Chapter 03

Trigonometric Ratios

RANKER'S STUFF

SINGLE CORRECT QUESTIONS

Q.1 Which of the following is correct-

(1) $\sin 1° > \sin 1$ (2) $\sin 1° < \sin 1$

(3) $\sin 1° = \sin 1$ (4) $\sin 1° = \frac{\pi}{180} \sin 1$

Q.2 If $\sin A = \sin B$ and $\cos A = \cos B$, then-

(1) $\sin \frac{A-B}{2} = 0$ (2) $\sin \frac{A+B}{2} = 0$

(3) $\cos \frac{A-B}{2} = 0$ (4) $\cos (A + B) = 0$

Q.3 If $\cos x + \cos y + \cos \alpha = 0$ and $\sin x + \sin y + \sin \alpha = 0$, then $\cot \left(\frac{x+y}{2}\right) =$

(1) $\sin \alpha$ (2) $\cos \alpha$ (3) $\cot \alpha$ (4) $2 \sin \alpha$

Q.4 If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = \lambda$ then $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ is equal to-

(1) $\frac{1}{\lambda}$ (2) λ (3) $1 - \lambda$ (4) $1 + \lambda$

Q.5 In a triangle ABC, if $\tan A + 2\tan B = 0$, then which one is correct ?

(1) $0 < \tan^2 C \le \frac{1}{8}$ (2) $\frac{1}{8} < \tan^2 C \le \frac{1}{2}$

(3) $\frac{1}{2} < \tan^2 C < 1$ (4) none of these

Q.6 The ratio of the maximum value to minimum value of $2\cos^2\theta + \cos\theta + 1$ is -

(1) 32 : 7 (2) 32 : 9 (3) 4 : 1 (4) 2 : 1

Q.7 The least value of

$\cos^2\theta - 6 \sin\theta . \cos\theta + 3\sin^2\theta + 2$ is-

(1) $4 + \sqrt{10}$ (2) $4 - \sqrt{10}$

(3) 0 (4) 4

Q.8 $\cos 1° . \cos 2° . \cos 3° .... \cos 179° =$

(1) 0 (2) 1 (3) 2 (4) 3

Q.9 If ABCD is a cyclic quadrilateral, then the value of $\cos A - \cos B + \cos C - \cos D$-

(1) 0 (2) 1

(3) 2 (cos B – cos D) (4) 2 (cos A – cos C)

Q.10 The value of $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$ is-

(1) 1 (2) –1 (3) 1/2 (4) 0

Q.11 Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ & $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is-

(1) $-\frac{3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$

(3) $\frac{6}{65}$ (4) $\frac{-6}{65}$

Q.12 If $y = (1 + \tan A)(1 - \tan B)$ where $A - B = \frac{\pi}{4}$, then $(y + 1)^{y+1}$ is equal to-

(1) 9 (2) 4 (3) 27 (4) 81

Q.13 If $\tan\left(\frac{\alpha}{2}\right)$ and $\tan\left(\frac{\beta}{2}\right)$ are the roots of the equation $8x^2 - 26x + 15 = 0$, then $\cos(\alpha + \beta)$ is equal to-

(1) $-\frac{627}{725}$ (2) $\frac{627}{725}$ (3) $-\frac{725}{627}$ (4) –1

Q.14 In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$, then-

(1) a + b = c (2) b + c = a

(3) a + c = b (4) b = c

Q.15 If θ and ϕ are acute angles such that $\sin\theta = \frac{1}{2}$ and $\cos\phi = \frac{1}{3}$ then $\theta + \phi$ lies in-

(1) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (2) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

(3) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (4) $\left(\frac{\pi}{6}, \pi\right]$

Q.16 If sin A, cos A and tan A are in G.P., then $\cos^3 A + \cos^2 A$ is equal to-

