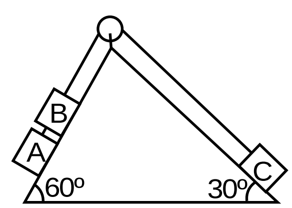
$$a_1 = \frac{2mg - mg}{m} = g; a_2 = \frac{2m - m}{2m + m} g = \frac{g}{3}$$

$$a_3 = \frac{mg + mg - mg}{2m} = \frac{g}{2}$$; Clearly $a_1 > a_3 > a_2$

Example 19: In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively. Find

(a) the acceleration of the system

(b) the tensions in the strings (Neglect friction).(g = 10 m/s^2)



Solution: (a) In this case net pulling force

$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$= (m_A + m_B) g \sin 60° - m_C g \sin 30°$

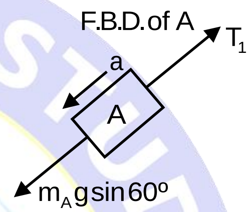
$= (1 + 3) \times 10 \times \frac{\sqrt{3}}{2} - 2 \times 10 \times \frac{1}{2} = 20\sqrt{3} - 10$

$= 20 \times 1.732 - 10 = 24.64$ N

Total mass being pulled = 1 + 3 + 2 = 6 kg

∴ Acceleration of the system a

$= \frac{24.64}{6} = 4.1 \text{ m/s}^2$

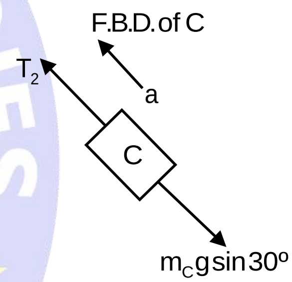


(b) For the tension in the string between A and B.

$m_A \sin 60° - T_1 = (m_A)(a)$

$\therefore T_1 = m_A g \sin 60° - m_A a = m_A (g \sin 60° - a)$

$\therefore (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56\text{N}$



For the tension in the string between B and C.

$T_2 - m_C g \sin 30° = m_C a$

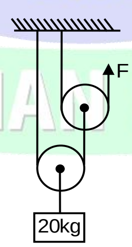
$\therefore T_2 = m_C(a + g \sin 30°)$

$= 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2$ N

Example 20: A student is able to lift a bag containing books of 20 kg-wt by applying a force of 5 kg-wt. Find the mechanical advantage.

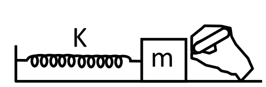
(1) 2 (2) 3 (3) 4 (4) 5

Solution: W = 20 kg-wt F = 5 kg-wt



$\text{M.A.} = \frac{20}{5} = 4$

Example 21: Consider a spring attached to a fixed support at one of its ends and to a box at other end, which rest on a smooth floor as shown in figure. Denote mass of the box by m, force constant of the spring by k and acceleration due to gravity by g. The box is pushed horizontally displacing it by a distance x towards the fixed support and held at rest.

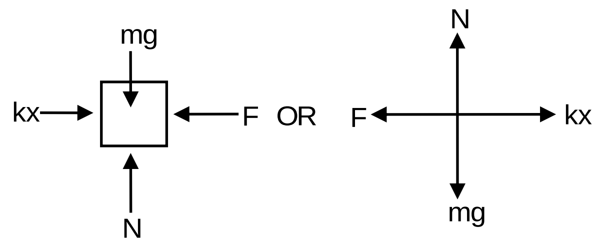


(a) Draw the free body diagram of the box.

(b) Find the force exerted by the hand on the box.

(c) Write all the action-reaction pairs corresponding to Newton's III[rd] law.

Solution: (a)F is push by hand.



(b)Since the block is in equilibrium

$\sum F_x = 0 \Rightarrow F = kx$

(c) (i)Force on box by hand and force on handby box

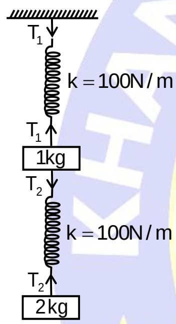
(ii) Force on box by spring and force on springby box.

(iii) Normal reaction by box on floor and normal reaction by floor on box.

(iv) Weight of the box and the gravitational force by which box pulls the earth.

(v) Force by spring on support and force by support on spring.

Example 22: Find extension in both the springs



Solution: Force on 2 kg block

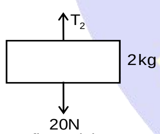


figure (a)

$T_2 - 20 = 0 \Rightarrow T_2 = 20$ N ....(1)

Force on 1 kg block

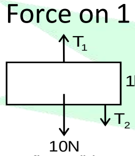


figure (b)

$T_1 = T_2 + 10$ ....(2)

From equation (1) & (2)

$T_1 = 20 + 10 = 30$ N

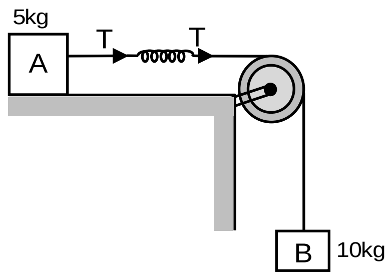
For 1[st] Spring

$T_1 = kx_1 \Rightarrow 30 = 100 \times x_1 \Rightarrow x_1 = 0.3$ m

For 2[nd] Spring

$T_2 = kx_2 \Rightarrow 20 = 100 \times x_2 \Rightarrow x_2 = 0.2$ m

Example 23: In the system shown in figure all surfaces are smooth, string is massless and inextensible. (in steady state) Find the



(a)acceleration of the system

(1) $\frac{20}{3}$ m/s^2 (2) $\frac{22}{5}$ m/s^2 (3) $\frac{3}{20}$ m/s^2 (4) $\frac{5}{22}$ m/s^2

(b)tension in the string and

(1) $\frac{100}{3}$ N (2) $\frac{50}{3}$ N (3) $\frac{25}{3}$ N (4) $\frac{40}{3}$ N

(c)extension in the spring if force constant of spring is k = 50 N/m (Take g = 10 m/s^2)

(1) $\frac{2}{3}$ m (2) $\frac{3}{4}$ m (3) $\frac{5}{2}$ m (4) $\frac{3}{2}$ m

Solution: (a) $100 - T = 10a$ ...(i)

$T = 5a$ ...(ii)

From (i) & (ii)

$$a = \frac{100}{15} = \frac{20}{3}\ \text{m/s}^2$$

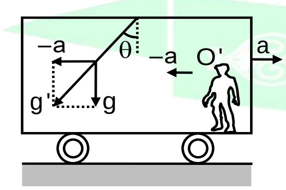
(b) From eqn. (ii)

$$T = 5 \times \frac{20}{3} = \frac{100}{3}\text{N}$$

(c) $T = Kx$

$$x = \frac{100}{3 \times 50} = \frac{2}{3}\ \text{m}$$

Example 24: A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in the equilibrium position. Also calculate tension in the string.



(1) $\tan^{-1}\left(\frac{a}{g}\right)$, $m\sqrt{a^2 + g^2}$ (2) $\sin^{-1}\left(\frac{a}{g}\right)$, $m\sqrt{a^2 - g^2}$

(3) $\cos^{-1}\left(\frac{a}{g}\right)$, $m\sqrt{a^2 + g^2}$ (4) None of these

Solution: With respect to train, the bob is in equilibrium

$\therefore\ \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ ...(1)

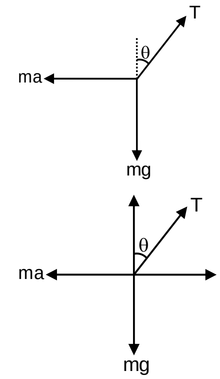
and $\Sigma F_x = 0 \Rightarrow T\sin\theta = ma$ ...(2)

Dividing eqns. (2) / (1), $\tan\theta = \frac{a}{g}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$

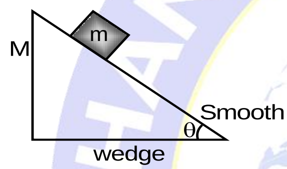
Squaring and adding eqns. (1) and (2)

$T = m\sqrt{a^2 + g^2}$



Example25: What horizontal acceleration should be provided to the wedge so that the block of mass m kept on wedge remains at rest w.r.t. wedge?

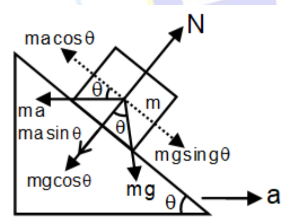
(1) $g\sin\theta$ (2) $g\cos\theta$ (3) $g\cot\theta$ (4) $g\tan\theta$



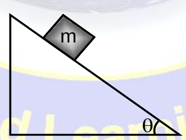
Solution: For equilibrium along wedge

$ma\cos\theta = mg\sin\theta$

$\Rightarrow a = g\frac{\sin\theta}{\cos\theta} \Rightarrow \boxed{a = g\tan\theta}$



Example 26: What horizontal acceleration should be provided to the wedge so that the block of mass m placed on the wedge falls freely?

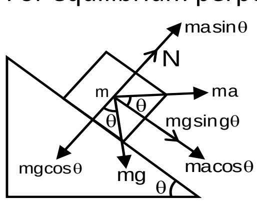


(1) $g\tan\theta$ (2) $\frac{g}{\tan\theta}$ (3) $g\cos\theta$ (4) None of these

Solution: For free fall, normal reaction should be zero

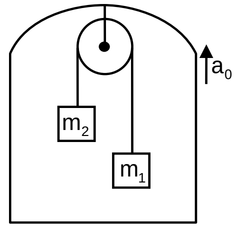
N = 0

For equilibrium perpendicular to the wedge



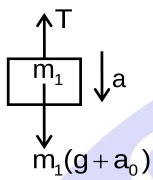
$0 + ma\sin\theta = mg\cos\theta \Rightarrow \boxed{a = \frac{g}{\tan\theta}}$

Example 27: Determine the acceleration of the masses and tension in the string if the whole system is moving vertically upwards with uniform acceleration a_0. ($m_1 > m_2$)

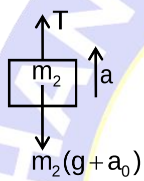


Solution: In the system

(i) For a mass m_1



$m_1(g + a_0) - T = m_1 a$ ...(i)



(ii) For a mass m_2

$T - m_2(g + a_0) = m_2 a$

$$a = \frac{(m_1 - m_2)(g + a_0)}{m_1 + m_2}$$

$$T = \frac{2m_1 m_2}{m_1 + m_2}(g + a_0)$$

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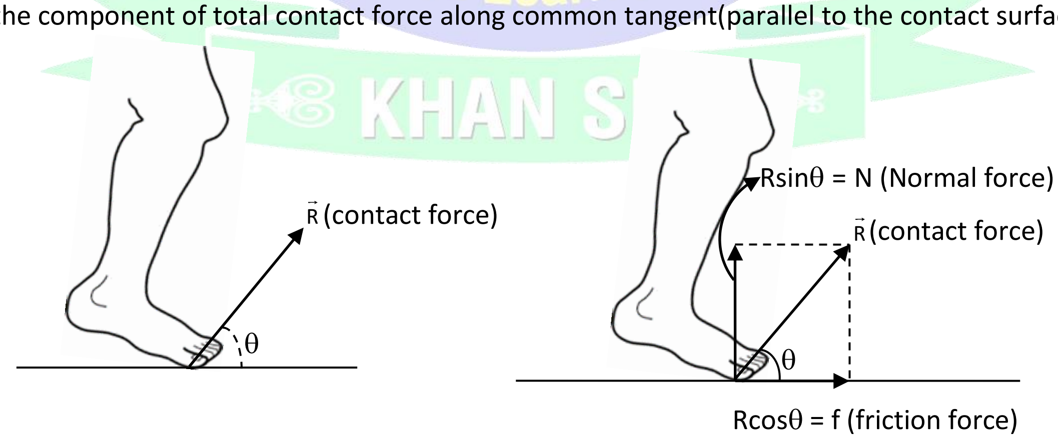
9. FRICTION

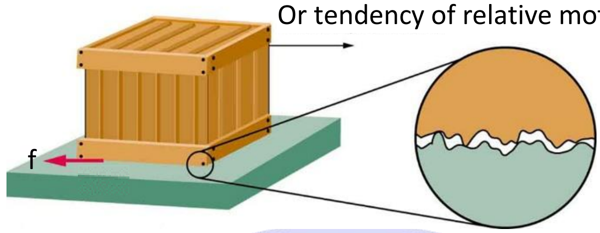
➢ It is the component of total contact force along common tangent(parallel to the contact surface).



$$\vec{R} = \vec{N} + \vec{f}$$

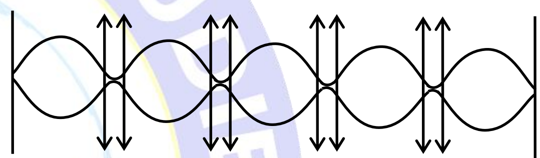
➢ Friction force always opposes the relative motion or tendency of relative motion between the two contacting surfaces.

Old view : When two bodies are in contact with each other, the irregularities in the surface of one body get interlocked with the irregularities in the surface of the other. This interlocking opposes the tendency of relative motion.



