

# Chapter 02

## Sets and Relations

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### 1. DEFINITION OF SET :

A set is a collection of well defined objects which are distinct from each other. Set are generally denoted by capital letters A, B, C, .... etc. and the elements of the set by a, b, c .... etc.

If a is an element of a set A, then we write  $a \in A$  and say a belongs to A. If a does not belong to A then we write  $a \notin A$ , e.g. The collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

#### Some important number sets :

$N$  = Set of all natural numbers  
= {1, 2, 3, 4, ....}

$W$  = Set of all whole numbers  
= {0, 1, 2, 3, ....}

$Z$  or  $I$  set of all integers  
= {.... -3, -2, -1, 0, 1, 2, 3, ....}

$Z^+$  = Set of all +ve integers  
= {1, 2, 3, ....} =  $N$ .

$Z^-$  = Set of all -ve integers  
= {-1, -2, -3, ....}

$Z_0$  = The set of all non-zero integers.  
= {±1, ±2, ±3, ....}

$Q$  = The set of all rational numbers.  
=  $\left\{ \frac{p}{q} : p, q \in I, q \neq 0 \right\}$

$Q^+$  = The set of all positive rational numbers.

$Q^-$  = The set of all negative rational numbers.

$Q_0$  = The set of all non-zero rational numbers.

$R$  = The set of all real numbers.

$R^+$  = The set of all positive real numbers.

$R^-$  = The set of all negative real numbers.

$R_0$  = The set of all non-zero real numbers.

$R-Q$  = The set of all irrational numbers

e.g.  $\sqrt{2}, \sqrt{3}, \sqrt{5}, \dots, \pi, e, \log 2$  etc. are all irrational numbers.

$C$  = the set of complex number

$$= \{x + iy : x, y \in \mathbb{R}, y \neq 0, i = \sqrt{-1}\}$$

## 2. REPRESENTATION OF SETS

### (1) Set Listing Method (Roster Method) :

In this method a set is described by listing all the elements, separated by commas, within brackets.

### (2) Set builder Method (Set Rule Method) :

In this method, a set is described by characterizing property  $P(x)$  of its elements  $x$ . In such case the set is described by  $\{x : P(x) \text{ holds}\}$  or  $\{x \mid P(x) \text{ holds}\}$ , which is read as the set of all  $x$  such that  $P(x)$  holds. The symbol ' $\mid$ ' or ':' is read as such that.

## 3. TYPES OF SETS

### (A) Finite set :

A set  $X$  is called a finite set if its elements can be listed by counting or labeling with the help of natural numbers and the process terminates at a certain natural number  $n$ . i.e.  $n(X) = \text{finite no.}$

eg (A) A set of English Alphabets (B) Set of soldiers in Indian Army

### (B) Infinite set :

A set whose elements cannot be listed or counted by the natural numbers  $(1, 2, 3, \dots, n)$  for any number  $n$ , is called a infinite set.

e.g. (a) A set of all points in a plane

$$(b) X = \{x : x \in \mathbb{R}, 0 < x < 0.0001\}$$

$$(c) X = \{x : x \in \mathbb{Q}, 0 \leq x \leq 0.0001\}$$

### (C) Singleton set :

A set consisting of a single element is called a singleton set. i.e.  $n(X) = 1$ ,

e.g.  $\{x : x \in \mathbb{N}, 1 < x < 3\}$ ,  $\{\{\}\}$ :Set of null set,  $\{\phi\}$  is a set containing alphabet  $\phi$ .

### (D) Null set :

A set is said to be empty, void or null set if it has no element in it, and it is denoted by  $\phi$ . i.e.  $X$  is a null set if  $n(X) = 0$ .

e.g. :  $\{x : x \in \mathbb{R} \text{ and } x^2 + 2 = 0\}$ ,  $\{x : x > 1 \text{ but } x < 1/2\}$ ,  $\{x : x \in \mathbb{R}, x^2 < 0\}$ .

