Sequence and series DPPS 01

① $T_1, T_2, T_3, T_4 - - - - T_{98}, T_{99}$.

$T_1+T_3+T_5+ \dots + T_{99} = 2550$

$a+(a+2d)+(a+4d)+ \dots +(a+98d)=2550$

$99 = 1+(n-1)2$

$\boxed{n=50}$

$\frac{50}{2}[a+a+98d] = 2550$

$25[2a+98d] = \cancel{2550} = 102$

$\boxed{a+49d=51}$

$S_{99} = \frac{99}{2}[2a+98d]$

$= \frac{99}{\cancel{2}} \times \cancel{2}[a+49d]$

$= 99\times 51 = 5049$

② $\frac{S_n}{S_n'} = \frac{5n+4}{9n+6}$

$\frac{t_{18}}{t_{18}'} = \frac{5\times(2\times18-1)+4}{9\times[2\times18-1]+6} = \frac{5\times35+4}{9\times35+6}$

$= \frac{179}{321}$

③ $\frac{a}{b}, \frac{b}{c}, \frac{c}{a} \to HP$

$\frac{b}{a}, \frac{c}{b}, \frac{a}{c} \to AP$

$\frac{2c}{b} = \frac{b}{a}+\frac{a}{c}$

$\frac{2c}{b} = \frac{bc+a^2}{ac}$

$2ac^2 = b^2c+ba^2$

$a^2b, c^2a, b^2c \to AP$

④ $25, 22\frac{3}{4}, 20\frac{1}{2}, 18\frac{1}{4} - - - -$

$25, \frac{91}{4}, \frac{41}{2}, - - - -$

$d = \frac{91}{4}-25 = \frac{-9}{4}$

Let $t_n > 0$

$25+(n-1)\frac{-9}{4} > 0$

$25 > \frac{9}{4}(n-1)$ $\quad n > 12 \dots$

$T_{12} = 25+11\times\frac{-9}{4}$

$= \boxed{1/4} \to$ numerically smallest

$T_{13} = 25+12\times\frac{-9}{4} = -2$

⑤ $T_1, T_2, T_3 \dots \quad T_{n-2}, T_{n-1}, T_n$

$T_1+T_2+T_3+T_4 = 56$

$\frac{4}{2}[2a+3d] = 56$

$2a+3d=28$ — ①

$T_n+T_{n-1}+T_{n-2}+T_{n-3} = 112$

$\frac{4}{2}[2\times(a+(n-1)d)+3\times(-d)] = 112$

$\boxed{2a+d(2n-5) = 56}$ — ②

$\boxed{a=11}$

$2\times11+3d=28$

$3d=6$

$\boxed{d=2}$

From 2

$2\times11+2(2n-5)=56$

$22+2(2n-5)=56$

$2(2n-5)=34$

$2n-5=17$

$2n=22$

$\boxed{n=11}$

⑥ $\log_2(5\times2^x+1), \log_4(2^{1-x}+1), 1 \to AP$

$2\log_4(2^{1-x}+1) = \log_2(5\cdot2^x+1)+\log_2 2$

$2\log_{2^2}(2^{1-x}+1) = \log_2(5\cdot2^x+1)+\log_2 2$

$\cancel{\frac{2}{2}}\log_2(2^{1-x}+1) = \log_2 2(5\cdot2^x+1)$

$2^{1-x}+1 = 10\cdot2^x+2$

$2^x = t \quad \frac{2}{t}+1 = 10t+2$

$2+t = 10t^2+2t$

$10t^2+\cancel{t}-2=0$

$10t^2+5t-4t-2=0$

$5t(2t+1)-2(2t+1)=0$

$(2t+1)(5t-2)=0$

$t \ne -1/2 \quad t=\frac{2}{5}$

$2^x = \frac{2}{5}$

$\log_2 2^x = \log_2\left(\frac{2}{5}\right)$

$x = \log_2 2 - \log_2 5$

$\boxed{x = 1-\log_2 5}$

(7) $20 + 19\frac{1}{3} + 18\frac{2}{3} + \cdots$

$20 + \frac{58}{3} + \frac{56}{3} + \cdots$

$t_n > 0$

$20 + (n-1)\left(-\frac{2}{3}\right) > 0$

$20 > \frac{2}{3}(n-1)$

$30 > n-1$

$n-1 < 30$

$\boxed{n < 31}$

$T_{30} \rightarrow$ last positive terms

maximum sum = sum of positive terms

$S_{30} = \frac{30}{2}[2a + 29d]$

$= 15\left[2 \times 20 + 29 \times -\frac{2}{3}\right]$

$= 15\left[40 - \frac{58}{3}\right]$

$= 15 \times \frac{120-58}{3}$

$= 5 \times 62 = 310$

(8) $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$

$S_{n+3} - S_{n+2} - 2S_{n+2} + 2S_{n+1} + S_{n+1} - S_n$