Modern View :

Friction arises on account of strong inter atomic or inter molecular forces of attraction between the two surfaces at the point of actual contact.

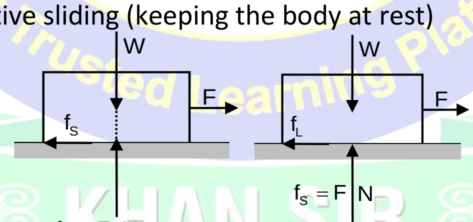


SPOT LIGHT

- For friction to be present both contacting surfaces to be rough. If either of them is smooth, friction force will be absent $\left(\vec{f}=0\right)$ and in that case $\vec{R}=\vec{N}$.

9.1 Static friction:

It a opposes tendency of relative sliding (keeping the body at rest)



- It is the frictional force which is effective before relative motion begins between two surface in contact with each other.
- It's nature is self adjusting (in direction and magnitude upto certain limit)
- Numerical value of static friction is equal to the tangential external force tendency to generate the relative motion of the body.
- Maximum value of static friction is called limiting friction.

9.1.1 Limiting Friction :

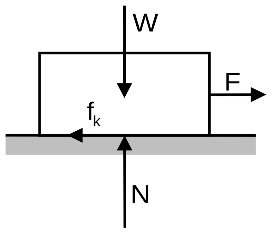
- The magnitude of limiting friction (f_L) is directly proportional to the normal reaction (N).

  $f_L \propto N \Rightarrow f_L = \mu_s N$
- The direction of limiting friction is opposite to the direction in which the body is on the verge of moving over the other.

- The force of limiting friction is independent of the apparent contact area, as long as normal reaction between the two bodies in contact remains the same.
- Limiting friction depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Its value is more than other types of friction force.

9.2 Kinetic/dynamic/Sliding friction :

The body is in relative motion, the friction opposing its relative motion is called dynamic or kinetic friction.



Kinetic friction

9.2.1 Laws of Kinetic Friction

- This is always slightly less than the limiting friction
- It depends on N.
- Its value does not depend on types of motion of body such as accelerated motion, retarded motion or moving with constant velocity because it is a constant friction.
- Its numerical values is $f_k = \mu_k N$, where μ_k= coefficient of kinetic friction.

9.3 Coefficient of friction :

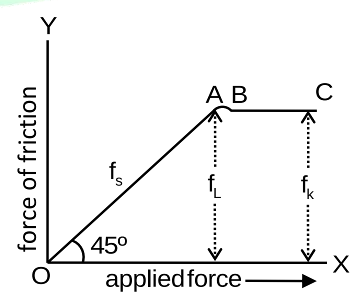
$\mu_s = \frac{f_L}{N}$ $\qquad$ $\mu_k = \frac{f_k}{N}$

- The values of μ_s and μ_k depend on the nature of both the surfaces in contact.
- The values of μ depend on material of the surfaces in contact.
- μ_k and μ_s are dimensionless and unitless.

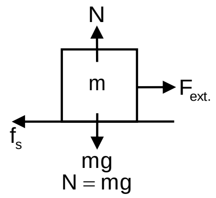
9.4 Graph Between Applied Force and Force of Friction

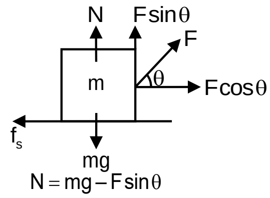
- The part OA of the curve represents static friction (f_s) which goes on increasing, with the applied force. At A, the static friction is maximum. This represents the limiting friction. Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve, therefore, represents the kinetic friction (f_k).

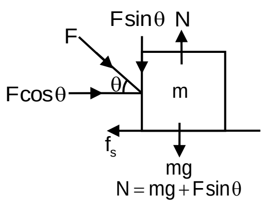
  As the portion BC of the curve is parallel to OX, therefore, kinetic friction does not change with the applied force. It remains constant whatever be the applied force.

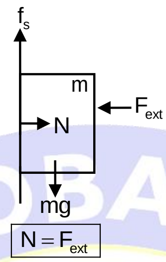


9.5 Normal reaction for some special case :

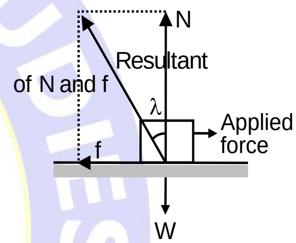








9.6 Angle of friction (λ) :



➢ The angle which the resultant of the force of limiting friction f_L and normal reaction N makes with the direction of normal reaction N.

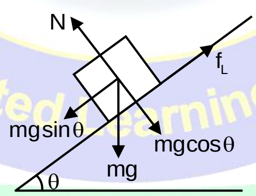
$$\tan\lambda = \frac{f_L}{N} = \mu \Rightarrow \lambda = \tan^{-1}\mu$$

Net contact force = $\sqrt{f_L^2 + N^2} = N\sqrt{\mu^2 + 1}$

9.7 ANGLE OF REPOSE OR ANGLE OF SLIDING

➢ It is defined as the minimum angle of inclination of a plane with the horizontal at which a body placed on it just beings to slide down or equivalently the maximum angle of inclination of plane with the horizontal at which a body placed on it does not slide.

$f_L = mg\sin\theta$ ....(i) and $N = mg\cos\theta$ ...(ii)



Dividing (i) by (ii)

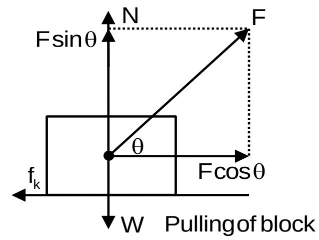
So $\mu_s = \frac{f_L}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta \Rightarrow \theta = \tan^{-1}\mu_s$

This fact is used for finding the coefficient of static friction in the laboratory.

DETECTIVE MIND

Angle of repose (θ) = Angle of friction (λ)

9.8 PULLING IS EASIER THAN PUSHING:



Case of pulling:

Force F is applied to pull a block of weight W.

F can be resolved into two rectangular components:

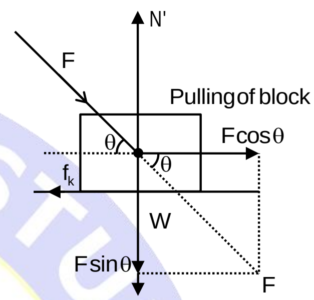
F cosθ and F sinθ the normal reaction

N = W – F sin θ

Force of kinetic friction

$f_k = \mu_k N \quad f_k = \mu_k\,(W - F\sin\theta)$ ......(i)

Case of pushing:



Force F is applied to push a block normal reaction N′ = W + F sinθ

Force of kinetic friction $f_k' = \mu_k N'$

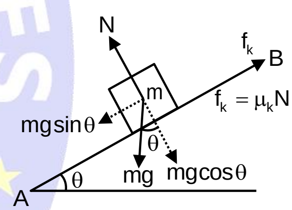
or $f_k' = \mu_k (W + F\sin\theta)$ .....(ii)

from (i) and (ii) $f'_k > f_k$

The opposing frictional force is more in the case of push.

Hence, it is easier to pull than to push a body.

9.9 ACCELERATION OF A BODY DOWN ON A ROUGH INCLINED PLANE



➢ If angle of inclination is greater than the angle of repose, then the body accelerates down the incline.

Net force on the body down the inclined plane.

$F_{net} = mg\sin\theta - f_k$

applying Newton's second law of motion

$ma = mg\,[\sin\theta - \mu_k\cos\theta]$

hence, a < g acceleration of a body down a rough inclined plane is always less than 'g'

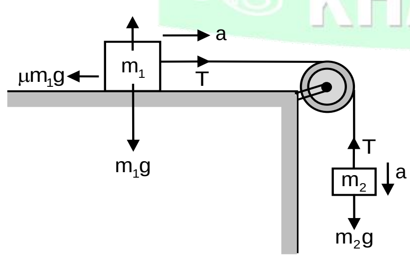
SPOT LIGHT

➢ If we want to prevent the downward slipping of body then minimum upward force required is $= mg\sin\theta - \mu_k\, mg\cos\theta$.

➢ If a body is projected in upward direction along the inclined plane then retardation of body is $a = (\sin\theta - \mu_k\cos\theta)$ retardation of a body up a rough inclined place may be greater then 'g'

10. PULLEY WITH FRICTION BETWEEN BLOCK AND SURFACE :

Case-I :



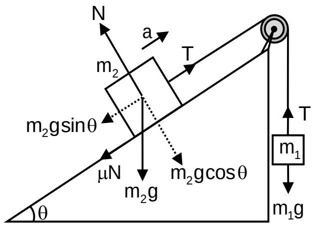
For mass $m_1 : T - \mu m_1 g = m_1 a$

For mass $m_2 : m_2 g - T = m_2 a$

Acceleration $a = \dfrac{(m_2 - \mu m_1)g}{(m_1 + m_2)}$

$$\Rightarrow T = \frac{m_1 m_2 (1+\mu)g}{(m_1 + m_2)}$$

Case II :



For mass m_1 : $m_1 g - T = m_1 a$ and $N = m_2 g \cos\theta$

$\Rightarrow \mu N = \mu m_2 g \cos\theta$

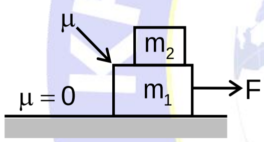
For mass on solving,

Acceleration $a = \left[\frac{m_1 - m_2(\sin\theta + \mu\cos\theta)}{m_1 + m_2}\right] g$

Tension $T = \frac{m_1 m_2 (1 + \sin\theta + \mu\cos\theta) g}{(m_1 + m_2)}$

10.1 TWO BLOCKS SYSTEM IN FRICTION

Case-I: As shown in figure what should be maximum value of F so that both blocks are moving together.



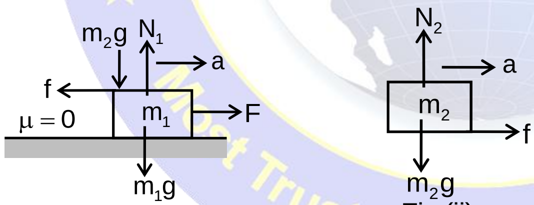


Fig.(i) Fig.(ii)

From figure (i)

$F - f = m_1 a \Rightarrow F = f + m_1 a$ ...(i)

From figure (ii) $\Rightarrow f = m_2 a$ ...(ii)

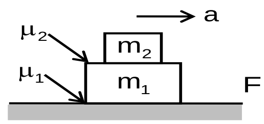
$\Rightarrow f = \mu N_2 \Rightarrow f = \mu m_2 g$ ...(iii)

From (ii) & (iii) $a = \mu g$

Putting value of f and a in equation (i)

$F = \mu m_2 g + m_1 \mu g = \mu g(m_1 + m_2)$

Case-II: When both surface are rough



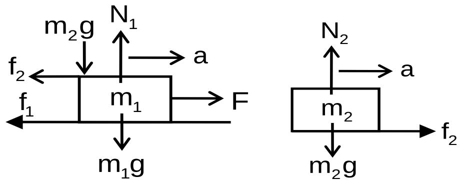


Fig.(i) Fig.(ii)

from fig.(i) $F - f_1 - f_2 = m_1a$

$F = f_1 + f_2 + m_1a$ ...(i)

$f_1 = \mu_1N_1$

$\Rightarrow f_1 = \mu_1(m_1 + m_2)g$ ...(ii)

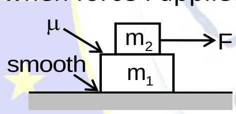
from figure. (ii) $f_2 = m_2a$

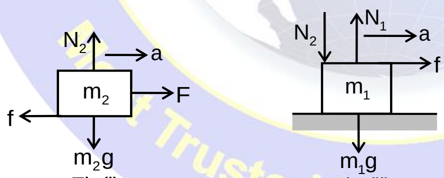
$\mu_2 m_2g = m_2a$

$$\boxed{a = \mu_2 g}$$ ...(iii)

$$\boxed{F = (\mu_1 + \mu_2)(m_1 + m_2)g}$$

Case-III: when force f applied on upper block





Fig(i) Fig.(ii)

from figure (i)

$F - f = m_2a$ ....(i)

$f = \mu N_2 = \mu m_2g$ ...(ii)

from figure (ii)

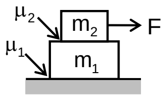
$f = m_1a \Rightarrow \mu m_2g = m_1a$

$$\boxed{a = \frac{\mu m_2}{m_1}g}$$ ...(iii)

put the value of f and a in equation (i)

$$\boxed{F = (m_1 + m_2)\frac{\mu m_2 g}{m_1}}$$

Case-IV: when both surface are rough



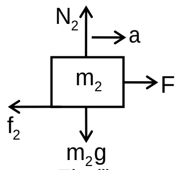


Fig.(i)

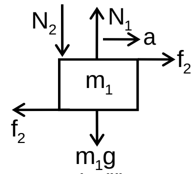


Fig.(ii)

From figure (i)

$F - f_2 = m_2a$ ...(i)

$f_2 = \mu_2 N_2 \Rightarrow f_2 = \mu_2 m_2 g$ ...(ii)

From figure (ii)

$f_1 = \mu_1(m_1 + m_2)g$

$f_2 - f_1 = m_1a$ ...(iii)

Put value of f_1 and f_2 in eqn. ...(iii)

$$a = \frac{(\mu_2 m_2 - \mu_1 m_1 - \mu_1 m_2)g}{m_1} \quad ...(iv)$$

Put value of f_1, f_2 and a in eqn ...(i)

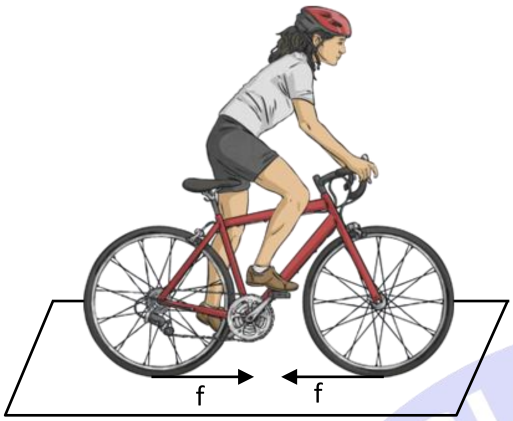
$$F = \frac{m_2 g(m_1 + m_2)(\mu_2 - \mu_1)}{m_1}$$

SPOT LIGHT

- As increases smoothness of two contacting surfaces, friction force decreases but if smoothness increases too much then the friction increases up to infinite hence, the value of coefficient of friction (μ_s or μ_k) ranges from zero to infinite.