### (E) Equivalent Sets :

Two finite sets  $A$  and  $B$  are equivalent if their cardinal numbers are same i.e.  $n(A) = n(B)$ .

### (F) Equal Sets :

Two sets  $A$  and  $B$  are said to be equal if every element of  $A$  is a member of  $B$  and every element of  $B$  is a member of  $A$ . i.e.  $A = B$ .

## 4. UNIVERSAL SET

It is a set which includes all the sets under considerations i.e. It is a super set of each of the given set. Thus, a set that contains all sets in a given context is called the universal set. It is denoted by  $U$ .

E.g. If  $A = \{1, 2, 3\}$ ,  $B = \{2, 4, 5, 6\}$  and  $C = \{1, 3, 5, 7\}$ , then  $U = \{1, 2, 3, 4, 5, 6, 7\}$  can be taken as the universal set.

## 5. DISJOINT SETS

Sets A and B are said to be disjoint iff A and B have no common element or  $A \cap B = \phi$ . If  $A \cap B \neq \phi$  Then A and B are said to be intersecting or overlapping sets.

**E.g.** (i) if  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6\}$  and  $C = \{4, 7, 9\}$  then A and B are disjoint set where as B and C are intersecting sets.

(ii) set of even natural numbers and odd natural numbers are disjoint sets.

## 6. COMPLEMENTARY SET

Complementary set of a set A is a set containing all those elements of universal set which are not in A. It is denoted by  $\bar{A}$  or  $A^c$  or  $A'$ . So  $A^c = \{x : x \in U \text{ but } x \notin A\}$ .

e.g. If set  $A = \{1, 2, 3, 4, 5\}$  and universal set  $U = \{1, 2, 3, 4, \dots, 50\}$  then  $\bar{A} = \{6, 7, \dots, 50\}$



### DETECTIVE MIND

all disjoint sets are not complementary sets but all complementary sets are disjoint.

## 7. SUBSET

Set A is said to be a subset of B if all the elements of A are present in B and is denoted by  $A \subset B$  (read as A is subset of B) and symbolically written as :  $x \in A \Rightarrow x \in B \Leftrightarrow A \subset B$

### (A) Number of subsets :

Consider a set X containing n elements i.e.  $\{x_1, x_2, \dots, x_n\}$  then the total number of subsets of  $x = 2^n$

**Proof :** number of subsets of above set is equal to the number of selections of elements taking any Number of them at a time out of the total n distinct elements and it is equal to  $2^n$

$$\therefore {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

### (B) Types of subsets :

Set A is said to be a proper subset of a set B if every element of A is an element of B and B has at least one element which is not an element of A and is denoted by  $A \subset B$ .

The sets A itself and the empty set is known as improper subset and is denoted as  $A \subseteq B$ .

E.g. If  $x = \{x_1, x_2, \dots, x_n\}$  then total number of proper sets =  $2^n - 2$  (excluding itself and the null set). The statement  $A \subset B$  can be written as  $B \supset A$ , then B is called the super set of A and is written as  $B \supset A$ .

## 8. POWER SETS

The collection of all subsets of set A is called the power set of A and is denoted by  $P(A)$

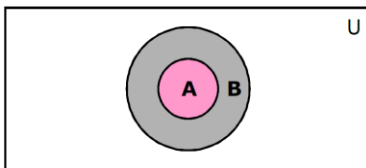
i.e.  $P(A) = \{x : x \text{ is a subset of } A\}$ . If  $X = \{x_1, x_2, x_3, \dots, x_n\}$  then  $n(P(X)) = 2^n$  ;  $n(P(P(X))) = 2^{2^n}$

## 9. VENN (EULER) DIAGRAMS

The diagrams drawn to represent sets are called venn diagram or euler-venn diagrams. Here We represents the universal set U as set of all points within rectangle and the subset A of the Set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle Representing A is drawn inside the circle

representing B. If A and B are not equal but they have Some common elements, then represent A and B by two intersecting circles.

E.g. If A is subset of B then it is represented diagrammatically in fig.