(1) 1 (2) 2

(3) 4 (4) None of these

Q.17 $\frac{\cos 20° + 8\sin 70° \sin 50° \sin 10°}{\sin^2 80°}$ is equal to -

(1) 1 (2) 2

(3) $\frac{3}{4}$ (4) None of these

Q.18 Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha$

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

Q.19 If $3\sin\theta + \cos\theta = 2$, then $3\cos\theta - \sin\theta$ is equal to-

(1) $-\sqrt{6}$ (2) $\sqrt{6}$ (3) $\sqrt{5}$ (4) $-\sqrt{5}$

Q.20 No. of solutions in the equation $x = 4\sin x$ when $x \in [0, 2\pi]$-

(1) 1 (2) 2 (3) 3 (4) 4

Q.21 The value of $\sum_{r=0}^{10} \cos^3\left(\frac{\pi r}{3}\right)$ is equal to-

(1) $\frac{-9}{2}$ (2) $\frac{-7}{2}$ (3) $\frac{-9}{8}$ (4) $\frac{-1}{8}$

Q.22 If $A = \cos^2\theta + \sin^4\theta$, then for all value of θ-

(1) $1 \leq A \leq 2$ (2) $\frac{3}{4} \leq A \leq 1$

(3) $\frac{13}{16} \leq A \leq 1$ (4) None of these

Q.23 The value of sin 10° + sin 20° + sin 30° + ... + sin 360° is equal to-

(1) 0 (2) 1 (3) $\sqrt{3}$ (4) 2

Q.24 cos 52° + cos 68° + cos 172° =

(1) 0 (2) 1

(3) 2 (4) None of these

Q.25 If $A = \sin^2 x + \cos^4 x$, then for all real x:

(1) $\frac{3}{4} \leq A \leq 1$ (2) $\frac{13}{16} \leq A \leq 1$

(3) $1 \leq A \leq 2$ (4) $\frac{3}{4} \leq A \leq \frac{13}{16}$

Q.26 In triangle ABC, $\angle C = \frac{2\pi}{3}$, then the value of $\cos^2 A + \cos^2 B - \cos A \cdot \cos B$ is equal to-

(1) $\frac{3}{4}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Q.27 If $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$, then range of $f(\theta)$ is-

(1) [–5, 11] (2) [–3, 9]

(3) [–2, 10] (4) [–4, 10]

Q.28 In a $\triangle PQR$ if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Q.29 $\cos^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$ is equal to -

(1) 4/5 (2) 5/2 (3) 5/4 (4) 3/4

Q.30 If θ_1 and θ_2 are two values lying in $[0, 2\pi]$ for which $\tan\theta = \lambda$, then $\tan\frac{\theta_1}{2} \cdot \tan\frac{\theta_2}{2}$ is -

(1) Zero (2) –1 (3) 2 (4) 1

Q.31 If A+B +C = π, then $\Sigma \tan\frac{A}{2}\tan\frac{B}{2}$ is equal to -

(1) 0 (2) –1 (3) 1/2 (4) 1

Q.32 ABCD is a trapezium such that AB and CD are parallel and BC $\perp$ CD. If $\angle$ADB = θ, BC = p and CD = q, then AB is equal to :

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

Q.33 If $\alpha+\beta+\gamma = 2\pi$, than -

(1) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(2) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\gamma}{2}\tan\frac{\alpha}{2}=1$

(3) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(4) None of these

Q.34 $\sin^2A + \sin^2(A - B) + 2\sin A\cos B\sin(B - A)$ is equal to -

(1) $\sin^2A$ (2) $\sin^2B$ (3) $\cos^2A$ (4) $\cos^2B$

Q.35 If $U_n = 2\cos n\theta$, then $U_1U_n - U_{n-1}$ is equal to -

(1) U_{n+2} (2) U_{n+1}

(3) U_2U_{n+1} (4) None of these

Q.36 The number of real solutions of the equation $\sin(e^x) = 2^x + 2^{-x}$ is -