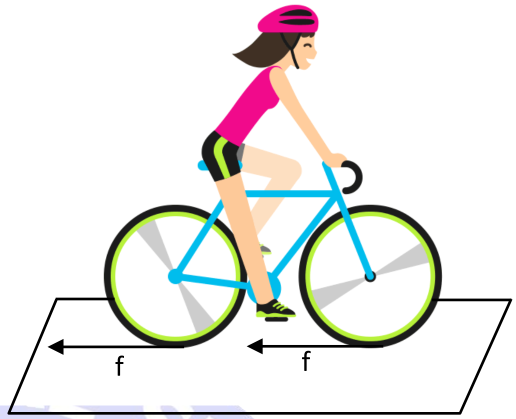
DIRECTION OF FRICTION IN CYCLING

When rear wheel moves by the force applying to it by pedaling.



While pedaling point of contact of rear wheel with the surface tends to move backward w.r.t. the surface so friction acts forward on the rear wheel and point of contact of front wheel tends to move forward w.r.t the surface so friction acts backward on the front wheel.

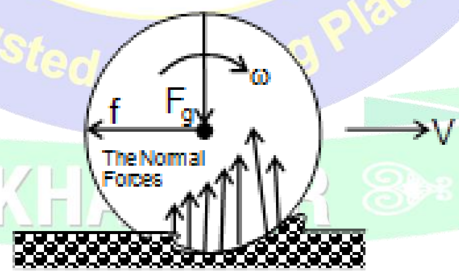
When rear wheel moves by the force applying to it by pedaling is stopped.



While without pedaling point of contact of rear wheel with the surface tends to move forward w.r.t. the surface so friction acts backward on the rear wheel and point of contact of front wheel tends to move forward w.r.t the surface so friction acts backward on the front wheel.

SPOT LIGHT

- If the brake is applied in such a way that, both front and rear wheel start slipping, in that case friction act backward on front and rear wheel.
- When a body rolls on a surface, the resistance offered by the surface is called rolling friction.



10.2 METHODS OF REDUCING FRICTION :

- By polishing the surface. (But extreme polishing increase friction)
- By lubrication
- By proper selection of materials
- By avoiding moisture
- By use of alloys
- By streamlining the shape
- By using ball bearings or roller bearings

10.3 ADVANTAGES AND DISADVANTAGES OF FRICTION:

Disadvantages

(i) A significant amount of energy of moving objects is wasted in the form of heat energy to overcome the force of friction.

(ii) Friction restricts the speed of moving vehicles like buses, trains, aeroplane, rockets etc.

(iii) The efficiency of machines decreases due to the presence of force of friction.

(iv) Friction causes lot of wear and tear in the moving parts of a machine.

(v) Sometimes, machines get burnt due to the friction between different moving parts.

Advantages

(i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think walking on the surface. That is why, we fall down while moving on a smooth surface.

(ii) The force of friction between the tip of pen and the surface of paper helps us to write on the paper. It is not possible to write on a glazed paper as there is no force of friction.

(iii) The force of friction between the tyre of a vehicle and the road aids the vehicle to stop when brakes are applied. In the absence of friction, the vehicle skids off the road after brakes are applied.

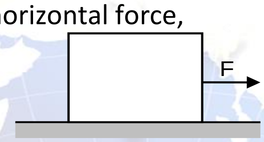
(iv) Moving belts remain on the rim of a wheel because of friction.

(v) The force of friction between a chalk and a black board helps us to write on the board. Thus, we observe that in spite of various disadvantages associated with friction, it is very difficult to part with it. Hence, it is commonly said that, friction is a necessary evil.

SOLVED EXAMPLES

Example 28: A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15.

Find the frictional force if a horizontal force,



(a) F = 1N (b) F = 1.96 N (c) F = 2.5 N is applied on the block

Solution: Maximum force of friction

$f_{max} = 0.2 \times 1 \times 9.8N = 1.96$ N

(a) For $F_{ext} = 1$ N

$F_{ext} < f_{max}$

So, body is at rest which implies that static friction is present and hence $f_s = F_{ext} = 1$ N

(b) For $F_{ext} = 1.96$ N

$F_{ext} = f_{max} = 1.96$ N

So, f = 1.96 N

(c) For $F_{ext} = 2.5$ N

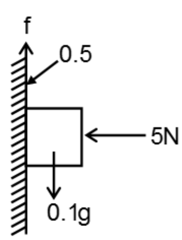
so $F_{ext} > f_{max.}$

Now the body is in motion

$\therefore f_{max.} = f_k = \mu_k N = \mu_k mg = 0.15 \times 1 \times 9.8 = 1.47$ N

Example 29: A body of mass 0.1 kg is pressed against a wall with a horizontal force 5 N. If coefficient of friction is 0.5 then find force of friction.

(1) 4 N (2) 3 N (3) 2 N (4) 1 N



Solution: $F_{app} < f_L$

$\therefore$ force of friction

f = mg

f = 0.1 × 10

f = 1 N

Example30: A horizontal force of 49 N is just able to move a block of wood weighing 10 kg on a rough horizontal surface. Calculate the coefficient of friction and angle of friction.

(1) 0.3, $\tan^{-1}$ (0.3) (2) 0.2, $\tan^{-1}$ (0.3) (3) 0.5, $\tan^{-1}$ (0.5) (4) 0.6, $\tan^{-1}$ (0.6)

Solution: Here, F = 49 N, N = W = mg = 10 × 9.8N

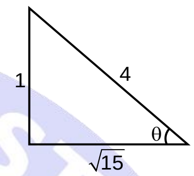
so $\mu = \frac{F}{N_s} = \frac{49}{10 \times 9.8} = 0.5$

$\Rightarrow \tan\theta = \mu = 0.5 \Rightarrow$ Angle of friction $\theta = \tan^{-1}(0.5)$

Example 31: A body placed on a rough inclined plane just begins to slide. Calculate the coefficient of friction if the gradient (slope) of the plane is 1 in 4.

(1) 0.653 (2) 0.329 (3) 0.138 (4) 0.258

Solution: Here $\sin\theta = \frac{1}{4}$



So, $\tan\theta = \frac{1}{\sqrt{15}}$

$\mu = \tan\theta = \frac{1}{\sqrt{15}} = 0.258$

Example32: A block of mass 2 kg slides down an inclined plane which makes an angle 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.

(i) What force must be applied to the block so that it moves down the plane without acceleration?

(ii) What force should be applied to the block so that it moves up without any acceleration?

Solution: Make a 'free-body' diagram of the block. Take the force of friction opposite to the direction of motion.

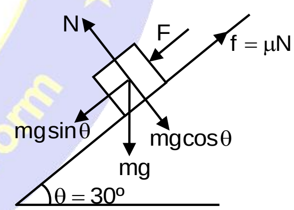
(i) Project forces along and perpendicular to the plane. Perpendicular to plane N = mg cos θ.

Along the plane F + mg sin θ – f = 0

(there is no acceleration along the plane)

F + mg sin θ– μN = 0

$\Rightarrow$ F + mg sinθ = μ mg cos θ



$F = mg(\mu\cos\theta - \sin\theta) = 2 \times 9.8(\frac{\sqrt{3}}{2}\cos 30º - \sin 30º)$

$= 19.6(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2}) = 19.6(\frac{3}{4} - \frac{1}{2}) = 4.9N$

(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

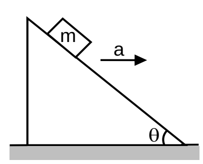
$\therefore N = mg\cos\theta;\ F = mg\sin\theta + f$

$\Rightarrow$ F = mg(μ cos θ + sinθ) = $19.6\left(\frac{3}{4} + \frac{1}{4}\right)$

= 24.5 N

Example 33: A block of mass 1 kg rests on an incline as shown in figure.

(a) What must be the frictional force between the block and the incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s^2 ?



(1) 3.48 N (2) 4.54 N (3) 5.20 N (4) 5.60 N

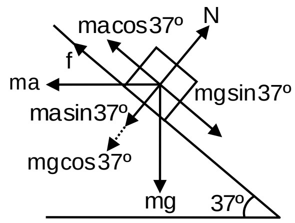
(b) What is the least value of μ_s which can have for this to happen?

(1) 0.20 N (2) 0.36 N (3) 0.46 N (4) 0.56 N

Solution: N = m(gcos37° + a sin37°)

= 1(9.8 × 0.8 + 3 × 0.6) = 9.64 N

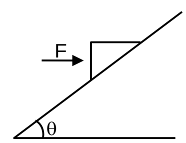
mg sin 37° = ma cos 37° + f



(a) f = 1 (9.8 × 0.6 – 3 × 0.8) = 3.48 N

(b) $\therefore$ f = μN, $\therefore \mu = \frac{f}{N} = \frac{3.48}{9.64} = 0.36$

Example 34: A block rests on a rough inclined plane as shown in fig. A horizontal force F is applied to it



(a) Find the force of normal reaction,

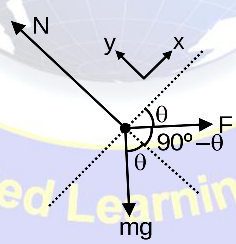
(b) can the force of friction be zero, if yes when ? and

(c) Assuming that friction is not zero find its magnitude and direction of its limiting value.

Solution: (a) $\sum F_y = 0 \rightarrow N = mg\cos\theta + F\sin\theta$

(b) $\sum F_x = 0 \rightarrow F\cos\theta = mg\sin\theta$

$\Rightarrow F = mg\tan\theta$



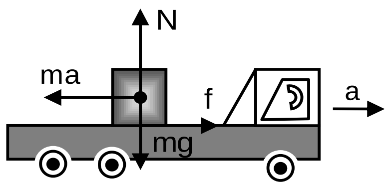
(c) Limiting friction $f_{sm} = \mu N = \mu(mg\cos\theta + F\sin\theta)$;

It acts down the plane if body has tendency to slide up and acts up the plane if body has tendency to slide down.

Example 35: A block of mass 1 kg lies on a horizontal surface of a truck. The coefficient of static friction between the block and the surface is 0.6. What is the force of friction on the block if the acceleration of the truck is 5 m/s^2.

Solution: Fictitious force (pseudo force) on the block opposite to the acceleration of the block

F = ma = 1 × 5 = 5 N



While the limiting friction force

$f_L = \mu_s N = \mu_s mg$

= 0.6 × 1 × 9.8 = 5.88 newton

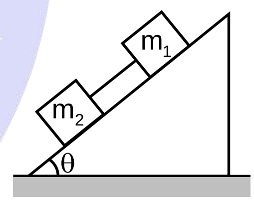
As applied force F is less than the limiting friction force, the block will remain at rest in the truck and the force of friction will be equal to 5 N and in the direction of acceleration of the truck.

Example 36: Two block with masses m_1 = 1kg and m_2 = 2 kg connected by a string, slide down an plane inclined at an angle θ = 45° with the horizontal. The coefficient of sliding friction between m_1 and plane is μ_1 = 0.4 and that between m_2 and plane is μ_2 = 0.2. Calculate the common acceleration of the two blocks and the tension in the string.

(1) $\frac{22}{2\sqrt{3}}$ m/s², $\frac{3}{2\sqrt{2}}$ N (2) $\frac{22}{3\sqrt{2}}$ m/s², $\frac{4}{3\sqrt{2}}$ N

(3) $\frac{22}{\sqrt{2}}$ m/s², $\frac{4}{5\sqrt{2}}$ N (4) $\frac{20}{3\sqrt{2}}$ m/s², $\frac{4}{3\sqrt{2}}$ N

Solution: As $\mu_2 < \mu_1$ block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But since they are connected they move together as a system with common acceleration.