E.g. if A is a set then the complement of A is represented in fig.



## 10. OPERATIONS ON SETS

### (A) Union of sets :

if A and B are two sets then union ( $\cup$ ) of A and B is the set of all those elements which belong either to A or to B or to both A and B. it is also defined as  $A \cup B = \{x : x \in A \text{ or } x \in B\}$ .

it is represented through venn diagram in fig.1 & fig.2

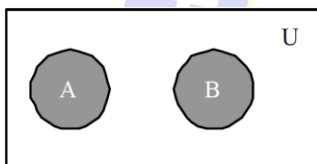


Fig.(1)

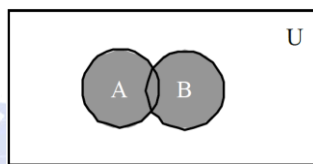
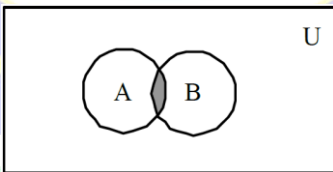


Fig.(2)

### (B) Intersection of sets :

if A and B are two sets then intersection ( $\cap$ ) of A and B is the set of all those elements which belong to both A and B. It is also defined as  $A \cap B = \{x : x \in A \text{ and } x \in B\}$  represented in venn diagram (see fig.)



### (C) Difference of two sets :

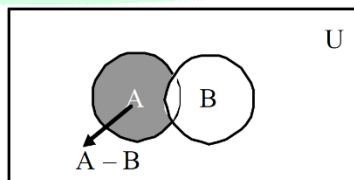
if A and B are two sets then the difference of A and B, is the set of all those elements of A which do not belong to B.

Thus,  $A - B = \{x : x \in A \text{ and } x \notin B\}$

or  $A - B = \{x \in A ; x \notin B\}$

Clearly  $x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$

It is represented through the Venn diagrams.



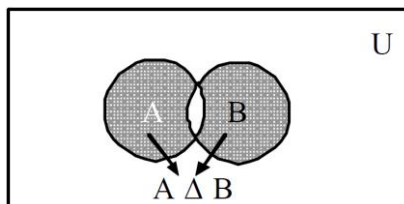
### (D) Symmetric difference of two sets :



Set of those elements which are obtained by taking the union of the difference of A & B is  $(A - B)$  & the difference of B & A is  $(B - A)$ , is known as the symmetric difference of two sets A & B and it is denoted by  $(A \Delta B)$ .

Thus  $A \Delta B = (A - B) \cup (B - A)$

Representation through the venn diagram is given in the fig.



### 11. NUMBER OF ELEMENTS IN DIFFERENT SETS

If A, B & C are finite sets and U be the finite universal set, then

- (i)  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- (ii)  $n(A \cup B) = n(A) + n(B)$  (if A & B are disjoint sets)
- (iii)  $n(A - B) = n(A) - n(A \cap B)$
- (iv)  $n(A \Delta B) = n[(A - B) \cup (B - A)] = n(A) + n(B) - 2n(A \cap B)$
- (v)  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- (vi)  $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- (vii)  $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

### 12. CARTESIAN PRODUCT OF TWO SETS

Cartesian product of A to B is a set containing the elements in the form of ordered pair  $(a, b)$  such that  $a \in A$  and  $b \in B$ . It is denoted by  $A \times B$ .

i.e.  $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If set  $A = \{a_1, a_2, a_3\}$  and  $B = \{b_1, b_2\}$  then

$A \times B$  and  $B \times A$  can be written as :

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$  and  $B \times A = \{(b, a) : b \in B \text{ and } a \in A\}$

$A \times B = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2), (a_3, b_1), (a_3, b_2)\}$

$B \times A = \{(b_1, a_1), (b_1, a_2), (b_1, a_3), (b_2, a_1), (b_2, a_2), (b_2, a_3)\}$

Clearly  $A \times B \neq B \times A$  until A and B are equal



### DETECTIVE MIND

1. If  $n(A) = m$  and  $n(B) = n$  then number of elements in  $(A \times B) = m \times n$
2. Since  $A \times B$  contains all such ordered pairs of the type  $(a, b)$  such that  $a \in A$  &  $b \in B$ , that means it includes all possibilities in which the elements of set A can be related with the elements of set B. Therefore,  $A \times B$  is termed as largest possible relation defined from set A to set B, also known as universal relation from A to B.

