26. $x^2-2(1+3k)x+7(3+2k)=0$

$\boxed{D=0}$

$(2(1+3k))^2-4\times7\times(3+2k)=0$

$4(1+9k^2+6k)-4\times7\times(3+2k)=0$

$4(1+9k^2+6k-21-14k)=0$

$9k^2-8k-20=0$

$9k^2-18k+10k-20=0$

$9k[k-2]+10[k-2]=0$

$(k-2)(9k+10)=0$

$k=2 \qquad k=-\frac{10}{9}$

27. $ix^2-2(1+i)x+2-i$

Let $\alpha=2-i$

$\alpha+\beta=-\frac{(-2(1+i))}{i}$

$2-i+\beta=2\frac{(1+i)}{i}$

$2-i+\beta=2\frac{(1+i)}{i}\times\frac{i}{i}$

$2-i+\beta=2\frac{(i+i^2)}{i^2}$

$2-i+\beta=2\frac{(-1+i)}{(-1)}$

$2-i+\beta=-2(-1+i)$

$\cancel{2}-i+\beta=\cancel{2}-2i$

$\beta=-2i+i=-i$

Quadratic DPPS-3 (Solution)

1. $x^2+4x+6=0$, $ax^2+bx+c=0$

$D=(4)^2-4\times6<0$

$D<0$ then Both roots are common (तब दोनों मूल उभयनिष्ठ हैं)

$\frac{a}{1}=\frac{b}{4}=\frac{c}{6}$

$a:b:c=1:4:6$

2. $x^2+bx+1=0$ —(1)

$x^2+x+b=0$ —(2)

$(b-1)x=b-1$

$\boxed{x=1}\to$ common root (उभयनिष्ठ मूल)

Hence $x=1\to$ satisfy $x^2+bx+1=0$

$1+b+1=0$

$\boxed{b=-2}$

3. $ax^2+bx+c=0$, $2x^2+3x+4=0$

$D=(3)^2-4\times2\times4<0$

Hence Both roots are common (तब दोनों मूल उभयनिष्ठ हैं)

$\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k \qquad a=2k \quad b=3k \quad c=4k$

$\cancel{f(x)=ax^2+bx+c}$

$\cancel{f(2)=4a+4b+c<0}$ ~~(given)~~

$\frac{a+c}{b}=\frac{2k+4k}{3k}=\frac{6k}{3k}=2$

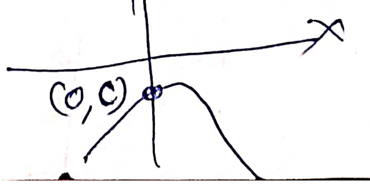
4. $f(x)=ax^2+2bx+c$

$f(2)=a\times2^2+2b\times2+c$

$f(2)=4a+4b+c<0$ (given) —(1)

roots are Imaginary (मूल काल्पनिक हैं) —(2)

According to (1) and (2)



$f(0)=0+0+c=c$

Pt $(0,c)$ lies on Negative y axis then $c<0$.

(5) Let $f(x) = 3ax^2 + 4bx + c$

$f(0) = 0 + 0 + c > 0$ (given)

$f(0) = c > 0$

$f(0) > 0$, Eqn has Imag. roots then graph lies above the x axis hence $f(x) > 0 \ \forall x$