(1) 1 (2) 0 (3) 2 (4) Infinite

Q.37 If $\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta$ then c is equal to -

(1) –5 (2) 1

(3) 5 (4) None of these

Q.38 If $f(x) = \frac{\sin 3x}{\sin x}$, $x \neq n\pi$, then the range of values of f (x) for real values of x is -

(1) [–1, 3] (2) $(-\infty, -1]$

(3) $(3, +\infty)$ (4) [–1, 3)

Q.39 If $\sin\theta = n\sin(\theta + 2\alpha)$, then $\tan(\theta + \alpha)$ is equal to -

(1) $\frac{1+n}{2-n}\tan\alpha$ (2) $\frac{1-n}{1+n}\tan\alpha$

(3) $\tan\alpha$ (4) $\frac{1+n}{1-n}\tan\alpha$

Q.40 The function $f(x) = \sin^4 x + \cos^4 x$ increases if

(1) $0 < x < \frac{\pi}{8}$ (2) $\frac{\pi}{4} < x < \frac{\pi}{2}$

(3) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (4) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

NUMERICAL VALUE TYPE QUESTIONS

Q.41 If $(1 - \cot 1°)(1 - \cot 2°)(1 - \cot 3°) \ldots\ldots (1 - \cot 44°) = 2^n$ then n equals_____.

Q.42 If the length of an arc of a circle of radius 5 cm subtending 15° angle of centre of circle is given by $\frac{k\pi}{12}$ cm. Then value of k is equal to _____.

Q.43 Let $0 \leq$ a, b, c, d $\leq \pi$, where b and c are not complementary, such that $2\cos a + 6\cos b + 7\cos c + 9\cos d = 0$ and $2\sin a - 6\sin b + 7\sin c - 9\sin d = 0$, then the value of $3\frac{\cos(a+d)}{\cos(b+c)}$ is _____.

Q.44 $16\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{3\pi}{8}\right)\left(\cos\theta-\cos\frac{5\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right) = \lambda\cos 4\theta$, then the value of λ is _____.

Q.45 Let f and g be function defined by $f(\theta) = \cos^2\theta$ and $g(\theta) = \tan^2\theta$, suppose α and β satisfy $2f(\alpha) - g(\beta) = 1$, then value of $2f(\beta) - g(\alpha)$ is_____.

Q.46 If $\frac{9x}{\cos\theta}+\frac{5y}{\sin\theta}=56$ and $\frac{9x\sin\theta}{\cos^2\theta}-\frac{5y\cos\theta}{\sin^2\theta}=0$ then value of $\left[(9x)^{\frac{2}{3}}+(5y)^{\frac{2}{3}}\right]^3$ is____.

Q.47 If $4\cos 36° + \cot\left(7\frac{1}{2}°\right) =$

$\sqrt{n_1}+\sqrt{n_2}+\sqrt{n_3}+\sqrt{n_4}+\sqrt{n_5}+\sqrt{n_6}$, then the value of $\sum_{i=1}^{6} n_i^2$ must be _____.

Q.48 The value of $f(x) = x^4 + 4x^3 + 2x^2 - 4x + 7$, when $x = \cot\frac{11\pi}{8}$ is _____.

Q.49 In any triangle, if

$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$, find the angle C (in degree).

Q.50 Let $x = \frac{\sum_{n=1}^{44} \cos n^\circ}{\sum_{n=1}^{44} \sin n^\circ}$ find the greatest integer that does not exceed.

Q.51 Find θ (in degree) satisfying the equation, $\tan 15^\circ \cdot \tan 25^\circ \cdot \tan 35^\circ = \tan\theta$, where $\theta \in (0, 45^\circ)$.

STATEMENT TYPE QUESTIONS

Each of the questions given below consist of Statement -I and Statement- II. Use the following key to choose the appropriate answer.

