

Euler's theorem

→ Totient method

$$\frac{N}{D} \propto$$

$N, D \rightarrow$ Co-prime no. (सह अभाज्य सं०)

Relatively prime no. (सापेक्षीत अभाज्य सं०)

$$\boxed{HCF=1}$$

\Rightarrow prime no = p

$$N, D \rightarrow \text{HCF}=1$$

$$\boxed{\phi = p-1}$$

पूर्वफल
 $\boxed{\text{Totient} \Rightarrow \phi}$

अभाज्य सं० का Totient

$$\begin{aligned} \textcircled{1} \quad 2 &\rightarrow \phi = 2 \times \left(1 - \frac{1}{2}\right) \\ &= 2 \times \frac{1}{2} = \textcircled{1} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 3 &\rightarrow \phi = 3 \times \left(1 - \frac{1}{3}\right) \\ &= \cancel{3} \times \frac{2}{\cancel{3}} = \textcircled{2} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 5 &\rightarrow \phi = 5 \times \left(1 - \frac{1}{5}\right) \\ &= \cancel{5} \times \frac{4}{\cancel{5}} = \textcircled{4} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad 7 &\rightarrow \phi = 7 \times \left(1 - \frac{1}{7}\right) \\ &= \cancel{7} \times \frac{6}{\cancel{7}} = \textcircled{6} \end{aligned}$$

Note: → अभाज्य सं० का totient उसी सं० से 1 कम होता है।

$$23 \rightarrow \phi = 23 - 1 = 22$$

$$29 \rightarrow \phi = 29 - 1 = 28$$

$$37 \rightarrow \phi = 37 - 1 = 36$$

Ex: →

$$\text{HCF}=1 \quad \textcircled{1} \quad \frac{(9)^{74}}{19} = \textcircled{2} = \frac{9^2}{19} = \frac{81}{19} \quad \boxed{R=5}$$

$$19 \rightarrow \phi = 19 - 1$$

$$\phi = 18$$

$$\text{HCF}=1 \quad \textcircled{3} \quad \frac{(7)^{62}}{11} = \textcircled{2} = \frac{7^2}{11} = \frac{49}{11} \quad \boxed{R=5}$$

$$\phi = 11 - 1$$

$$= 10$$

① Binomial theorem → संख्या को छोटा करता है।

② To find → power को छोटा है।

$$\text{HCF}=1 \quad \textcircled{2} \quad \frac{(15)^{89}}{23} = \textcircled{1} = \frac{15^1}{23} = \frac{15}{23} \quad \boxed{R=15}$$

$$23 \rightarrow \phi = 23 - 1$$

$$= 22$$

$$\textcircled{4} \quad \frac{(25)^{46}}{23} = \frac{2^{46}}{23} = \frac{2^2}{23} = \frac{4}{23}$$

$$\phi = 23 - 1 = 22$$

$$= \frac{4}{23}$$

$$\boxed{R=4}$$

भाज्यसं (composite no.) का totient

(i) $4 \rightarrow 2^2 \rightarrow \text{Base}$

$$\phi = 4 \times \left(1 - \frac{1}{2}\right) = \cancel{4}^2 \times \frac{1}{2} = 2$$

(v) $12 \rightarrow 2^2 \times 3$

$$\phi = \cancel{12}^4 \times \frac{1}{2} \times \frac{2}{3} = 4$$

(ii) $6 \rightarrow 2 \times 3$

$$\phi = 6 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) = \cancel{6} \times \frac{1}{2} \times \frac{2}{3} = 2$$

(vi) $14 \rightarrow 2 \times 7$

$$\phi = \cancel{14} \times \frac{1}{2} \times \frac{6}{7} = 6$$

(iii) $8 \rightarrow 2^3$

$$\phi = 8 \times \left(1 - \frac{1}{2}\right) = \cancel{8}^4 \times \frac{1}{2} = 4$$

(vii) $15 \rightarrow 3 \times 5$

$$\phi = \cancel{15}^4 \times \frac{2}{3} \times \frac{4}{5} = 8$$

(iv) $10 \rightarrow 2 \times 5$

$$\phi = \cancel{10} \times \frac{1}{2} \times \frac{4}{5} = 4$$

16.

Given that $N = 5^{20}$, what will be the remainder when N is divisible by 7 ?

दिया गया है $N = 5^{20}$, N को 7 से विभाजित करने पर क्या शेष प्राप्त होगा?

(a) 4

(b) - 4

(c) - 1

(d) None of these

$$\boxed{HCF=1} \leftarrow \frac{5^{\frac{20}{6}}}{7} = \frac{5^2}{7} = \frac{25}{7} \boxed{R \rightarrow 4}$$

$7 \rightarrow \phi = 6$



17.

If 17^{200} is divided by 18, then what will be the remainder?