So acceleration of the blocks is :

$$a = \frac{(m_1 + m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1 + m_2}$$

$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4 \times 1 \times 10 \times \frac{1}{\sqrt{2}} - 0.2 \times 2 \times 10 \times \frac{1}{\sqrt{2}}}{1+2}$$

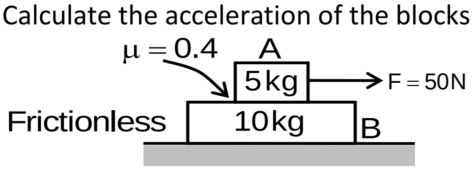
$$= \frac{22}{3\sqrt{2}} m/s^2$$

For block of mass m_2 :

$\mu_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a \Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$ = 2 × 10 ×

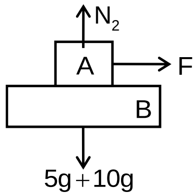
$$\frac{1}{\sqrt{2}} - 0.2 \times 2 \times 10 \times \frac{1}{\sqrt{2}} - 2 \times \frac{22}{3\sqrt{2}} = \frac{4}{3\sqrt{2}} N$$

Example 37: Calculate the acceleration of the blocks and the force of friction between them.



Solution: **Step-1:** Draw FBD of the combined blocks system.

Obviously, there is an unbalanced force (F=50N) in the horizontal direction.



$\therefore$ Movement occurs.

Step 2: Assuming that the blocks move together with common acceleration draw the FBD of block B (on which external force is not applied)

$$a_c = \frac{50}{5+10} = 3.33 m/s^2$$

Required frictional force f = $m_B a_c$

= 10 × 3.33 = 33.3 N

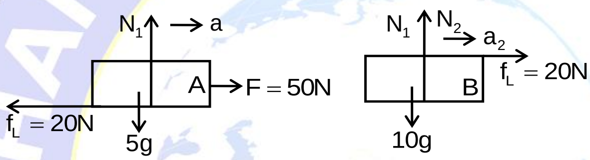
Now limiting friction (maximum available static friction)

$f_L = \mu N_1 = 0.4 \times 5 \times 10 = 20$ N

Obviously, f exceeds f_L.

The two blocks move separately.

Step 3: Draw the individual FBD's of the two blocks



Applying Newton's II law, in the horizontal direction

$$a_1 = \frac{50-20}{5} = 6 m/s^2 \text{ and } a_2 = \frac{20}{10} = 2 m/s^2$$

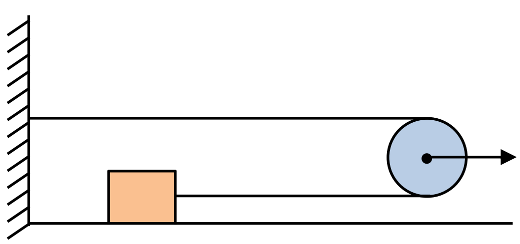
Force of friction between the blocks = 20 N

11. CONSTRAINT MOTION:

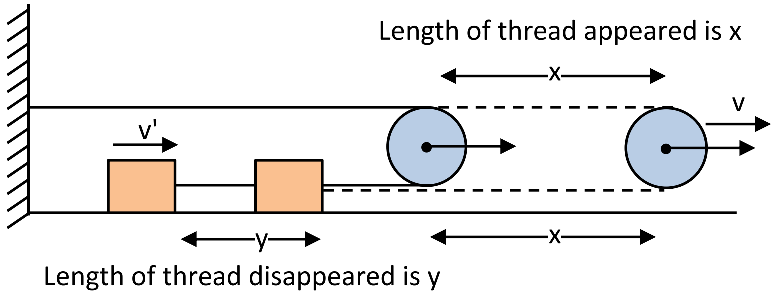
There are some constraints which do not change, for example length of an inextensible thread is constant and distance between two point on the rigid rod using such facts we can derive the relation among displacement velocity and acceleration of the movable parts of the system such relation are called constraint relation.

11.1 VISUAL ANALYSIS OF THREAD CONSTRAINT:

Example: Find the displacement and velocity of block if pully is displaced by x rightward with velocity v.



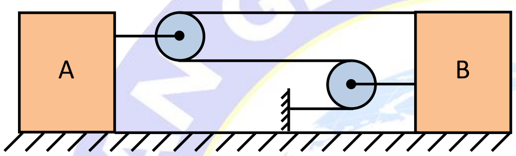
Solution:



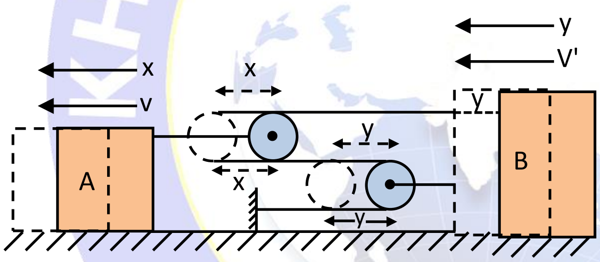
Length of thread appeared = Length of thread disappeared

$$y = 2x \Rightarrow \frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow v' = 2v$$

Example: Find the displacement of B and when A is displaced by x leftwards.

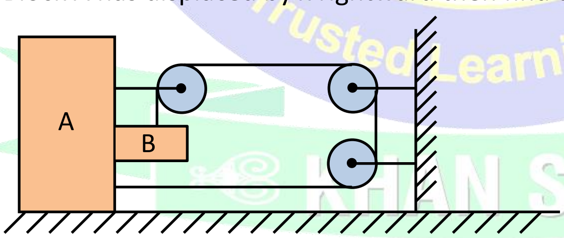


Solution:

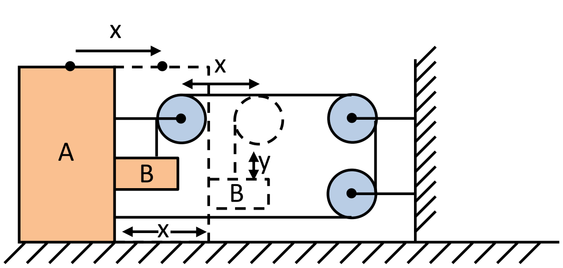


$$3y = 2x \Rightarrow y = \frac{2x}{3} \Rightarrow \frac{dy}{dt} = \frac{2}{3}\frac{dx}{dt} \Rightarrow v' = \frac{2}{3}v$$

Example: Block A has displaced by x rightward then find the displacement of block B.



Solution:



Horizontal displacement of block B, S_h = x

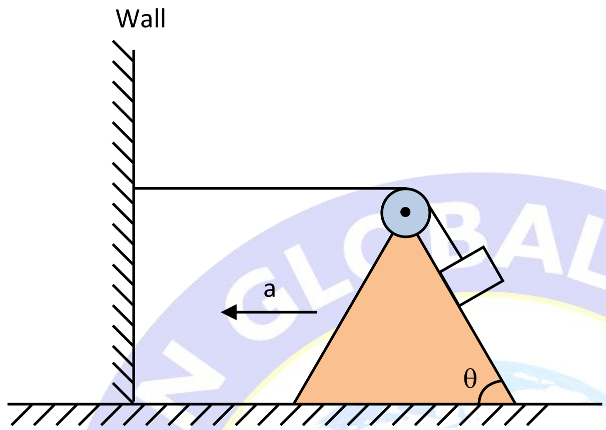
Vertical displacement of block B, $S_v = y$

Length of thread appeared = Length of thread disappeared

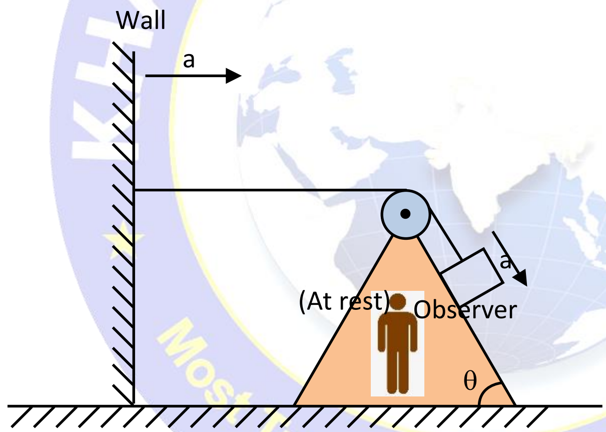
$y = 2x$

Now, net displacement of the block B = $\sqrt{s_v^2 + s_h^2} = \sqrt{2x^2 + x^2} = \sqrt{5}x$

Example: Find acceleration of block w.r.t. ground acceleration of wedge is a leftward.



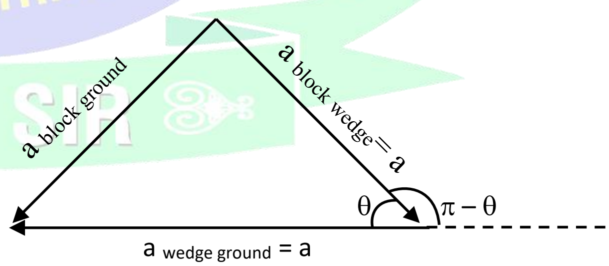
Solution:



Acceleration of wall w.r.t. wedge = a (rightward)

i.e. wall is accelerating with acceleration a rightwardand wedge (observer) is at rest

i.e. $a_{\text{block wedge}} = a$



$\vec{a}_{\text{block ground}} = \vec{a}_{\text{block wedge}} + \vec{a}_{\text{wedge ground}}$

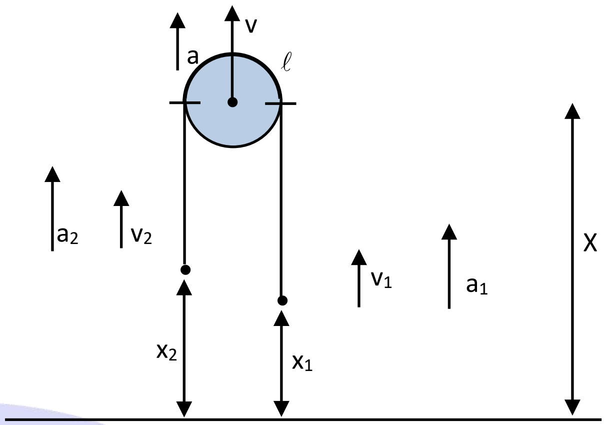
$\left|\vec{a}_{\text{block ground}}\right| = \sqrt{a^2 + a^2 + 2a^2\cos(\pi - \theta)}$

$\sqrt{2a^2 - 2a^2\cos\theta}$

$= 2a\sin\dfrac{\theta}{2}$

11.2 MATHEMATICAL ANALYSIS OF THREAD CONSTRAINT:

(a) Choose a point or line which is fixed (which is not moveable).



(b) Show all distance of different moveable parts or the system.

(c) Now using thread constraint established the relation betweendistance of moveable parts of the system and after differentiatingthis relation w.r.t. time we can get relation among velocities ofthe moveable parts and again differentiating this relation w.r.t. time we can get the relation among the acceleration of the moveable parts of the system.

Sign convention

$x - x_1 + x - x_2 + \ell$ (length of pulley)= total length of thread

$x - x_1 + x - x_2$ = constant

$2x - x_1 - x_2$ = constant ..... (1)

Now differentiating the equation (1) w.r.t. time

$$\frac{2dx}{dt} - \frac{dx_1}{dt} - \frac{dx_2}{dt} = 0$$

$2v - v_1 - v_2 = 0$ ..... (2)

$$\boxed{v = \frac{v_1 + v_2}{2}}$$

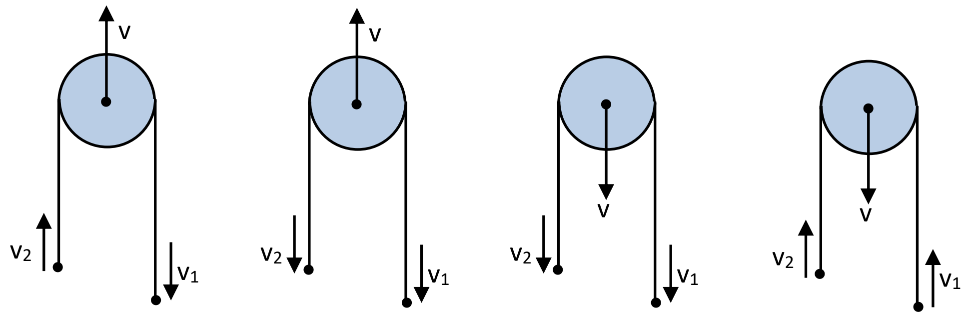
Again differentiating equation (2) we can get the relation between acceleration of the moveable parts.

$$\frac{2dv}{dt} - \frac{dv_1}{dt} - \frac{dv_2}{dt} = 0 \Rightarrow 2a - a_1 - a_2 = 0 \Rightarrow \boxed{a = \frac{a_1 + a_2}{2}}$$

SPOT LIGHT

- $v = \frac{v_1 + v_2}{2}$ are applicable when velocity of both end of the thread and pulley are in same direction and if these velocities are not in same direction then make them in same direction by using negative signs. Also apply the same for acceleration.