(A) Both Statement-I and Statement-II are true, and Statement-II is the correct explanation of Statement-I.

(B) Both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true.

Q.52 **Statement- I :** $\cos 36^\circ > \tan 36^\circ$

Statement- II : $\cos 36^\circ > \sin 36^\circ$

(1) A (2) B (3) C (4) D

Q.53 **Statement- I :** If A, B, C are the angles of a triangle such that angle A is obtuse, then $\tan B \tan C > 1$.

Statement- II : In any triangle, $\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$.

(1) A (2) B (3) C (4) D

Q.54 **Statement-I :** The number of roots of the equation $\sin px = x^2 - x + \frac{5}{4}$ is 2.

Statement-II : In $[0, 2\pi]$, $\sin x = \frac{1}{2}$ has exactly two solutions.

(1) A (2) B (3) C (4) D

Q.55 **Statement-I :** $\log_\pi \tan\left(-\frac{3\pi}{4}\right) = 0$

Statement-II : $\tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$

(1) A (2) B (3) C (4) D

Q.56 **Statement-I :** $\sec^2\theta . \operatorname{cosec}^2\theta = \sec^2\theta + \operatorname{cosec}^2\theta$

Statement-II : $1 + \tan^2\theta = \sec^2\theta$ & $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(1) A (2) B (3) C (4) D

MORE THAN ONE CORRECT TYPE QUESTIONS

Q.57 If $\frac{\sin\alpha}{\sin\beta} = \frac{\sqrt{3}}{2}$ and $\frac{\cos\alpha}{\cos\beta} = \frac{\sqrt{5}}{2}$, $0 < \alpha < \beta < \frac{\pi}{2}$, then

(1) $\tan\alpha = 1$ (2) $\tan\alpha = \frac{\sqrt{3}}{\sqrt{5}}$

(3) $\tan\beta = \frac{\sqrt{3}}{\sqrt{5}}$ (4) $\tan\beta = 1$

Q.58 Let $P = \sin25^\circ \sin35^\circ \sin60^\circ \sin85^\circ$ and $Q = \sin20^\circ \sin 40^\circ \sin75^\circ \sin80^\circ$. Which of the following relation(s) is (are) correct ?

(1) $P + Q = 0$ (2) $P - Q = 0$

(3) $P^2 + Q^2 = 1$ (4) $P^2 - Q^2 = 0$

Q.59 If A = sin 44° + cos 44° , B = sin 45° + cos 45° and C = sin 46° + cos 46°. Then, correct option(s) is/are

(1) A < B < C (2) C < B < A

(3) B > A (4) A = C

Q.60 If $\tan\left(\frac{x}{2}\right)$ = cosec x–sin x, then $\tan^2\left(\frac{x}{2}\right)$ is equal to

(1) $2-\sqrt{5}$ (2) $\sqrt{5}-2$

(3) $(9-4\sqrt{5})(2+\sqrt{5})$ (4) $(9+4\sqrt{5})(2-\sqrt{5})$

Q.61 Which of the following is possible in ΔABC?

(1) cos A + cos B + cos C = $\frac{3}{2}$

(2) cos A cos B cos C = 0

(3) sin A + sin B + sin C = $\sqrt{2}$ + 1

(4) sin A sin B sin C = $-\frac{3}{8}$

COMPREHENSION TYPE QUESTIONS

Q.62 In a ΔABC, if cos A. cos B. cos C = $\frac{\sqrt{3}-1}{8}$ and sin A. sin B. sin C = $\frac{3+\sqrt{3}}{8}$, then