Even no. (सम सं.) $\rightarrow +ve$
i) (-) $\rightarrow +ve$
Odd no. (विषम सं.) $\rightarrow -ve$
ii) (-) $\rightarrow -ve$

यदि 17^{200} को 18 से विभाजित किया जाए तो शेषफल क्या होगा?

~~(a) 1~~

(c) 16

(b) 2

(d) 17

$$\frac{(-1)^{200}}{18} = (-1)^{200} = +1$$

1 Ans.

18.

If 17^{207} is divided by 18, then what will be the remainder?

यदि 17^{207} को 18 से विभाजित किया जाए तो शेषफल क्या होगा?

(a) 1

(c) 16

(b) 2

(d) 17

$$\frac{(-1)^{207}}{18} = (-1) = \boxed{-1}$$

$$\text{Rem} \Rightarrow 18 - 1 = 17 \text{ Ans.}$$

19.

If 2^{31} is divided by 5, then what will be the remainder?

यदि 2^{31} को 5 से विभाजित किया जाए तो शेषफल क्या होगा?

- (a) 4
(c) 2

- (b) 3
(d) 1

$$\frac{2^{\frac{31}{4} = 7}}{5} = \frac{2^3}{5} = \frac{8}{5} \boxed{R \rightarrow 3}$$

$$5 \rightarrow \phi = 4$$

20.

$(7^{19} + 2)$ is divided by 6, the remainder will be :

$(7^{19} + 2)$ को 6 से विभाजित करने पर शेष प्राप्त होगा :

(a) 5

(b) 3

(c) 2

(d) 1

$$\begin{aligned} \frac{7^{19} + 2}{6} &= (1)^{19} + 2 \\ &= 1 + 2 \\ &= 3 \text{ Ans.} \end{aligned}$$



21.

$9^6 - 11$ is divided by 8 would leave a remainder of :

$9^6 - 11$ को 8 से विभाजित करने पर शेषफल प्राप्त होगा :

(a) 0

(b) 1

(c) 2

(d) 6

$$\begin{array}{r} \textcircled{+1}6 \quad \textcircled{3} \\ 9 - 11 \\ \hline 8 \end{array} = (+1)^6 - 3$$
$$= 1 - 3$$
$$= \textcircled{-2}$$

Rem $\rightarrow 8 - 2 = \underline{\underline{6 \text{ Ans}}}$



22.

The remainder when 3^{21} is divided by 5 is :

3^{21} को 5 से विभाजित करने पर शेषफल प्राप्त होगा :

(a) 1

(b) 3

(c) 2

(d) 4

$$\text{HCF} = 1 \quad \begin{array}{l} \nearrow 3^{\frac{21}{4}} = 1 \\ \searrow 5 \end{array} = \frac{3^1}{5} \boxed{R=3}$$

$$5 \rightarrow \phi = 4$$



23.

$N = 1234567\dots55$. Find the remainder when N is divided by 16.

$N = 1234567\dots55$, N को 16 से भाग देने पर शेषफल ज्ञात करें।

(a) 13

(c) 18

(b) 15

(d) 22

$$2^n | 5^n$$

$$\begin{array}{r} 123456\dots53\boxed{5455} \\ \hline 16 \end{array}$$

$$\begin{array}{r} 16 \overline{) 5455} \quad (340) \\ \underline{48} \\ 65 \\ \underline{64} \\ 15 \\ \underline{16} \\ 15 \end{array}$$

16 → 2⁴
16 → Last 4 digit



Factorial (फैक्टोरियल)

$n!$, $\lfloor n \rfloor$

$$\lfloor 0 \rfloor = 1$$

$$\lfloor 1 \rfloor = 1$$

$$\lfloor 2 \rfloor = 2 \times 1 = 2$$

$$\lfloor 3 \rfloor = 3 \times 2 \times 1 = 6$$

$$\lfloor 4 \rfloor = 4 \times 3 \times 2 \times 1 = 24$$

$$\lfloor 5 \rfloor = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$\lfloor 6 \rfloor = 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = 720$$

$R=0$

$$\textcircled{1} \frac{\lfloor 1 \rfloor + \lfloor 2 \rfloor + \lfloor 3 \rfloor + \lfloor 4 \rfloor + \lfloor 5 \rfloor + \dots + \lfloor 29 \rfloor}{8} = R$$

$\frac{9}{8} R=1$

24.

R = Remainder when $(1! + 2! + 3! + \dots + 120!)$ is divided by 15. find R.

$(1! + 2! + 3! + \dots + 120!)$ को जब 15 से भाग दिया जाता है तो R शेष बचता है। R का मान है :

(a) 0

(c) 2

(b) 1

(d) 3

$$\begin{array}{r}
 1 + 2 + 6 + 24 \\
 \hline
 1! + 2! + 3! + 4! + 5! + 6! + \dots + 120! \\
 \hline
 15
 \end{array}$$

$\frac{33}{15} R=3$

$R=0$