$\Rightarrow f(-1) = 3a(-1)^2 - 4b + c > 0$

$\boxed{3a + c > 4b}$

(6) $y = \dfrac{1}{4x^2 + 2x + 1}$

$4yx^2 + 2xy + y = 1$

$4yx^2 + 2xy + y - 1 = 0$

$\boxed{x \in R}$ then $D \geq 0$

$(2y)^2 - 4 \times 4y(y-1) \geq 0$

$4y^2 - 16(y^2 - y) \geq 0$

$4(y^2 - 4y^2 + 4y) \geq 0$

$4y - 3y^2 \geq 0$

$-3y^2 + 4y \geq 0$

$-y(3y - 4) \geq 0$

$y(3y - 4) \leq 0$

$y \in \left(0 \ \frac{4}{3}\right]$

max $= 4/3$

(7) $\alpha, \beta \xrightarrow{\text{roots}} ax^2 + bx + c = 0$,

~~finding eqn~~

finding eqn

roots $2+\alpha, 2+\beta$

$x = 2 + \alpha$

$\alpha = x - 2$

Replace x by $x-2$

$a(x-2)^2 + b(x-2) + c = 0$

$a(x^2 + 4 - 4x) + bx - 2b + c = 0$

$ax^2 + 4a - 4xa + bx - 2b + c = 0$

$= ax^2 + x(b - 4a) + 4a - 2b + c = 0$

(8) $\alpha, \beta \xrightarrow{\text{roots}} ax^2 + bx + c = 0$

$ax^2 - bx(x-1) + c(x-1)^2 = 0$

$\dfrac{ax^2}{(x-1)^2} + \dfrac{-bx(x-1)}{(x-1)^2} + \dfrac{c(x-1)^2}{(x-1)^2} = 0$

$a\left(\dfrac{x}{x-1}\right)^2 + \dfrac{bx}{(1-x)} + c = 0$

$\dfrac{x}{1-x} = \alpha$

$x = \alpha - \alpha x$

$\boxed{x = \dfrac{\alpha}{1+\alpha}}$ $\boxed{y = \dfrac{\beta}{1+\beta}}$

(9) $\alpha, \beta, \gamma \xrightarrow{\text{roots}} x^3 + 4x + 1 = 0$

$\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = 4$

$\alpha\beta\gamma = -1$

$\alpha + \beta = -\gamma$

$\dfrac{1}{\alpha+\beta} = \dfrac{-1}{\gamma}$ $\quad \dfrac{1}{\beta+\gamma} = \dfrac{-1}{\alpha}$ $\quad \dfrac{1}{\gamma+\alpha} = \dfrac{-1}{\beta}$

$(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$

$= -\dfrac{1}{\gamma} - \dfrac{1}{\alpha} - \dfrac{1}{\beta}$

$= -\left[\dfrac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right]$

$= -\left[\dfrac{4}{-1}\right] = 4$

(10) $x^2+5|x|+6=0$

let $x^2=|x|^2$

$|x|^2+5|x|+6=0$

$|x|=t$ $\quad t^2+5t+6=0$

$t=-2 \quad t=-3$

$|x|\neq-2 \quad |x|\neq-3$

No real roots.

(11) $|x-2|^2+|x-2|-2=0$

$|x-2|=t$ $\quad t^2+t-2=0$

$t^2+2t-t-2=0$

$t(t+2)-1(t+2)=0$

$(t+2)(t-1)=0$

$t\neq-2 \quad t=1$

$|x-2|=1$

$x-2=\pm1$

$x=2\pm1$

$x=3 \quad x=1$

sum of all roots $=3+1=4$

(12) $x^2+2(k+1)x+9k-5=f(x)$

$D\geq0$

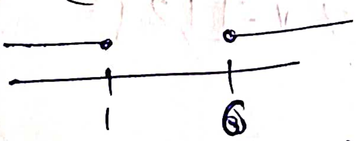
$[2(k+1)]^2-4\times(9k-5)\geq0$

$4(k^2+1+2k)-4(9k-5)\geq0$

$4(k^2+1+2k-9k+5)\geq0$

$4(k^2-7k+6)\geq0$

$(k-1)(k-6)\geq0$

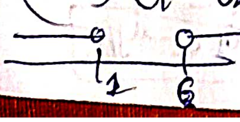


$k\in(-\infty,1]\cup[6,\infty)$

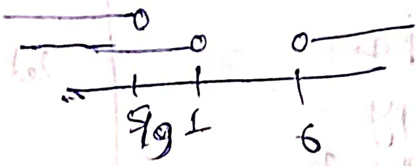
(13) roots of opposite sign

(1) $D>0$ $\quad$ (2) $P<0$

$(k-1)(k-6)>0$ $\qquad$ $9k-5<0$



$\boxed{k<5/9}$



$\boxed{k<5/9}$

(14) $f(x)=(x-k)(x-10)+1$

$=x^2-10x-kx+10k+1$

$=x^2-(10+k)x+10k+1$

$D=(10+k)^2-4(10k+1)$

$=100+k^2+20k-40k-4$

$=k^2-20k+96$

$D=(k-12)(k-8)$

$x=\dfrac{(10+k)\pm\sqrt{(k-12)(k-8)}}{2}$

$k=12$ or $k=8$

then $\boxed{D=0}$

(15) $x^2+ax+10=0$

$x^2+bx+10=0$

$(a-b)x+20=0$

$\boxed{x=\dfrac{-20}{a-b}}$

$x=\frac{-20}{a-b}$ satisfied both eqn.