(i) Value of tan A + tan B + tan C is

(1) $\frac{3+\sqrt{3}}{\sqrt{3}-1}$ (2) $\frac{\sqrt{3}+4}{\sqrt{3}-1}$

(3) $\frac{6-\sqrt{3}}{\sqrt{3}-1}$ (4) None of these

(ii) Value of Σ tan A.tan B =

(1) $5-4\sqrt{3}$ (2) $5+4\sqrt{3}$

(3) $6+4\sqrt{3}$ (4) $6-4\sqrt{3}$

(iii) Value of tan A, tan B and tan C are

(1) 1, $\sqrt{3}$, $\sqrt{2}$ (2) 1, $\sqrt{3}$, 2

(3) 1, 2, $\sqrt{3}$ (4) None of these

Q.63 If $\sqrt{1\pm\sin x}=\left|\cos\frac{x}{2}\pm\sin\frac{x}{2}\right|$ then

On the basis of above passage, answer the following questions :

(i) If $\frac{A}{2}=-140°$, then the value of $\sqrt{1+\sin A}-\sqrt{1-\sin A}$ is

(1) 2 sin $\frac{A}{2}$ (2) –2 sin $\frac{A}{2}$

(3) 2 cos $\frac{A}{2}$ (4) – 2 cos $\frac{A}{2}$

(ii) If $\frac{A}{2}=\frac{19\pi}{11}$, then 2 sin $\frac{A}{2}$ is

(1) $\sqrt{1-\sin A}+\sqrt{1+\sin A}$

(2) $\sqrt{1-\sin A}-\sqrt{1+\sin A}$

(3) $\sqrt{1+\sin A}-\sqrt{1-\sin A}$

(4) $-\sqrt{1+\sin A}-\sqrt{1-\sin A}$

(iii) The value of cos 9° – sin 9° is

(1) $\frac{\sqrt{5-\sqrt{5}}}{2}$ (2) $\frac{\sqrt{5+\sqrt{5}}}{4}$

(3) $\frac{\sqrt{5+\sqrt{5}}}{2}$ (4) $\frac{\sqrt{3-\sqrt{5}}}{4}$

MATCH THE COLUMN TYPE QUESTIONS

Q.64 Match the following :

Column -I	Column-II
(1) No. of solution for $\|x-2\| = [-2\pi]$ [.] denote greatest integer	(P) 1
(2) Value of $\sum_{r=0}^{20}\cos\frac{\pi r}{3}$ is	(Q) 3
(3) If $N = \sin^2\alpha + \cos\left(\frac{\pi}{3}-\alpha\right).\cos\left(\frac{\pi}{3}+\alpha\right)$ then $5 + \log_2 N$ equals	(R) 0
(4) If A,B,C,D are angles of a cyclic quadrilateral then cosA + cosB + cosC + cosD equals	(S) ϕ

Q.65 Match the following

Column -I	Column-II
(1) $\frac{\sqrt{3}}{\cos 290^\circ}+\frac{1}{\sin 250^\circ}=$	(P) $\cot\alpha$
(2) $\cos 20^\circ + 2\sin^2 55^\circ - \sqrt{2}\sin 65^\circ =$	(Q) $\sin 2\alpha$
(3) The expression $\frac{1}{4\sin^2\alpha\cos^2\alpha}-\frac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$ reduces to	(R) 1
(4) The expression $\sin^2\alpha\left(1+\frac{1}{\sin\alpha}+\cot\alpha\right)\left(1-\frac{1}{\sin\alpha}+\cot\alpha\right)$ reduces to	(S) 4
	(T) 0

ANSWER KEY

RANKER'S STUFF

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	1	3	2	1	1	2	1	1	2	1	3	1	1	2
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	1	2	1	2	2	4	2	1	1	2	1	4	2	4	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	4	1	1	2	2	2	3	4	4	2	22	5	7	2	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3136	91	6	60	241	5	2	4	4	3	1	2,4	2,4	3,4	2,3
Que.	61	62(i)	62(ii)	62(iii)	63(i)	63(ii)	63(iii)								
Ans.	1,2,3	1	2	4	2	4	1								

Q.64 A → R, S; B → P; C → Q; D → R **Q.65** A → S; B → R; C → R; D → Q