$(S_{n+3} - S_{n+2}) - 2(S_{n+2} - S_{n+1}) + (S_{n+1} - S_n) = 0$

$t_n = S_n - S_{n-1}$

$t_{n+3} - 2t_{n+2} + t_{n+1}$

$t_{n+3} - t_{n+2} - t_{n+2} + t_{n+1}$

$(t_{n+3} - t_{n+2}) - (t_{n+2} - t_{n+1})$

$d - d = 0$

(9) $a, b, c \rightarrow AP$

$2b = a + c$

$\boxed{a + c - 2b = 0}$

$a^3 + c^3 + (-2b)^3$

$= 3a.c.(-2b) = -6abc$

(10) $T_1, T_2, T_3 \cdots T_{2n-1}, T_{2n}$

$T_1 + T_3 + T_5 + \cdots T_{2n-1} = 24$ — (i)

$T_2 + T_4 + T_6 + \cdots + T_{2n} = 30$ — (ii)

$T_{2n} = a + 10\frac{1}{2}$ — (iii)

$a + a + 2d + a + 4d + \cdots + a + (2n-2)d = 24$

$\frac{n}{2}[2 \times a + (n-1)2d] =$

$n[a + (n-1)d] = 24$

$an + (n^2 - n)d = 24$ — (a)

$T_2 + T_4 + T_6 + \cdots + T_{2n} = 30$

$\frac{n}{2}[2 \times \cancel{(a+d)} + (n-1)2d] = 30$

$n[a + \cancel{d} + dn - \cancel{d}] = 30$

$n[a + nd] = 30$

$na + n^2 d = 30$ — (b)

(a) − (b) $\cancel{na} + (n^2 - n)d - \cancel{na} - n^2 d = -30 + 24$

$\cancel{n^2 d} - nd - \cancel{n^2 d} = -6$

$\boxed{nd = 6}$

from (iii) $T_{2n} = a + 21/2$

$\cancel{a} + (2n-1)d = \cancel{a} + \frac{21}{2}$

$(2n-1) \times \frac{6}{n} = \frac{21}{2}$

$2(12n - 6) = 21n$

$24n - 12 = 21n$

$3n = 12 \quad n = 4$

Hence No of terms $2n = 2 \times 4 = 8$.

(11) $a_3 + a_5 + a_8 = 11$

$a + 2d + a + 4d + a + 7d = 11$

$3a + 13d = 11$ — (i)

$a_4 + a_2 = -2$

$a + 3d + a + d = -2$

$2a + 4d = -2$

$a + 2d = -1$ — (ii)

$3a + 13d = 11$
$\cancel{3}a + 6d = -3$
$\underline{- \quad - \quad +}$
$7d = 14$
$\boxed{d = 2}$
$a + 4 = -1$
$\boxed{a = -5}$

$a_1 + a_6 + a_7$
$= a + a + 5d + a + 6d$
$= 3a + 11d$
$= 3 \times -5 + 11 \times 2 = 7$

(12) $a_1, a_2, a_3 - - - - a_n \to AP$
$a_p, a_q, a_r \to AP$
$a_q - a_p = a_r - a_q$
$a + (q-1)d - (a + (p-1)d) = a + (r-1)d - (a + (q-1)d)$
$d(q - \cancel{1} - p + \cancel{1}) = d(r - \cancel{1} - q + \cancel{1})$
$\cancel{d}(q-p) = \cancel{d}(r-q)$
$\boxed{2q = p + r}$
$p, q, r \to AP$.

(13) $\frac{S_{nx}}{S_x} = \frac{\frac{n\cancel{x}}{\cancel{2}}[2a + (nx-1)d]}{\frac{\cancel{x}}{\cancel{2}}[2a + (x-1)d]}$
$= n\left[\frac{2a - d + nxd}{2a - d + xd}\right]$
if $\boxed{2a - d = 0}$ ⇒ $d = 2a$
then $\frac{S_{nx}}{S_x} = n\left(\frac{nx\cancel{d}}{\cancel{x}\cancel{d}}\right) = n^2$
↳ is indep of x

$S_p = \frac{p}{2}[2a + (p-1)d]$
$= \frac{p}{2}[2a - d + pd]$
$= \frac{p}{2} \times pd$
$= \frac{p^2}{2} \times 2a = ap^2$

(14) $x, y, z \to GP$
$K = a^x = b^y = c^z \to GP$
$a^x = K$, $b^y = K$, $c^z = K$
$x = \log_a K$ $\quad y = \log_b K$ $\quad z = \log_c K$
$\because x, y, z \to GP$
$y^2 = xz$
$(\log_b K)^2 = (\log_a K \times \log_c K)$
$\frac{\cancel{(\log K)^2}}{(\log b)^2} = \frac{\cancel{\log K}}{\log a} \times \frac{\cancel{\log K}}{\log c}$
$\boxed{\log a \log c = (\log b)^2}$
c option satsfied above result.

