Concep-I

$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\cdots\cdots\cdots\times\left(1-\frac{1}{n}\right)=\frac{1}{n}$

$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\cdots\cdots\times\frac{n-1}{n}$

eg:→ ① $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\times\cdots\cdots\times\left(1-\frac{1}{49}\right)=\frac{1}{49}$ Ans.

② $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{5}\right)\times\cdots\cdots\times\left(1-\frac{1}{79}\right)=\frac{1}{79}$ Ans.

Concept – II

$$\# \left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\times\cdots\cdots\cdots\times\left(1+\frac{1}{n}\right)=\frac{n+1}{2}$$

$$\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\frac{6}{5}\times\cdots\cdots\times\frac{n+1}{n}$$

Eg:- ① $\left(1+\frac{1}{2}\right)\times\left(1+\frac{1}{3}\right)\times\left(1+\frac{1}{4}\right)\times\cdots\cdots\times\left(1+\frac{1}{59}\right)=\frac{59+1}{2}=\frac{60}{2}=30$ Ans.

② $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\cdots\cdots\times\left(1+\frac{1}{80}\right)=\frac{80+1}{2}=\frac{81}{2}=40.5$

$a^2-b^2=(a+b)(a-b)$

concept-III

$$\# \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\times\cdots\cdots\times\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

$$\left(1+\frac{1}{2}\right)\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1-\frac{1}{4}\right)\cdots\cdots\times\left(1+\frac{1}{n}\right)\left(1-\frac{1}{n}\right)$$

$$\underbrace{\left[\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\times\cdots\cdots\times\left(1+\frac{1}{n}\right)\right]}_{\frac{n+1}{2}}\times\underbrace{\left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\times\cdots\times\left(1-\frac{1}{n}\right)\right]}_{\frac{1}{n}}$$

$$=\frac{n+1}{2}\times\frac{1}{n}=\frac{n+1}{2n}$$

10. $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{120^2}\right)=?$

(a) $\frac{119}{240}$ (b) $\frac{119}{120}$ (c) $\frac{120}{240}$ (d) $\frac{121}{240}$

Ans $\longrightarrow \frac{n+1}{2n}=\frac{120+1}{2\times120}=\frac{121}{240}$

Eg: $\rightarrow \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\times\cdots\cdots\times\left(1-\frac{1}{60^2}\right)=\frac{n+1}{2n}=\frac{61}{120}$ Ans.

11. If $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\ldots\ldots\left(1-\frac{1}{N^2}\right)=\frac{85}{168}$, then the value of N is :

(a) 84 (b) 82 (c) 81 (d) 80

$$\frac{N+1}{2N}=\frac{85}{168}$$

$N+1=85$

$N=84$

Eg: $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\times\ldots\ldots\left(1-\frac{1}{x^2}\right)=\frac{21}{40}$

then Find $= x^2$

$= 20^2$

$= 400$ Ans.

$$\frac{x+1}{2x}=\frac{21}{40}$$

$x=20$

12. When simplified, theproduct $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\cdots\left(2-\frac{997}{999}\right)$ equals

(a) $\frac{5}{999}$ (b) $\frac{5}{3}$ (c) $\frac{1001}{999}$ (d) $\frac{1001}{3}$

$\frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \cdots \times \frac{1001}{999}$

$= \frac{1001}{3}$ Ans.

13. If $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{6}\right)\left(1+\frac{1}{8}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)=1+\frac{1}{x}$, then what is the value of x ?

यदि $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{6}\right)\left(1+\frac{1}{8}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)=1+\frac{1}{x}$ हो, तो x का मान क्या है ?

(a) 6 (b) 8 (c) 5 (d) 7

$$\frac{3}{2}\times\frac{5}{4}\times\frac{7}{6}\times\frac{9}{8}\times\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}=\frac{x+1}{x}$$