### 13. ALGEBRAIC OPERATIONS ON SETS

#### (A) Idempotent operation :

For any set A, we have (i)  $A \cup A = A$  and (ii)  $A \cap A = A$

**Proof :** (i)  $A \cup A = \{x : x \in A \text{ or } x \in A\} = \{x : x \in A\} = A$

(ii)  $A \cap A = \{x : x \in A \text{ & } x \in A\} = \{x : x \in A\} = A$

#### (B) Identity operation :

For any set A, we have

(i)  $A \cup \phi = A$  and

(ii)  $A \cap U = A$  i.e.  $\phi$  and  $U$  are identity elements for union and intersection respectively

**Proof:** (i)  $A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$

$$= \{x : x \in A\} = A$$

(ii)  $A \cap U = \{x : x \in A \text{ and } x \in U\}$

$$= \{x : x \in A\} = A$$

**(C) Commutative operation :**

For any set A and B, we have

(i)  $A \cup B = B \cup A$  and

(ii)  $A \cap B = B \cap A$

i.e. union and intersection are commutative.

**(D) Associative operation :**

If A, B and C are any three sets then

(i)  $(A \cup B) \cup C = A \cup (B \cup C)$

(ii)  $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

**(E) Distributive operation :**

If A, B and C are any three sets then

(i)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

**(F) De-Morgan's Principle :**

If A and B are any two sets, then

(i)  $(A \cup B)' = A' \cap B'$

(ii)  $(A \cap B)' = A' \cup B'$

**Proof :** (i) Let x be an arbitrary element of  $(A \cup B)'$ . Then  $x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$$\Rightarrow x \notin A \text{ and } x \notin B \Rightarrow x \in A' \cap B'$$

Again let x be an arbitrary element of  $A' \cap B'$ . Then  $x \in A' \cap B'$

$$\Rightarrow x \in A' \text{ and } x \in B' \Rightarrow x \notin A \text{ and } x \notin B$$

$$\Rightarrow x \notin (A \cup B) \Rightarrow x \in (A \cup B)'$$

$$\therefore A' \cap B' \subseteq (A \cup B)'$$

Hence  $(A \cup B)' = A' \cap B'$  Similarly (ii) can be proved.

### SOLVED EXAMPLES

**Example: 1** If a set  $A = \{a, b, c\}$  then find the number of subsets of the set A and also mention the set of all the subsets of A.

**Solution:**

Since  $n(A) = 3$

$$\therefore \text{number of subsets of A is } 2^3 = 8$$

and set of all those subsets is  $P(A)$  named as power set

$$P(A) : \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

**Example: 2** Show that  $n\{P[P(\phi)]\} = 4$

**Solution:**

We have  $P(\phi) = \{\phi\}$

$$\therefore P(P(\phi)) = \{\phi, \{\phi\}\}$$

$$\Rightarrow P[P(P(\phi))] = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$$

$$\text{Hence, } n\{P[P(\phi)]\} = 4$$

**Example: 3** If  $A = \{x : x = 2n + 1, n \in \mathbb{Z}\}$  and  $B = \{x : x = 2n, n \in \mathbb{Z}\}$ , then find  $A \cup B$ .

**Solution:**

$$A \cup B = \{x : x \text{ is an odd integer}\} \cup \{x : x \text{ is an even integer}\} = \{x : x \text{ is an integer}\} = \mathbb{Z}$$

**Example: 4** If  $A = \{x : x = 3n, n \in \mathbb{Z}\}$  and

$B = \{x : x = 4n, n \in \mathbb{Z}\}$  then find  $A \cap B$ .