$\left(\frac{-20}{a-b}\right)^2+a\times\frac{-20}{a-b}+10=0$

$\frac{400}{(a-b)^2}-\frac{20a}{a-b}+10=0$

$40-2a(a-b)+1(a-b)^2=0$

$40-2a^2+\cancel{2ab}+a^2+b^2-\cancel{2ab}=0$

$b^2-a^2=-40$

$\boxed{a^2-b^2=40}$

(16) $y = \dfrac{x^2-6x+4}{x^2+2x+4}$

$yx^2+2xy+4y = x^2-6x+4$

$(y-1)x^2+(2y+6)x+4(y-1)=0$

$D \geq 0$

$(2y+6)^2-16(y-1)^2 \geq 0$

$[2y+6+4(y-1)](2y+6-4(y-1)) \geq 0$

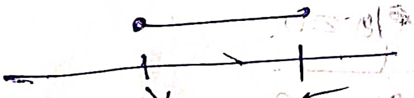
$(6y+2)(10-2y) \geq 0$

$(3y+1)(10-2y) \geq 0$

$(3y+1)(2y-10) \leq 0$

$(3y+1)\,2(y-5) \leq 0$

$(3y+1)(y-5) \leq 0$



$-1/3 \qquad 5 \qquad y \in \left[-\frac{1}{3}, 5\right]$

min value $= -\frac{1}{3}$

max value $= 5$

(17) α, β roots $ax^2+bx+c=0$

$\alpha+\beta = -b/a \qquad \alpha\beta = c/a$

$1+\alpha\beta+|\alpha+\beta|$

given $\alpha < -1$ and $\beta > 1$

let $\alpha = -2 \quad \beta = 2$

$1+-2\times 2+|-2+2|$

$1-4+0 = -3 < 0$

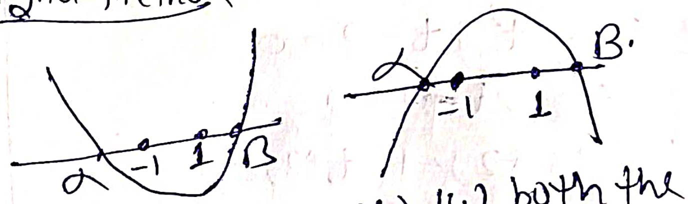
let $\alpha = -3 \quad \beta = 4$

$1+(-3\times 4)+|-3+4|$

$1-12+1 = -10 < 0$

Hence $1+\frac{c}{a}+\left|\frac{b}{a}\right| < 0$

(18) 2nd method



$\Rightarrow$ 1 and -1 lies b/w both the roots then

$f(1)<0$ and $f(-1)<0$

$f(x) = ax^2+bx+c$

$f(1) = a+b+c < 0$

$\frac{a}{a}+\frac{b}{a}+\frac{c}{a} < \frac{0}{a}$

$1+\frac{c}{a}+\frac{b}{a} < 0$ — (i)

$f(-1) = a-b+c < 0$

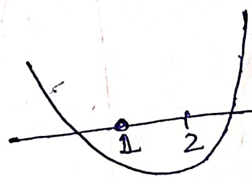
$\frac{a}{a}-\frac{b}{a}+\frac{c}{a} < 0$

$1+\frac{c}{a}-\frac{b}{a} < 0$ — (ii)

from (i) and (ii)

$1+\frac{c}{a}+\left|\frac{b}{a}\right| < 0$

(18) $x^2+2ax+a<0 \quad \forall x \in [1,2]$

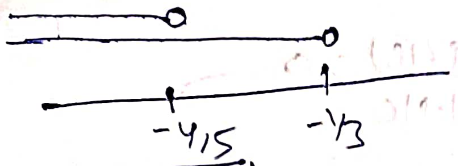


$f(1)<0$ and $f(2)<0$

$1+2a+a<0$ and $2^2+4a+a<0$

$3a<-1$ and $5a<-4$

$a<-1/3$ and $\boxed{a<-4/5}$



$\boxed{a < -4/5}$

$a \in (-\infty, -4/5)$

(19) $\alpha, \beta \xrightarrow{\text{roots}} ax^2+bx+c=0$

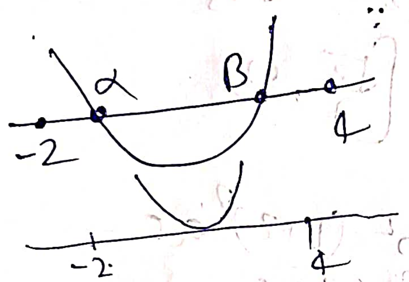
$\alpha+\beta = -\frac{b}{a} > 0$ $\quad \alpha\beta = \frac{c}{a} < 0$