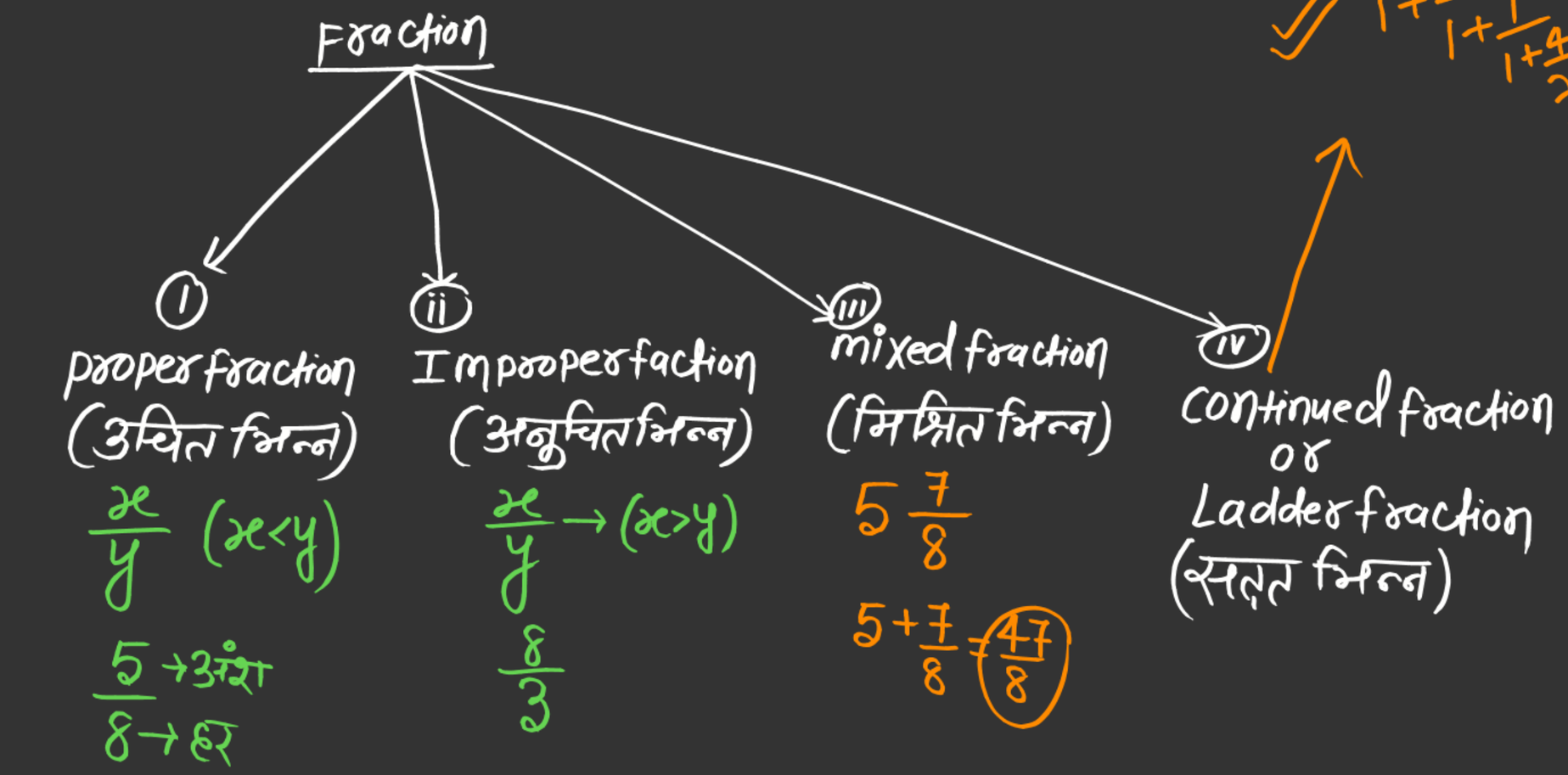
$$\frac{9}{8}=\frac{x+1}{x}$$

$$x=8$$

concept

Fraction (भिन्न)

$\frac{x}{y}$ → अंश
$\frac{x}{y}$ → हर



continued fraction
or
Ladder fraction

Eg: ①

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{5}{8}}}}$$

$1+\frac{5}{8}=\frac{13}{8}$

$$1+\cfrac{1}{1+\frac{8}{13}}$$

$1+\frac{8}{13}=\frac{21}{13}$

$$1+\frac{13}{21}=\frac{34}{21}$$

II-method

$$1+\cfrac{1}{1+\cfrac{1}{1+\frac{5}{8}}}$$

5, 8, 13, 21, 34

$\frac{34}{21}$ Ans.

(2) $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{7}{6}}}}$

7, 6

13 ✓ 19 ✓ 32 ✓ 51 ✓

$\frac{51}{32}$ Ans.

③ $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{3}}}}}$

2, 3 ✓ 5 ✓ 8 ✓ 13 ✓ 21 ✓ 34

$\frac{34}{21}$ Ans.