**Solution:**

We have,

$$x \in A \cap B \Leftrightarrow x = 3n, n \in \mathbb{Z} \text{ and } x = 4n, n \in \mathbb{Z}$$

$\Leftrightarrow x$  is a multiple of 3 and  $x$  is a multiple of 4

$\Leftrightarrow x$  is a multiple of 3 and 4 both

$\Leftrightarrow x$  is a multiple of 12  $\Leftrightarrow x = 12n, n \in \mathbb{Z}$

Hence  $A \cap B = \{x : x = 12n, n \in \mathbb{Z}\}$

**Example: 5**

If  $A$  and  $B$  be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in  $A \cup B$ ? Find also, the maximum number of elements in  $A \cup B$ .

**Solution:**

We have,  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .

This shows that  $n(A \cup B)$  is minimum or maximum according as  $n(A \cap B)$  is maximum or minimum respectively.

**Case-I**

When  $n(A \cap B)$  is minimum, i.e.,  $n(A \cap B) = 0$

This is possible only when  $A \cap B = \phi$ .

In this case,

$$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9.$$

So, maximum number of elements in  $A \cup B$  is 9.

**Case-II**

When  $n(A \cap B)$  is maximum.

This is possible only when  $A \subseteq B$ . In this case,  $n(A \cap B) = 3$

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= (3 + 6 - 3) = 6$$

So, minimum number of elements in  $A \cup B$  is 6.

**Example: 6**

If  $A = \{2, 3, 4, 5, 6, 7\}$  and  $B = \{3, 5, 7, 9, 11, 13\}$  then find  $A - B$  and  $B - A$ .

**Solution:**

$$A - B = \{2, 4, 6\} \text{ \& } B - A = \{9, 11, 13\}$$

**Example: 7**

If the number of elements in  $A$  is  $m$  and number of element in  $B$  is  $n$  then find

(i) The number of elements in the power set of  $A \times B$ .

(ii) Number of relation defined from  $A$  to  $B$

**Solution:**

(i) Since  $n(A) = m$ ;  $n(B) = n$

$$\text{Then } n(A \times B) = mn$$

$$\text{So number of subsets of } A \times B = 2^{mn}$$

$$\Rightarrow n(P(A \times B)) = 2^{mn}$$

(ii) Number of relation defined from  $A$  to  $B = 2^{mn}$

Any relation which can be defined from set  $A$  to set  $B$  will be subset of  $A \times B$

$$\therefore A \times B \text{ is largest possible relation } A \rightarrow B$$

$$\therefore \text{no. of relation from } A \rightarrow B = \text{no. of subsets of set } (A \times B)$$

**Example: 8**

Let  $A$  and  $B$  be two non-empty sets having  $n$  elements in common, then prove that  $A \times B$  and  $B \times A$  have  $n^2$  elements in common.

**Solution:**

$$\text{We have } (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

On replacing  $C$  by  $B$  and  $D$  by  $A$ , we get

$$\Rightarrow (A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$$

It is given that  $A \cap B$  has  $n$  elements so

$(A \cap B) \times (B \cap A)$  has  $n^2$  elements

$$\text{But } (A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$$

$$\therefore (A \times B) \cap (B \times A) \text{ has } n^2 \text{ elements}$$

Hence  $A \times B$  and  $B \times A$  have  $n^2$  elements in common.

**Example: 9** Let  $A = \{x : x \text{ is a digit in the number } 3591\}$ ,  $B = \{x : x \in \mathbb{N}, x < 10\}$ . Which of the following is false ?

- (1)  $A \cap B = \{1, 3, 5, 9\}$  (2)  $A - B = \phi$   
 (3)  $B - A = \{2, 4, 6, 7, 8\}$  (4)  $A \cup B = \{1, 2, 3, 5, 9\}$

**Solution:** We have  $A = \{1, 3, 5, 9\}$  and  $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Now find  $A \cup B$ ,  $A \cap B$ ,  $A - B$ ,  $B - A$ .