Example:



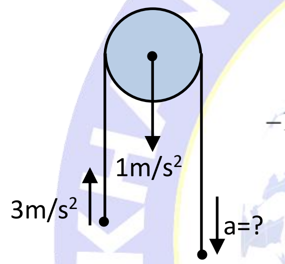
$$v = \frac{v_2 - v_1}{2}$$

$$v = \frac{-v_2 - v_1}{2}$$

$$-v = \frac{-v_2 - v_1}{2}$$
or
$$v = \frac{v_1 + v_2}{2}$$

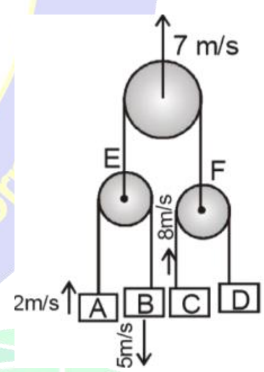
$$-v = \frac{v_2 + v_1}{2}$$
or
$$-v = \frac{v_1 + v_2}{2}$$

Example:



$$-1 = \frac{3-a}{2} \Rightarrow a = 5m/s^2$$

Example: Find out the velocity of block D



Solution: from 2nd format of constrained motion

$$v_E = \frac{v_A + v_B}{2}$$

$$v_E = \frac{2-5}{2} = -\frac{3}{2}$$ (If upward direction is taken to be positive)

$$v_E = -\frac{3}{2} m/s$$

Now $\frac{v_E + v_D}{2} = v_F \Rightarrow \frac{8 + v_D}{2} = \frac{31}{2} \Rightarrow v_D = 31 - 8$

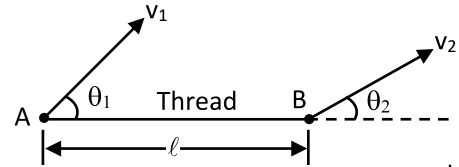
v_D = 23 m/s (upward direction)

DETECTIVE MIND

If the thread is inextensible and taut then the component of velocities of any two points on the thread along the thread are equal.

Explanation:

$\frac{d\ell}{dt} \Rightarrow \frac{d}{dt}$ (distance between the point A and B) = Rate of change of distance between points A and B w.r.t. time $\frac{d\ell}{dt}$ is not the speed but also it is rate of increase or decrease of distance between points A and BSo, for the component of velocity which increases the distance (ℓ), $\frac{d\ell}{dt}$ = positive = $v_2 \cos\theta_2$And for the component of velocity which decreases the distance

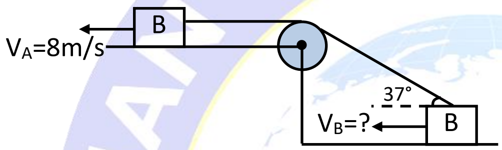


$(\ell), \frac{d\ell}{dt}$ = negative = $-v_1 \cos\theta_1$ Now, $\frac{d\ell}{dt} = v_2 \cos\theta_2 - v_1 \cos\theta_1$

Since thread is taut it means ℓ is constant i.e. $\frac{d\ell}{dt} = 0 \Rightarrow v_2 \cos\theta_2 - v_1 \cos\theta_1 = 0 \Rightarrow v_2 \cos\theta_2 = v_1 \cos\theta_1$

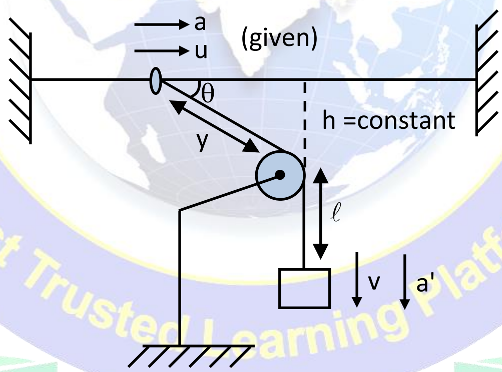
i.e. in a taut thread component of velocity of any two points of thread on the thread along the thread is same.

Example:



Solution: $V_B \cos 37° = 8 \Rightarrow V_B = \frac{8}{\cos 37°} = 10 m/s$

Example: Find velocity v' and acceleration a' of the block.



Solution: For velocity v = u cosθ

For acceleration differentiating v' = u cosθ we get acceleration of block

$$\frac{dv}{dt} = u\sin\theta \frac{d\theta}{dt} + \cos\theta \frac{du}{dt} \Rightarrow a' = u\sin\theta \frac{d\theta}{dt} + \cos\theta \frac{du}{dt}$$

But a' ≠ a cosθ as here θ is variable

SPOT LIGHT

- If thread is inextensible and taut then component of acceleration of any two points on the thread along the thread is same only if thread is not rotating.

Rigid Body :

If the distance between any two points on the body does not change then the body is said to be rigid body Example: Rod is Rigid Body, chain is not a rigid body

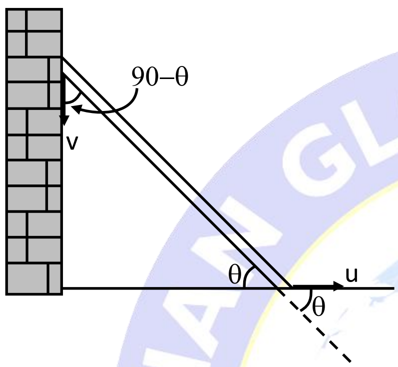
11.3 Rod Constraint:

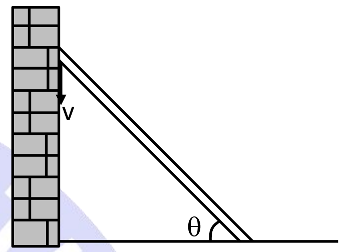
Components of velocities of any two points on the rod along the rod is same

Example: Velocity of point of contact of the rod leaning to the wall is v as on in figurethen find the velocity of point of contact of the rod on the horizontal surface.

Solution: Component of velocity of the two end of the rod along the rod is same v cos (90°–θ) = u cosθ⇒ vsinθ = u cosθ⇒ u = vtanθ

$$v\cos(90^\circ-\theta) = u\cos\theta \Rightarrow v\sin\theta = u\cos\theta \Rightarrow u = v\tan\theta$$



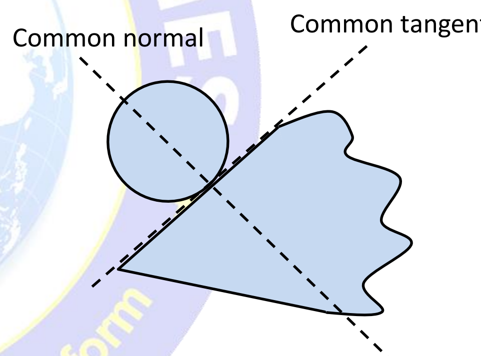


11.4 WEDGE CONSTRAINT:

If two bodies are in contact then components of their velocities at thepoint of contact along common normal is same provided there is not deformation.

Common tangent:A tangent common for both bodies

Common normal:A straight line perpendicular to common tangent



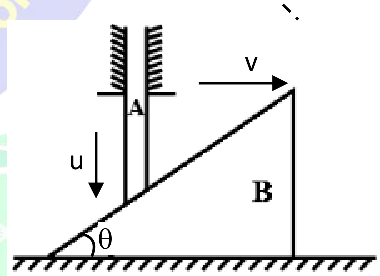
Example: Find the relation between velocity of rod A and that of the wedge B at any instant in the figure shown.

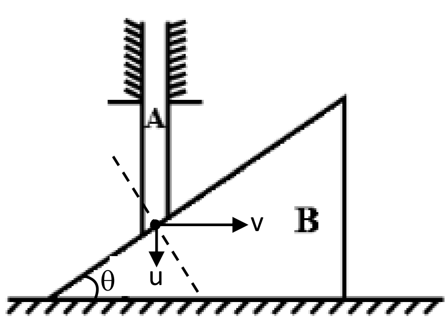
Solution: Component of velocity of rod along perpendicular to inclined (commonnormal) surface is equal to velocity of wedge along that direction.

$u\cos\theta = v\sin\theta$

$\frac{u}{v} = \tan\theta$

$u = v\tan\theta$

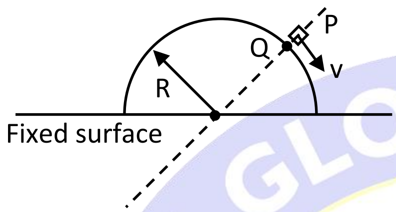




SPOT LIGHT

- Components of acceleration of points of contact along the common normal (perpendicular to the surface) is same if contacting surfaces are flat. Also if contacting surfaces are curved then components of accelerations of points of contact will not be

Example:



Acceleration of contact point 'Q' of surface along common normal = 0.

But acceleration of contact point 'P' on body along the common normal $=\frac{v^2}{R}$

Hence, we can conclude that acceleration of P and Q are not same

CIRCULAR MOTION :

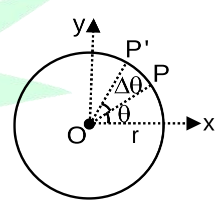
12. KINEMATICS OF CIRCULAR MOTION

If a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed or moving point.

That fixed point is called the centre and the corresponding distance is called the radius of circular path. The vector joining the centre of the circle and the particle performing circular motion, directed towards the later is called the radius vector. It has constant magnitude but variable direction.

TERMINOLOGY RELATED TO CIRCULAR MOTION :

12.1 Angular position :



The angle made by the position vector w.r.t. origin, with the reference line is called angular position.

Clearly angular position depends on the choice of the origin as well as the reference line.

The angular position of particle P at a given instant may be described by the angle θ between OP and Ox. This angle θ is called the angular position of the particle.

12.1.1 Angular Displacement:

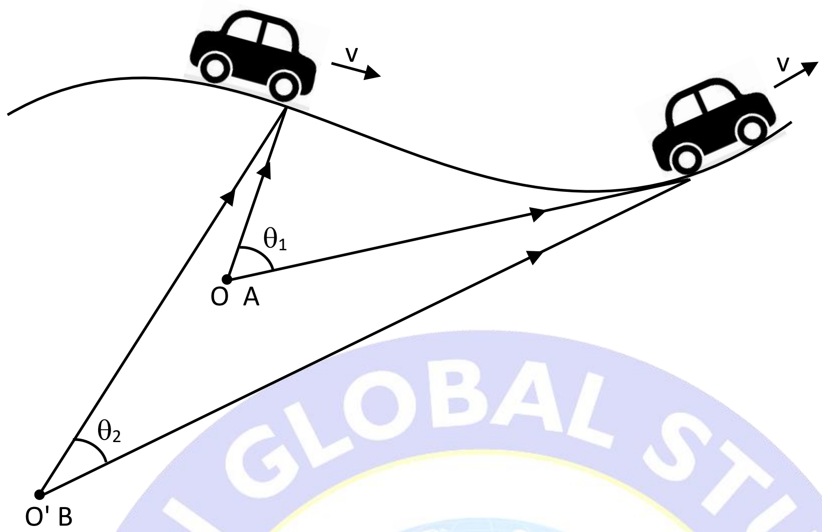
Angle traced by the position vector of particle moving w.r.t. some fixed point is called angular displacement.

$\Delta\theta$ = angular displacement angle = $\frac{\text{arc}}{\text{radius}}$ $\Delta\theta = \frac{\text{arc pp}'}{r}$

SPOT LIGHT

- Unlike linear displacement in same frame angle displacement may be different.

Example:

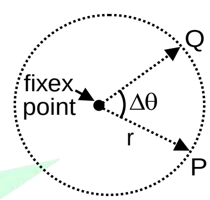


For observer O angular displacement of car is θ_1 while that for observer O' is θ_2

- Small angular displacement is a vector quantity, but large angular displacement θ is not a vector quantity.
  $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ But $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$
- Direction of angular displacement is perpendicular to the plane of rotation and is given by right hand thumb rule.
- Angular displacement is dimensionless and its S.I. unit is radian while other units are degree and revolution.
  2π radian = 360° = 1 revolution

12.1.2 Angular Velocity (ω)

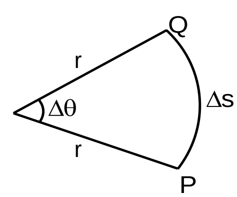
It is defined as the rate of change of angular displacement of a moving particle, w.r.t. to time.



$$\omega = \frac{\text{angle traced}}{\text{time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\Rightarrow$ Its unit is rad/s and dimensions is $[T^{-1}]$. It is a axial vector.

12.1.3 Relation between linear and Angular velocity :



Angle ($\Delta\theta$) = $\frac{\text{arc}}{\text{radius}} = \frac{\Delta s}{r} \Rightarrow \Delta s = r\Delta\theta$

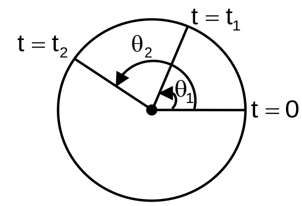
$\therefore \frac{\Delta s}{\Delta t} = \frac{r\Delta q}{\Delta t}$ if $\Delta t \to 0$ then $\frac{ds}{dt} = r\,\frac{d\theta}{dt} \Rightarrow v = \omega\, r$

In vector form $\vec{v} = \vec{\omega} \times \vec{r}$ (direction of $\vec{v}$ is according to right hand thumb rule)

12.1.4 Instantaneous Angular Velocity:

The angular velocity at some particular instant $\vec{\omega} = \lim_{\Delta t - 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

12.1.5 Average Angular Velocity (ω_{av}):



$$\vec{\omega}_{av} = \frac{\text{total angle rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are the angular positions of the particle at instants t_1 and t_2.