(15) $a^2 + b^2, ab + bc, b^2 + c^2 \to GP$
$(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$
$\cancel{a^2b^2} + \cancel{b^2c^2} + 2ab^2c = \cancel{a^2b^2} + a^2c^2 + b^4 + \cancel{b^2c^2}$
$b^4 + a^2c^2 - 2ab^2c = 0$
$(b^2 - ac)^2 = 0$
$b^2 = ac \Rightarrow a, b, c \to GP$

(16) $a_1, a_2, a_3 - - - - a_n, a_{n+1} \to AP$
$d = a_2 - a_1 = a_3 - a_2 = - - \quad = a_{n+1} - a_n = d$
$\left(\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \frac{1}{a_3a_4} + - - - + \frac{1}{a_na_{n+1}}\right)$
$\frac{1}{d}\left[\frac{d}{a_1a_2} + \frac{d}{a_2a_3} + \frac{d}{a_3a_4} + - - + \frac{d}{a_na_{n+1}}\right]$
$\frac{1}{d}\left[\frac{a_2 - a_1}{a_1a_2} + \frac{a_3 - a_2}{a_2a_3} + \frac{a_4 - a_3}{a_3a_4} + - - + \frac{a_{n+1} - a_n}{a_na_{n+1}}\right]$
$\frac{1}{d}\left[\frac{1}{a_1} - \cancel{\frac{1}{a_2}} + \cancel{\frac{1}{a_2}} - \cancel{\frac{1}{a_3}} + \cancel{\frac{1}{a_3}} - \cancel{\frac{1}{a_4}} + \cdot \cdot + \cancel{\frac{1}{a_n}} - \frac{1}{a_{n+1}}\right]$
$\frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_{n+1}}\right]$
$\frac{1}{d}\left[\frac{a_{n+1} - a_1}{a_1 a_{n+1}}\right] = \frac{1}{\cancel{d}}\left[\frac{\cancel{a_1} + n\cancel{d} - \cancel{a_1}}{a_1 a_{n+1}}\right]$
$= \frac{n}{a_1 a_{n+1}}$

(17) $\boxed{a=4}$ $T_9 = a+8d = 20$

$4+8d = 20$

$\boxed{d=2}$

$T_{15} = a+14d$

$= 4+14\times 2 = 32$

(18) $a, b, c \to AP$

$a=1 \quad b=25 \quad c=49$

$\sqrt{a}=1 \quad \sqrt{b}=5 \quad \sqrt{c}=7$

$\frac{1}{5+7}, \frac{1}{1+7}, \frac{1}{1+5}$

$\frac{1}{12}, \frac{1}{8}, \frac{1}{6}$

$\frac{1}{8}-\frac{1}{12} = \frac{1}{6}-\frac{1}{8}$

$\frac{1}{24} = \frac{1}{24} \to \underline{AP}$

(19) $\frac{S_n}{S_n'} = \frac{2n+3}{6n+5}$

$\frac{t_{13}}{t_n'} = \frac{2(2n-1)+3}{6(2n-1)+5}$

$\frac{t_{13}}{t_{13}'} = \frac{2(2\times 13-1)+3}{6(2\times 13-1)+5}$

$= \frac{53}{155}$

(20) $1, \log_9(3^{1-x}+2), \log_3(4\cdot 3^x-1) \to AP$

$2\log_{3^2}(3^{1-x}+2) = \log_3 3 + \log_3(4\cdot 3^x-1)$

$\cancel{\frac{2}{2}}\log_3(3^{1-x}+2) = \log_3 3\times[4\cdot 3^x-1]$

$3^{1-x}+2 = 12\cdot 3^x-3$

$\frac{3}{3^x}+2 = 12\cdot 3^x-3$

$3^x=t \quad \frac{3}{t}+2 = 12t-3$

$3+2t = 12t^2-3t$

$12t^2-5t-3=0$

$12t^2-9t+4t-3=0$

$3t(4t-3)+1(4t-3)=0$

$t=3/4 \quad \cancel{t=-1/3}$

$3^x = \frac{3}{4}$

$\log_3 3^x = \log_3\left(\frac{3}{4}\right)$

$x = \log_3 3 - \log_3 4$

$\boxed{x = 1-\log_3 4}$