$\alpha<\beta$ $\quad \boxed{\alpha = -ve}$ $\quad \beta = +ve$.

But $|\alpha|<\beta$

d Ans

(20) $\because a>0$



① $D\ge0$ ② $f(-2)>0$ and $f(4)>0$

③ $-2 < -\frac{b}{2a} < 4$ $\quad f(x) = x^2-2kx+k^2-1$

(i) $(-2k)^2 - 4\times(k^2-1) \ge 0$

$4k^2-4k^2+4 \ge 0$

$4\ge0$ (always)

(ii) (a) $f(-2) = (-2)^2+4k+k^2-1>0$

$= k^2+4k+3>0$

$=(k+3)(k+1)>0$

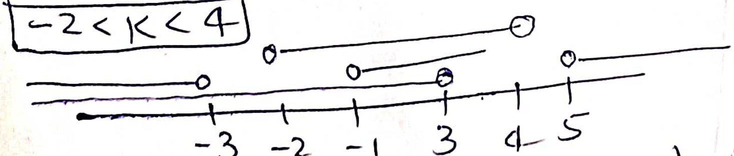
(b) $f(4) = 4^2-8k+k^2-1>0$

$= k^2-8k+15>0$

$(k-5)(k-3)>0$

(iii) $-2 < \frac{2k}{2\times1} < 4$

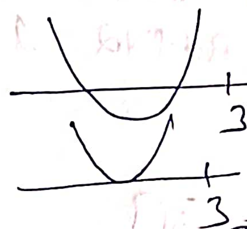
$\boxed{-2<k<4}$



$k\in(-1,3) \to$ common part

(21) $x^2-2ax+a^2+a-3=0$

$f(x) = x^2-2ax+a^2+a-3$



① $D\ge0$ ② $3 > -\frac{b}{2a}$ ③ $f(3)>0$

① $(2a)^2 - 4\times[a^2+a-3] \ge 0$

$4[\cancel{a^2}-\cancel{a^2}-a+3]\ge0$

$3-a\ge0$

$\boxed{a\le3}$

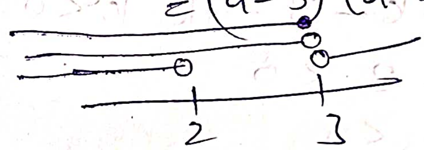
② $3 > -\frac{(-2a)}{2\times1}$

$3>a \Rightarrow \boxed{a<3}$

③ $f(3) = 3^2-6a+a^2+a-3$

$=a^2-5a+6>0$

$=(a-3)(a-2)>0$



$\boxed{a<2}$

(22) $x^3-px^2+qx+r=0$

Let roots of above eqn be

$\alpha, -\alpha, \beta$

$\alpha+(-\alpha)+\beta = p$

$\boxed{\beta=p}$

$\alpha\times-\alpha + \cancel{-\alpha\times\beta} + \cancel{\alpha\times\beta} = q$

$\boxed{-\alpha^2=q}$

$\alpha\times-\alpha\times\beta = +r$

$-\alpha^2\times\beta = +r$

$q\times p = r$

$\boxed{pq=r}$

(23) α, β, γ roots $\rightarrow x^3 + x + 1 = 0$

$\alpha+\beta+\gamma = 0$, $\alpha\beta\gamma = -1$

$\alpha+\beta = -\gamma$ $\quad$ $\alpha\beta+\beta\gamma+\gamma\alpha = 1$

$\alpha+\beta = -\gamma$

$(\alpha+\beta)^3 = (-\gamma)^3$

$\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta) = -\gamma^3$

$\alpha^3+\beta^3+\gamma^3 = -3\alpha\beta(\alpha+\beta)$

$= -3 \times \left(\frac{-1}{\gamma}\right) \times (-\gamma)$

$\boxed{\alpha^3+\beta^3+\gamma^3 = -3}$

Que 24:

$(1+k)x^2 - 2(1+2k)x + 3+k = 0$

$D = [2(1+2k)]^2 - 4(1+k)(3+k) < 0$

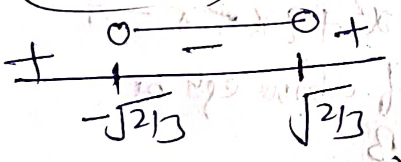
$4[1+4k^2+4k] - 4[3+3k+k+k^2] < 0$

$1+4k^2+4k - 3 - 4k - k^2 < 0$

$3k^2 - 2 < 0$

$k^2 - 2/3 < 0$

$(k+\sqrt{2/3})(k-\sqrt{2/3}) < 0$



$k \in (-\sqrt{2/3}, \sqrt{2/3})$

$\boxed{k=0}$ one value of k exist

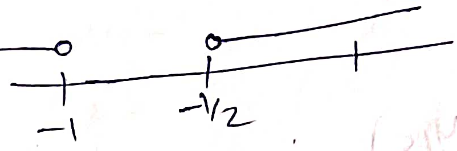
(25) roots are +ve

(a) $D \geq 0$ (b) $S > 0$ (c) $P > 0$

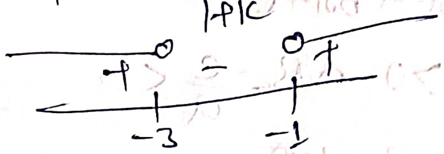
from 24 Question

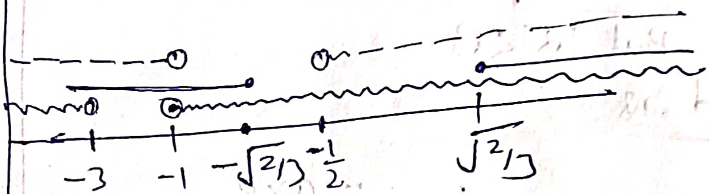
$D = 3k^2 - 2 \geq 0$

$S = +\frac{2(1+2k)}{1+k} > 0$



$P = \frac{3+k}{1+k} > 0$





Ans $(-\infty, -3) \cup \left[\sqrt{\frac{2}{3}}, \infty\right)$

(26) $\alpha : \beta = 1:2$

then $1 \times 2\, b^2 = (1+2)^2 ac$

$2b^2 = 9ac$

$2 \times [-2(1+2k)]^2 = 9 \times (1+k)(3+k)$

$8(1+4k^2+4k) = 9(3+3k+k+k^2)$

$8+32k^2+32k = 9k^2+36k+27$

$23k^2 - 4k - 19 = 0$

$23k^2 - 23k + 19k - 19 = 0$

$23k(k-1) + 19(k-1) = 0$

$(k-1)(23k+19) = 0$

$k=1$ $\quad$ $k = -\frac{19}{23}$

two values of k

27) $$y = \frac{x^2-3x-4}{x^2-3x+4}$$

$yx^2-3xy+4y = x^2-3x-4$

$(y-1)x^2+3x-3xy+4y+4=0$

$(y-1)x^2+3x(1-y)+4(y+1)=0$

$\because x\in R \Rightarrow D\geq 0$

$D=[3(1-y)]^2-4\times4(y^2-1)\geq0$

$=9(1+y^2-2y)-16(y^2-1)\geq0$

$=9+9y^2-18y-16y^2+16\geq0$

$=-7y^2-18y+25\geq0$

$7y^2+18y-25\leq0$

$7y^2+25y-7y-25\leq0$

$y(7y+25)-1(7y+25)\leq0$

$(7y+25)(y-1)\leq0$

$+ \quad - \quad +$

$-25/7 \qquad 1$

$y\in\left[\frac{-25}{7}, 1\right)$

y = 1 is not possible

$-\frac{25}{7} \qquad 1$

$y=-3,-2,-1,0$ → possible range.