12.1.6 Frequency (n) :

Number of revolutions described by particle per second is its frequency. Its unit is revolutions per second (rps) or revolutions per minute (rpm).

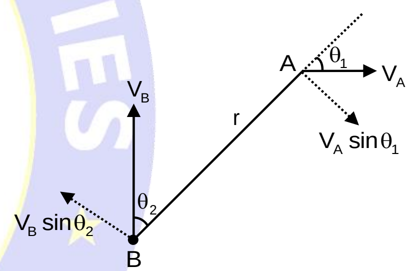
Note : 1 rps = 60 rpm

12.1.7 Time Period (T) :

It is the time taken by particle to complete one revolution. i.e. $T = 1/n$

12.1.8 Relative Angular Velocity :

Relative angular velocity of a particle 'A' w.r.t. an other moving particle B is the angular velocity of the position vector of 'A' w.r.t. B. It means the rate at which the position vector of 'A' w.r.t. B rotates at the instant.

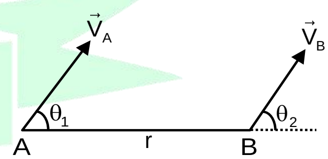


$$w_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t B perpendicular to line AB}}{\text{separation between A and B}}$$

Here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$

$$\therefore \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

- Instantaneous angular velocity is an axial vector quantity.
- Direction of angular velocity is same as that of angular displacement i.e., perpendicular to the plane of rotation and along the axis according to right hand screw rule or right hand thumb rule.
- If particles A and B are moving with a velocity $\vec{v}_A$ and $\vec{v}_B$ and are separated by a distance r at a given instant then



(i) $\frac{dr}{dt} = v_B \cos\theta_2 - v_A \cos\theta_1$

(ii) $\frac{d\theta_{AB}}{dt} = \omega_{AB} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$

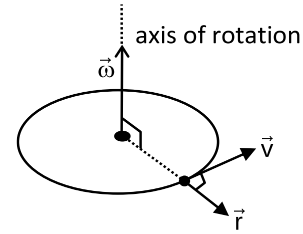
12.1.9 Angular Acceleration (α) :

Rate of change of angular velocity is called angular acceleration i.e., $\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$

12.1.10 Average Angular Acceleration:

$$\vec{\alpha}_{avg} = \frac{\text{change in angular velocity}}{\text{time taken}} \Rightarrow \frac{\Delta\vec{\omega}}{\Delta t} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$$

12.1.11 Relation between Angular and Linear Acceleration:



Velocity $\vec{v} = \vec{\omega} \times \vec{r}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega}\frac{d\vec{r}}{dt}$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a} = \vec{a}_T + \vec{a}_C$$

($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration & $\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are the two component of net linear acceleration)

As $\vec{a}_T \perp \vec{a}_C$ so $|\vec{a}| = \sqrt{a_T^2 + a_C^2}$

12.1.12 Tangential Acceleration:

$a_T = \vec{\alpha} \times \vec{r}$, its direction is parallel (or antiparallel) to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ **both parallel (or antiparallel) and along the axis.**

Magnitude of tangential acceleration in case of circular motion :

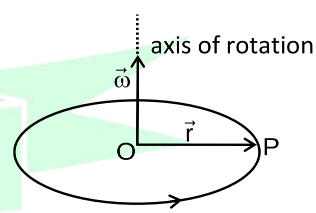
$a_T = \alpha$ r sin 90° = αr ($\vec{\alpha}$ is axial, $\vec{r}$ is radial so that $\vec{\alpha} \perp \vec{r}$)

As $\vec{a}_\perp$ is along the direction of motion (in the direction of $\vec{v}$) so $\vec{a}_\perp$ is responsible for change in speed of the particle. Its magnitude is the rate of change of speed of the particle.

DETECTIVE MIND

- Centripetal acceleration is always perpendicular to the velocity at each point.
- If a particle is moving on a circular path with constant speed then tangential acceleration is zero.

12.1.13Centripetal acceleration:



$\vec{a}_c = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) (\because \vec{v} = \vec{\omega} \times \vec{r})$

Let $\vec{r}$ be in $\hat{i}$ direction and $\vec{\omega}$ be in $\hat{j}$ direction then the direction of is along $\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) - \hat{i}$, opposite to the direction of $\vec{r}$ i.e., from P to O and it is centripetal in direction.

Magnitude of centripetal acceleration, $a_c = \omega v = \frac{v^2}{r} = \omega^2 r$ therefore

$$\vec{a}_c = \frac{v^2}{r}(-\vec{r})$$

Note: Centripetal acceleration is always perpendicular to the velocity at each point.

- Angular acceleration is an axial vector quantity. It's direction is along the axis according to the right hand thumb rule or right hand screw rule.
- **Important difference between projectile motion and uniform circular motion :**
  In projectile motion, both the magnitude and the direction of acceleration (g) remains constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

12.2 UNIFORM AND NON-UNIFORM CIRCULAR MOTION:

12.2.1 Uniform Circular Motion:

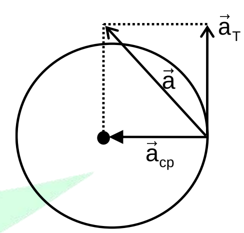
If a particle moves with a constant speed in a circle, the motion is called uniform circular motion. In uniform circular motion a resultant non-zero force acts on the particle. This force is provided by some external such as friction, magnetic force, coulomb force, gravitational force, tension. etc.

- Speed = constant
- Velocity $\neq$ constant (because its direction continuously changes). But magnitude of velocity is constant.
- Velocity = constant (because magnitude and direction, both are constants)
- Tangential acceleration $a_T = 0 \left[\because a_T = \frac{dv}{dt} = \frac{d(\text{constant})}{dt} = 0\right]$
- Angular acceleration $\alpha = 0$
- Direction of acceleration of particle is towards centre and its magnitude

  $= |\vec{a}| = |\vec{a}_{cp}| = \omega v = \omega^2 r = \frac{v^2}{r}$ = constant
- $= \vec{a} = \vec{a}_{cp} \neq$ constant

  (because the direction of a_{cp} is toward the centre of circle which changes as the particle revolves)
- The direction of the resultant force F is therefore, towards the centre and its magnitude is (as $v = r\omega$)
- F = $\frac{mv^2}{r} = mr\omega^2 (\text{as } v = r\omega)$
- K.E. = $\frac{1}{2}mv^2$ = constant
- Uniform circular motion is usually executed in a horizontal plane.

12.2.2 Non-Uniform Circular Motion :



If a particle moves with variable speed in a circle, then the motion is called non uniform circular motion. Circular motion in vertical plane is an example of non-uniform circular motion.

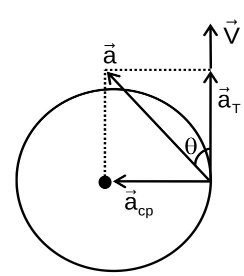
Acceleration (a) has two components :-

$\vec{a}_{CP}$ = responsible for change in direction only.

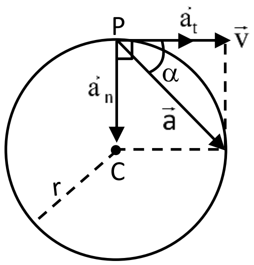
$\vec{a}_T$ = responsible for change in speed only.

Hence due to a_T speed = | velocity | is variable,

- As speed variable hence given physical quantities are also variable.
- $|\vec{\omega}| = \frac{v}{r}$
- $a_T \neq 0$
- $\alpha \neq 0$
- $|\vec{a}_{cp}| = \omega v = \omega^2 r = \frac{v^2}{r}$
- $\vec{a} = \vec{a}_T + \vec{a}_{cp}$
- $|\vec{a}| = \sqrt{a_{T^2} = a_{cp^2}} = \sqrt{(\alpha r)^2 + \left(\frac{v^2}{r}\right)^2}$
- $\vec{F} = \vec{F}_T + \vec{F}_{cp}$



- $K.E. = \frac{1}{2}mv^2$
- Angle between velocity and acceleration is given by : $\tan\theta = \frac{a_{cp}}{a_T} = \frac{F_{cp}}{F_T}$
- If $\vec{a}_t$ and $\vec{v}$ are parallel to each other then magnitude of velocity i.e.speed increases.

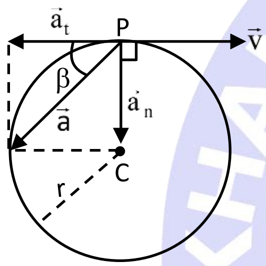


$\vec{a} = \vec{a}_n + \vec{a}_t$

$\Rightarrow |\vec{a}| = \sqrt{|\vec{a}_n|^2 + |\vec{a}_t|^2}$

Angle between $\vec{a}$ and $\vec{a}_t$, $\alpha = \tan^{-1}\frac{|\vec{a}_n|}{|\vec{a}_t|}$

- If $\vec{a}_t$ and $\vec{v}$ are antiparallel to each other then magnitude of velocity i.e.speed decreases.

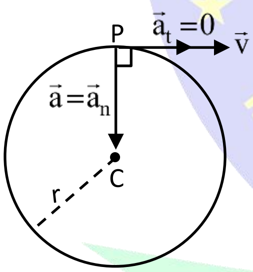


$\vec{a} = \vec{a}_n + \vec{a}_t$

$\Rightarrow |\vec{a}| = \sqrt{|\vec{a}_n|^2 + |\vec{a}_t|^2}$

Angle between $\vec{a}$ and $\vec{a}_t$, $\beta = \tan^{-1}\frac{|\vec{a}_n|}{|\vec{a}_t|}$

- If $\vec{a}_t$ = 0 then $\vec{v}$ are perpendicular to normal acceleration then magnitude of velocity i.e. speed remains constant but direction changes continuously.



$\vec{a} = \vec{a}_n + \vec{a}_t$

$\Rightarrow |\vec{a}| = |\vec{a}_n| \quad (\because \vec{a}_t = 0)$

Angle between $\vec{a}(\vec{a}_n)$ and $\vec{v} = 90^\circ$

DETECTIVE MIND

$\vec{a}_n$ & $\vec{a}_t$ are two rectangular component of acceleration $(\vec{a})$

In 1-D motion $\vec{a}_n$ is zero so $\vec{a}_t$ becomes $\vec{a}$

12.2.3 Equations of circular motion

Initial angular velocity (ω_0) | Final angular velocity (ω)

Angular displacement (θ) | Angular acceleration (α)

If α = constant, then $\omega = \omega_0 + \alpha t \Rightarrow \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \theta_{n^{th}} = \omega_0 + \frac{1}{2}\alpha(2n-1) \Rightarrow \theta = \left(\frac{\omega_0 + \omega}{2}\right)t$

SOLVED EXAMPLES

Example 38: A particle moving on 7m radius circular path. If it covers 21 m distance on circular path then find angular displacement.

(1) $\frac{420^\circ}{\pi}$ (2) $\frac{520^\circ}{\pi}$ (3) $\frac{535^\circ}{\pi}$ (4) $\frac{540^\circ}{\pi}$

Solution: $\Delta\theta$ = angular displacement = $\frac{\text{arc}}{\text{radius}} = \frac{21}{7}$ = 3 radin degree $\Delta\theta = 3 \times \frac{180}{\pi} = \frac{540º}{\pi}$

Example 39: The position vector of a particle changes from $3\hat{i}+4\hat{j}$ to $4\hat{i}-3\hat{j}$. What is the angular displacement.

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{2\pi}{3}$

Solution: Angular displacement is the angle subtended between the two position vector.

So, $\cos\theta = \frac{\vec{r}_f \cdot \vec{r}_i}{|\vec{r}_f||\vec{r}_i|} = 0 \Rightarrow \theta = 90° = \frac{\pi}{2}$

Example 40: A particle moves on circular track of 7 cm radius its angular displacement is 225° in 50 sec find angular velocity of a particle.

(1) $\frac{\pi}{20}$ rad/sec (2) $\frac{\pi}{40}$ rad/sec (3) $\frac{\pi}{45}$ rad/sec (4) $\frac{\pi}{50}$ rad/sec

Solution: $\omega = \frac{225º}{50} \times \frac{\pi}{180} = \frac{\pi}{40}$ rad/sec

Example 41: A particle revolving in a circular path completes first one third of the circumference in 2 s, while next one third in 1s. Calculate its average angular velocity.