$(-3)+(-2)+(-1)+0=-6$

28)

-1

① $D\geq0$ ② $-\frac{b}{2a}>-1$ ③ $f(-1)>0$

① $(b+1)^2-4\times(b-1)\geq0$

$b^2+1+2b-4b+4\geq0$

$b^2-2b+5\geq0$

always ≥0 because $D<0$ — ①

② $-\frac{b}{2a}>-1$

$-\frac{(-(b+1))}{2\times1}>-1$

$2\times\frac{b+1}{2}>-1\times2$

$b+1>-2$

$\boxed{b>-3}$ — ②

③ $f(-1)>0$

$1+b+\cancel{1}+b-\cancel{1}>0$

$2b+1>0$

$\boxed{b>-1/2}$ — ③

from ①, ② and ③

$-3 \qquad -1/2$

$\boxed{b>-1/2}$

$b\in\left(-\frac{1}{2}, \infty\right)$

29) $g(x)>-2$

$x^2-(b+1)x+b-1>-2$

$x^2-(b+1)x+b-1+2>0$

$x^2-(b+1)x+b+1>0$

$a>0$ and $D<0$

$D=(b+1)^2-4(b+1)<0$

$=(b+1)(b+1-4)<0$

$=(b+1)(b-3)<0$

$-1 \qquad 3$

$b\in(-1,3)$

largest natural no $b=2$

(30) $\alpha, \beta \xrightarrow{\text{roots}} ax^2+bx+c=0$

$a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0$

$a\frac{(2x+1)^2}{(x-1)^2} + b\frac{(2x+1)}{x-1} + \frac{c\,\cancel{(x-1)^2}}{\cancel{(x-1)^2}} = 0$

$a\left[\frac{2x+1}{x-1}\right]^2 + b\left[\frac{2x+1}{x-1}\right] + c = 0$

Let $\alpha = \frac{2x+1}{x-1}$

$\alpha x - \alpha = 2x+1$

$(\alpha-2)x = \alpha+1$

$\boxed{x = \frac{\alpha+1}{\alpha-2}}$ Roots are $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$

(31) $\alpha, \beta \xrightarrow{\text{roots}} 2x^2+4x-5=0$

find ~~roots~~ eqn whose roots are reciprocal of $2\alpha-3$ and $2\beta-3$, ie

$\Rightarrow$ find ~~roots to~~ eqn whose roots are $\frac{1}{2\alpha-3}$ and $\frac{1}{2\beta-3}$

Let $x = \frac{1}{2\alpha-3}$

$2x\alpha - 3x = 1$

$\boxed{\alpha = \frac{1+3x}{2x}}$

Replace x by $\frac{1+3x}{2x}$

$2\times\left(\frac{1+3x}{2x}\right)^2 + 4\times\left(\frac{1+3x}{2x}\right) - 5 = 0$

$2(1+3x)^2 + 4(1+3x)\cdot 2x - 5\cdot(2x)^2 = 0$

$2(1+9x^2+6x) + 8x + 24x^2 - 20x^2 = 0$

$2+18x^2+12x + 8x + 24x^2 - 20x^2 = 0$

$22x^2 + 20x + 2 = 0$

$\boxed{11x^2+10x+1 = 0}$

(32) $\alpha, \beta \xrightarrow{\text{roots}} px^2 - qx + r = 0 \quad \alpha\beta = r/p \quad \alpha + \beta = q/p$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(\frac{q}{p}\right)^2 - 2 \times \frac{r}{p} = \frac{q^2 - 2pr}{p^2}$$

$$x^2 - \left[\alpha^2 + \frac{r}{p} + \beta^2 + \frac{r}{p}\right]x + \left(\alpha^2 + \frac{r}{p}\right)\left(\beta^2 + \frac{r}{p}\right) = 0$$

$$x^2 - \left[\alpha^2 + \beta^2 + \frac{2r}{p}\right]x + (\alpha\beta)^2 + \frac{r}{p}(\alpha^2 + \beta^2) + \frac{r^2}{p^2} = 0$$

$$x^2 - \left[\frac{q^2 - 2pr}{p^2} + \frac{2r}{p}\right]x + \left(\frac{r}{p}\right)^2 + \frac{r}{p}\left[\frac{q^2 - 2pr}{p^2}\right] + \frac{r^2}{p^2} = 0$$

$$p^3x^2 - pq^2x + \cancel{r^2p} + rq^2 - \cancel{2pr^2} + \cancel{pr^2} = 0$$

$$\boxed{p^3x^2 - pq^2x + rq^2 = 0}$$