(1) $\frac{8\pi}{3}$ rad/sec (2) $\frac{7\pi}{6}$ rad/sec (3) $\frac{3\pi}{8}$ rad/sec (4) $\frac{4\pi}{9}$ rad/sec

Solution: $\theta_1 = \frac{2\pi}{3}$ and $\theta_2 = \frac{2\pi}{3}$ total time T = 2 + 1 = 3 s

$$\therefore <\omega_{av}> = \frac{\theta_1+\theta_2}{T} = \frac{\frac{2\pi}{3}+\frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9}\text{rad/s}$$

Example 42: Angular position of a line of a disc of radius r = 6 cm is given by $\theta = 10 - 5t + 4t^2$ rad. The average angular speed between 1 and 3 s is :

(1) 11 rad/sec (2) 12 rad/sec (3) 14 rad/sec (4) 15 rad/sec

Solution: $\theta = 10 - 5t + 4t^2$ rad $\Rightarrow \theta_1 = 10 - 5 + 4 = 9$ rad

$\theta_3 = 10 - 5 \times 3 + 4 \times 9 = 10 - 15 + 36 = 31 \Rightarrow \omega_{avg.} = \frac{\Delta\theta}{\Delta t} = \frac{31-9}{2} = \frac{22}{2} = 11\text{rad/sec}$

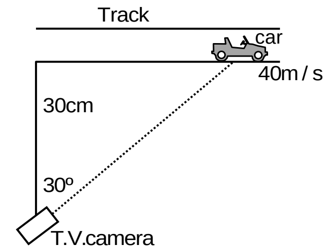
Example 43: In the above question linear speed of the point on the rim at 2s is :

(1) 14 rad/sec (2) 12 rad/sec (3) 11 rad/sec (4) 10 rad/sec

Solution: $\theta = 10 - 5t + 4t^2 \Rightarrow \omega = = 0 - 5 + 8t = 8t - 5$

'ω' at t = 2 $\Rightarrow 8 \times 2 - 5 = 16 - 5 = 11$ rad/sec {$v = \omega r = 11 \times 6 = 66$ cm/sec}

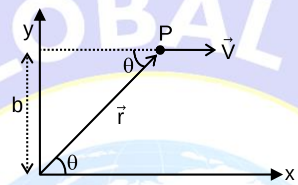
Example 44: A racing car is travelling along a track at a constant speed of 40 m/s. A T.V. camera men is recording the ever from a distance of 30 m directly away from the track as shown is figure. In order to keep the car under view in the position shown, the angular speed with which the camera should be rotated, is :



(1) $\frac{2}{5}$ rad/sec (2) $\frac{2}{7}$ rad/sec

(3) $\frac{4}{5}$ rad/sec (4) $\frac{3}{5}$ rad/sec

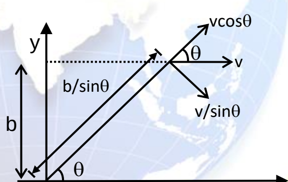
Solution: $\omega = \frac{v_{\perp}}{r} = \frac{2}{5}$ rad/sec



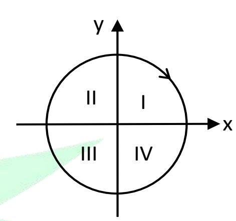
Example 45: A particle is moving parallel to x-axis as shown in figure such that they component of its position vector is constant at all instants and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes an angle θ with the x-axis.

(1) $\frac{v}{b}\sin^2\theta$ (2) $\frac{v}{b}\cos^2\theta$ (3) $\frac{v}{b}\tan^2\theta$ (4) $\frac{b}{v}\sin^2\theta$

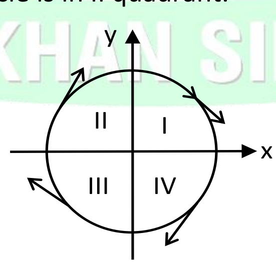
Solution: $\therefore \omega_{PQ} = \frac{v\sin\theta}{\frac{b}{\sin\theta}} = \frac{v}{b}\sin^2\theta$



Example 46: A particle is moving in clockwise direction in a circular path shown in figure. The instantaneous velocity of particle at a certain instant is $\vec{v} = (3\hat{i} + 3\hat{j})\text{m/s}$. Then in which quadrant does the particle lie at that instant? Explain your answer.



Solution: II quadrant. According to following figure x & y components of velocity are positive when the particle is in II quadrant.



Example 47: A stone tied to the end of a 80 cm long string is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, the magnitude of its acceleration is :

(1) 20 ms^{-2} (2) 12 ms^{-2} (3) 9.9 ms^{-2} (4) 8 ms^{-2}

Solution: Angular speed = $\frac{14\times 2\pi}{25} = \frac{28\pi}{25}$ Acceleration = $\frac{v^2}{r} = \omega^2 r = \left(\frac{28\pi}{25}\right)^2 \times \frac{80}{100}$ = 9.9 m/sec^2.

Example 48: A particle is performing circular motion of radius 1m. Its speed is v = $(2t^2)$ m/s. What will be the magnitude of its acceleration at t = 1s?

(1) $4\sqrt{2}$ m/s^2 (2) $5\sqrt{3}$ m/s^2 (3) $3\sqrt{5}$ m/s^2 (4) $5\sqrt{7}$ m/s^2

Solution: Tangential acceleration = $a_T = \frac{dv}{dt} = 4t$ at t = 1s, a_T = 4 m/s^2

Centripetal acceleration $a_C = \frac{v^2}{r} = \frac{4t^4}{1} = 4t^4$ at t = 1s, a_C = 4 m/s^2

Net acceleration $(a) = \sqrt{a_T^2 + a_C^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2} m/s^2$

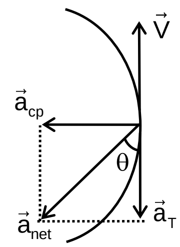
Example 49: A cyclist is riding with a speed of 18 km/h. As he approaches a circular turn on the road of radius $25\sqrt{2}m$, he applies brakes which reduces his speed at a constant rate of 0.5 m/s every second. Determine the magnitude and direction of his net acceleration on the circular turn.

(1) 0.86 ms^{-2}, $\tan^{-1}\left(\sqrt{2}\right)$ from tangential direction

(2) 0.86 ms^{-2}, $\tan^{-1}\left(\sqrt{2}\right)$ from radial direction

(3) 0.86 ms^{-2}, $\tan^{-1}\left(\sqrt{3}\right)$ from radial direction

(4) 0.76 ms^{-2}, $\tan^{-1}\left(\sqrt{2}\right)$ from tangential direction



Solution: v = $8 \times \frac{5}{18}$ = 5 ms^{-1}anda_{cp} = $\frac{v^2}{R} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}} ms^{-2}$,

$a_T = -\frac{dv}{dt} = -\frac{1}{2} ms^{-2} \Rightarrow a_{net} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2}$,= $\frac{\sqrt{3}}{2}$ = 0.86 ms^{-2}

$\tan\theta = \frac{a_{CP}}{a_T} = \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\theta = \tan^{-1}\left(\sqrt{2}\right)$ from tangential direction

Example 50: A disc starts from rest and gains an angular acceleration given by $\alpha = 3t - t^2$ (where t is in seconds) upon the application of a torque. Calculate its angular velocity after 2 s.

(1) $\frac{11}{3}$ rad/s (2) $\frac{11}{4}$ rad/s (3) $\frac{10}{3}$ rad/s (4) $\frac{12}{5}$ rad/s

Solution: $\alpha = \frac{d\omega}{dt}$ = $3t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^{1} (3t - t^2)dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow$ at t = 2 s, $\omega = \frac{10}{3} rad/s$

Example 51: The tangential and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively it will be at a distance from the axis of rotation:

(1) 8 m (2) 6 m (3) 4 m (4) 2 m

Solution: a_t = 10 m/s^2, α = 5 rad/sec^2 $\Rightarrow a_t = \alpha R \Rightarrow 10 = 5 \times R \Rightarrow R = 2$ m

Example 52: A grinding wheel attained a angular velocity of 20 rad/sec in 5 sec starting from rest. Find the number of revolutions made by the wheel.

(1) $\frac{22}{\pi}$ (2) $\frac{24}{\pi}$ (3) $\frac{25}{\pi}$ (4) $\frac{28}{\pi}$

Solution: $\omega_f = \omega_0 + \alpha t \Rightarrow 20 = 0 + \alpha \times 5 \Rightarrow \alpha = 4 rad/sec^2 \Rightarrow \theta = \omega_0 t \times \alpha t^2$

$\Rightarrow \theta = 0 + \frac{1}{2} \times 4 \times 25$ {θ = 2 × 25 = 50 rad}{No. of revolution = $\frac{\theta}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi}$ }

13. DYNAMIC CIRCULAR MOTION

13.1 CENTRIPETAL FORCE :

- A force causing a centripetal acceleration at towards the centre of the circular path and causes a change in the direction of velocity vector is called centripetal force $F = \frac{mv^2}{r} = m\omega^2 r$

13.2 CENTRIFUGAL FORCE :

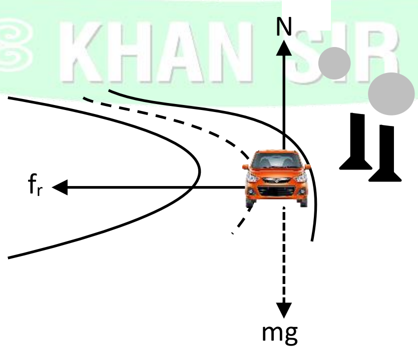
- It gives a physical sensation of being centrifuged (pushed radially away).
- It's magnitude is same as centripetal but direction is opposite to centripetal force.
- It never exists in inertial frame
- It is a pseudo force. It has no action-reaction pair.

13.3 CIRCULAR TURNING OF ROADS :

- When vehicles travel along a nearly circular arc. There must be some force which provides the required centripetal acceleration. In real life the necessary centripetal force is provided by friction and banking of roads both. The necessary centripetal force is being provided to the vehicles by the following :

13.3.1 By Friction only

- Suppose a car of mass m is moving with a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force will be provided to the car by the force of friction f acting towards center of the circular path.

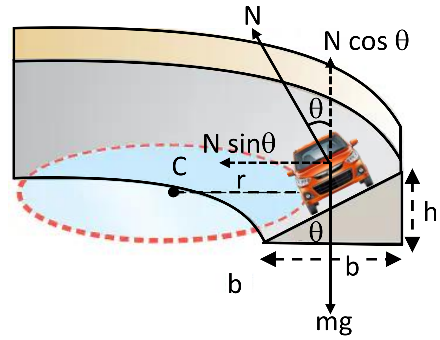


Thus, $f = \frac{mv^2}{r}$ $\therefore$ $f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without skidding $\frac{mv^2}{r} \le f_{max} \Rightarrow \frac{mv^2}{r} \le \mu mg \Rightarrow \boxed{v \le \sqrt{\mu rg}}$

13.3.2 By Banking of Roads only

Friction is not always reliable at turns particularly when high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn in the sense that the outer part of the road is somewhat lifted compared to the inner part.



and $N \sin\theta = \frac{mv^2}{r}$ ...... (i)

and $N \cos\theta = mg$ ...... (ii)

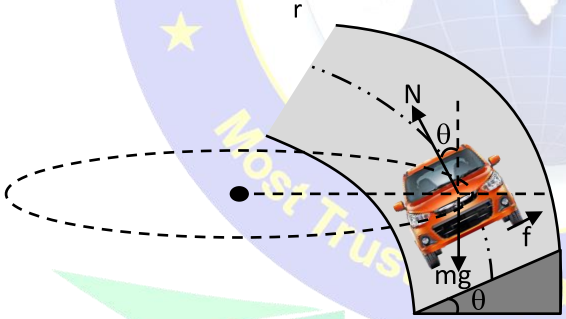
Dividing equation (ii) by equation (i)

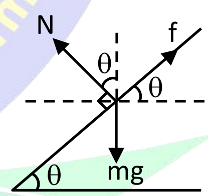
$\Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$ $\because \tan\theta = \frac{h}{b}$

$$\boxed{\tan\theta = \frac{v^2}{rg} = \frac{h}{b}}$$

13.3.3 Friction and Banking of Road both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, direction and the magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.





(a) If speed of the vehicle is small then friction acts outwards.

In this case, $N\cos\theta + f\sin\theta = mg$ .....(i)

and $N\sin\theta - f\cos\theta = \frac{mv^2}{R}$ ....(ii)

For minimum speed $f = \mu N$

So, by dividing equation (i) by equation (ii)

$$\frac{N\cos\theta + \mu N\sin\theta}{N\sin\theta - \mu N\cos\theta} = \frac{mg}{mv_{min}^2 / R};$$

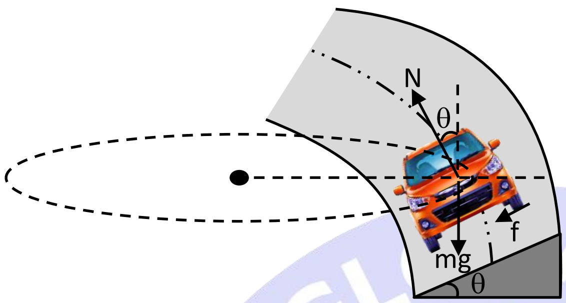
Therefore

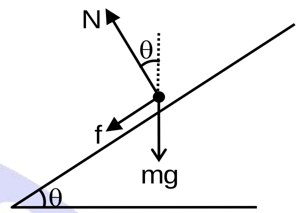
$$\boxed{v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}}$$

If we assume $\mu = \tan\phi$, then

$$v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \tan\phi}{1+\tan\phi\tan\theta}\right)} = \sqrt{Rg\tan(\theta-\phi)}$$

(b) If speed of vehicle is high then friction force act inwards.





in this case for maximum speed

$N\cos\theta - \mu N\sin\theta = mg$

and $N\sin\theta + \mu N\cos\theta = \frac{mv_{max}^2}{R}$

which gives $v_{max} = \sqrt{Rg\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)}$

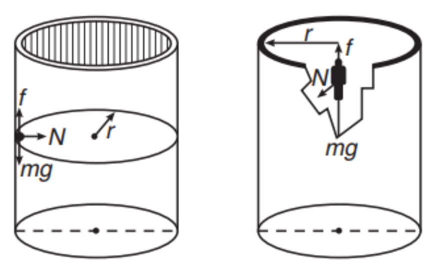
$= \sqrt{Rg\tan(\theta+\phi)}$

Hence for successful turning on a rough banked road, velocity of vehicle must satisfy following relation

$\sqrt{Rg\tan(\theta-\phi)} \le v \le \sqrt{Rg\tan(\theta-\phi)}$ where θ = banking angle and $\phi = \tan^{-1}(\mu)$.

14. DEATH WELL:

In case of 'death well' a person drives a bicycle on a vertical surface of a large wooden well while in case of a rotor, at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotate. In both cases, friction force balances the weight of person while reaction provides the centripetal force for circular motion, i.e.



(A) Death well (B) Rotor

$f = mg$ and $N = \frac{mv^2}{r} = mr\omega^2$ $\qquad (v = r\omega)$

SOLVED EXAMPLES

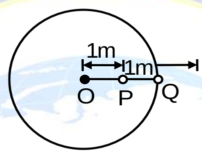
Example 53: The breaking tension of a string is 100 N. A particle of mass 0.1 kg tied to it is rotated along horizontal circle of radius 0.5 meter. The maximum speed with which the particle can be rotated without breaking the string is :

(1) $\sqrt{5}\text{m / sec}$ (2) $\sqrt{(50)}\text{m / sec}$ (3) $\sqrt{(500)}\text{m / sec}$ (4) $\sqrt{(1000)}\text{m / sec}$

Solution: T = $T = \frac{mv^2}{r}$

$$\Rightarrow v = \sqrt{\frac{Tr}{m}} = \sqrt{50}\ \text{m/s}$$

Example 54: Two sphere of equal mass are attached to a string of length 2 m as shown in figure. The string and the spheres are then rotated in horizontal circle about 'O' at constant rate. The value of ratio of tension in string PQ & PO:

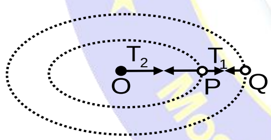


(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 2

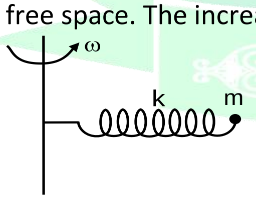
Solution: $T_1 = m\omega^2R$

$T_1 = m\omega^2\,(2)$

$T_2 - T_1 = m\omega^2$ $\Rightarrow T_2 = 3m\omega^2$ so $\frac{T_1}{T_2} = \frac{2}{3}$



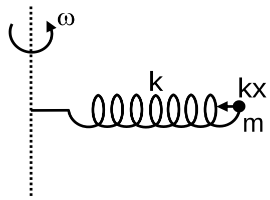
Example 55: A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length L. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be :



(1) $\frac{m\omega^2 l}{k}$ (2) $\frac{m\omega^2 l}{k - m\omega^2}$ (3) $\frac{m\omega^2 l}{k + m\omega^2}$ (4) None of these

Solution: $kx = m\omega^2R \Rightarrow kx = m\omega^2(L + x)$

$$\Rightarrow (k - m\omega^2)x = m\omega^2L \Rightarrow x = \frac{m\omega^2L}{k - m\omega^2}$$



Example 56: Find the maximum speed at which a car can turn round a curve of 30 m radius on a levelled road if the coefficient of friction between the tyres and the road is 0.4
[acceleration due to gravity = 10 m/s^2]
(1) 6 m/s (2) 10 m/s (3) 11 m/s (4) 12 m/s

Solution: Here centripetal force is provided by friction so

$$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 ms^{-1}$$

Example 57: A car starts from rest with a constant tangential acceleration a_0 in a circular path of radius r. At time t_0, the car skids, find the value of coefficient of friction.

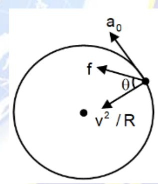
(1) $\frac{a_0}{g}\sqrt{1+\frac{a_0^2 t_0^4}{r^2}}$ (2) $\frac{a_0}{g}\sqrt{1-\frac{a_0^2 t_0^4}{r^2}}$

(3) $\frac{g}{a_0}\sqrt{1+\frac{a_0^2 t_0^2}{r^2}}$ (4) $\frac{a_0}{g}\sqrt{1+\frac{a_0 t_0}{r^2}}$

Solution: The tangential and centripetal acceleration is provided only by the frictional force.
Thus, f sinθ = ma_0 and

$$f\cos\theta = \frac{mv^2}{r} = \frac{m(a_0 t_0)^2}{r}$$

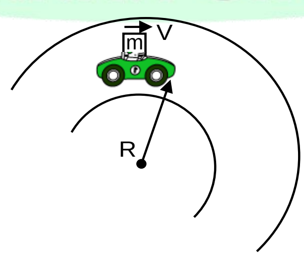
$$f = m\sqrt{a_0^2 + \frac{(a_0 t_0)^4}{r^2}} \Rightarrow ma_0\sqrt{1+\frac{a_0^2 t_0^4}{r^2}} = f_{max}$$



(since the car skids beyond this speed)

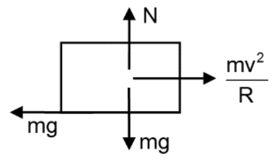
$$\mu mg = \mu a_0\sqrt{1+\frac{a_0^2 t_0^4}{r^2}} \Rightarrow \mu = \frac{a_0}{g}\sqrt{1+\frac{a_0^2 t_0^4}{r^2}}$$

Example 58: A car is carry a box of mass m = 50 kg on its flat horizontal rough surface with coefficient of friction μ = 0.3. It is crossing a circular track of radius 27m. What is the maximum speed of the car so that the box does not slide from the car while moving on the circular path ?



(1) 18 km/hr (2) 36 km/hr (3) 32.4 km/hr (4) None of these

Solution: F.B.D. of the particle in car travel.



Here $\frac{mv^2}{R} = \mu\, mg$

$v^2 = \mu\, mg\, R = 0.3 \times 50 \times 10 \times 27$

$v = 45\sqrt{2}$

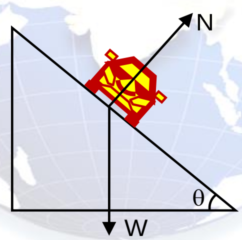
Example 59: For traffic moving at 60 km/h, if the radius of the curve is 0.1 km, what is the correct banking angle of the road ? (g = 10 m/s²)

(1) $\tan^{-1}\left(\frac{5}{18}\right)$ (2) $\tan^{-1}\left(\frac{6}{19}\right)$ (3) $\tan^{-1}\left(\frac{18}{5}\right)$ (4) $\tan^{-1}\left(\frac{7}{9}\right)$

Solution: In case of banking $\tan\theta = \frac{v^2}{rg}$. Here v = 60 km/h, $60 \times \frac{5}{18} ms^{-1} = \frac{50}{3} ms^{-1}$ $r = 0.1 km$ = 100 m

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

Example 60: A car is moving along a banked road laid out as a circle of radius r. (a) What should be the banking angle θ so that the car travelling at speed υ needs no frictional force from the tyres to negotiate the turn? (b) The coefficients of friction between tyres and road are $\mu_s = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding toward the top edge of the banked turn?



Solution:

(a) the banking angle θ so that car travelling at speed υ needs no frictional force from the tyres to negotiate the turn $\Rightarrow \tan\theta = v^2 / rg$

(b) The coefficients of friction between tyres and road are $\mu_s = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding toward the top edge of the banked turn

$$\Rightarrow V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)} = \sqrt{rg\tan(\theta + \phi)}$$

Example 61: When the string of 5/m of conical pendulum makes an angle 30° with vertical. Find its time period?

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

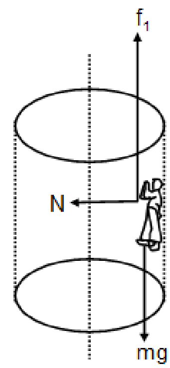
Solution: $T = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2 \times \pi\sqrt{\frac{5/\sqrt{3} \times \sqrt{3}/2}{10}} = \pi$

Example 62: A person with a mass of M kg stands in contact against the wall of the cylindrical drum of radius r rotating with an angular velocity ω. The coefficient of friction between the wall and the clothing is μ. The minimum rotation speed of the cylinder which enables the person to remain stuck to the wall when the floor is suddenly removed is :

(1) $\omega_{min} = \sqrt{\frac{g}{\mu r}}$ (2) $\omega_{min} = \sqrt{\frac{\mu r}{g}}$ (3) $\omega_{min} = \sqrt{\frac{2g}{\mu r}}$ (4) $\omega_{min} = \sqrt{\frac{gr}{\mu}}$

Solution: $N = m\omega^2 R$ and $f_r = mg$

$mg \leq \mu(m\omega^2 R) \Rightarrow \omega \geq \sqrt{\frac{g}{\mu r}}$.



Newton's Laws

Newton's 1st law:
A body continues its state of motion until unless an external force is applied on it.

Newton's 2nd law:
The rate of change of linear momentum of a body is directly proportional to the external force applied on $\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$

if $\vec{v} = \text{constant} \Rightarrow \vec{F} = \vec{v}\frac{dm}{dt}$

Newton's 3rd law:
Every action has equal and opposite reaction
Action – reaction forces are of same type.
Action & Reaction act on different bodies and not on the same body.

Conditions for friction-reaction pairs

- Equal in magnitude and nature
- Act along the same line
- Act in opposite direction on different bodies
- Act simultaneously

Impulse:
$\vec{I} = \vec{F}_{avg}\Delta t = \Delta\vec{P} \Rightarrow \vec{I} = \Delta\vec{P} = \int\vec{F}\cdot dt$
= area under F – t curve

Forces

Spring force
Force applied by spring
$F_{spring} = kx$
x is elongation or compression
Unit of k: N/m

Weight force
Force applied by earth
Pulling force
Always acts towards centre of earth
Weight of a body acts on body itself
Weight force is applied by earth but not by body

Normal

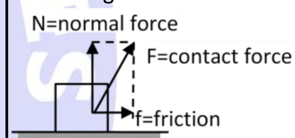
- Component of contact forcealong common.
- Always acts along the Common normal of two surfaces in contact.
- Always directed towards the system.

Tension

- Acts along the string and away from the contact point.
- Tension in a massless string remains constant throughout the string if no tangential force acts along the string.
- This is force applied by a string on an object or force applied by one section of string on other section of the string.

Frictional

- Component of contact force along common tangent
- Opposes relative motion of two contacting surfaces



- Depends upon the nature of Surfacesin contact
- It can support or oppose the motion
- Smoothness increases friction decreases
- Too much smoothness increases frictionsuddenly upto infinity.
- μ ranges from zero to infinite.

Static friction:
Opposes tendency of relative motion.
$f_s < \mu_s N$
$f_{s\,max} = \mu N$
$f_{s\,max}$ acts when a body is just at the verge of relative motion

Kinetic friction:
Opposes relative motion.
$f_k = \mu_k N$
$f_{s\,max} = \mu N$
$f_k < f_{smax}$ (experimentally)

Rolling friction:
The force of friction which comes into play when one body rolls or tends to roll on the surface of another body.

Inertia

Inertia:
Opposing property of an object while changing it's state of motion or rest is called Inertia.

Inertia of rest:
The property of a body due to which it cannot change its state of rest by itself.

Inertia of direction:
The property due to which a body cannot change its direction of motion by itself.

Inertia of motion:
The tendency of a body to remain in a state of uniform motion in a straight Line.

Kinematics of circular motion

$\theta \rightarrow$ angular displacement.

$\Delta\theta = \frac{\Delta\omega}{\Delta t}$

$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$ & $\alpha_{inst.} = \frac{d\omega}{dt}$

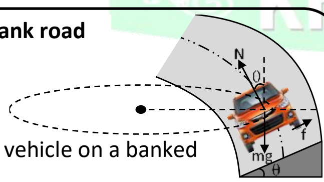
$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$a_t = \alpha r$ & $a_n = \frac{dv}{dt}$

Motion of a car on bank road



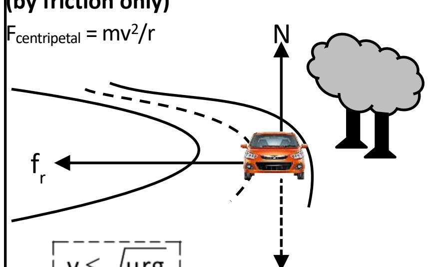
Optimum speed of a vehicle on a banked road $V = \sqrt{rg\tan\theta}$

Maximum safe speed on a banked frictional road $V_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$

Minimum safe speed on a banked frictional road $V_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$

Motion of a car on level road (by friction only)

$F_{centripetal} = mv^2/r$



$v \leq \sqrt{\mu rg}$