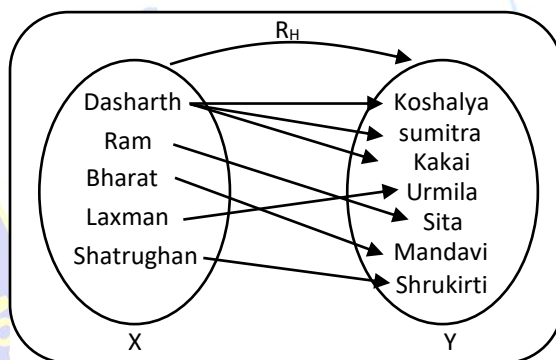
**Example: 10** If  $R$  be an anti-symmetric relation in a set  $A$  such that  $(a, b), (b, a) \in R$ , then

- (1)  $a \geq b$  (2)  $a \leq b$  (3)  $a = b$  (4) none of these

#### 14. DEFINITION OF RELATION

A relation  $R$  from set  $X$  to  $Y$  ( $R : X \rightarrow Y$ ) is a correspondence between set  $X$  to set  $Y$  by which some or more elements of  $X$  are associated with some or more elements of  $Y$ . Therefore a relation (or binary relation)  $R$ , from a non-empty set  $X$  to another non-empty set  $Y$ , is a subset of  $X \times Y$ . i.e.  $R : X \rightarrow Y$  is nothing but subset of  $A \times B$ .

e.g. Consider a set  $X$  and  $Y$  as set of all males and females members of a royal family of the kingdom Ayodhya  $X = \{\text{Dashrath, Ram, Bharat, Laxman, Shatrughan}\}$  and  $Y = \{\text{Koshaliya, Kakai, sumitra, Sita, Mandavi, Urmila, Shrutkirti}\}$  and a relation  $R$  is defined as "was husband of" from set  $X$  to set  $Y$ .



Then  $R_H = \{(\text{Dashrath, Koshaliya}), (\text{Ram, sita}), (\text{Bharat, Mandavi}), (\text{Laxman, Urmila}), (\text{Shatrughan, Shrutkirti}), (\text{Dashrath, Kakai}), (\text{Dashrath, Sumitra})\}$



#### DETECTIVE MIND

- (i) If  $a$  is related to  $b$  then symbolically it is written as  $a R b$  where  $a$  is pre-image and  $b$  is image  
 (ii) If  $a$  is not related to  $b$  then symbolically it is written as  $a \not R b$ .

#### (A) Domain, Co-domain & Range of Relation :

**Domain :** Domain of relation is collection of elements of the first set which are participating in the correspondence i.e. it is set of all pre-images under the relation  $R$ . e.g. Domain of  $R_H : \{\text{Dashrath, Ram, Bharat, Laxman, Shatrughan}\}$

**Co-Domain :** All elements of set  $Y$  irrespective of whether they are related with any element of  $X$  or not constitute co-domain. e.g.  $Y = \{\text{Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti}\}$  is co-domain of  $R_H$ .

**Range :** Range of relation is a set of those elements of set  $Y$  which are participating in correspondence i.e. set of all images. Range of  $R_H : \{\text{Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti}\}$ .



## 15. TYPES OF RELATIONS

### (A) Reflexive Relation

$R : X \rightarrow Y$  is said to be reflexive iff  $x R x \forall x \in X$ . i.e. every element in set  $X$ , must be related to itself therefore  $\forall x \in X; (x, x) \in R$  then relation  $R$  is called as reflexive relation.

**Example:** - Let  $A = \{1, 2, 3\}$  be a set. Then

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 1)\}$  is a reflexive relation on  $A$ .

But  $R_2 = \{(1, 1), (2, 2), (1, 3), (2, 1)\}$  is not a reflexive relation on  $A$ .

### (B) Identity Relation :

Let  $X$  be a set. Then the relation  $I_X = \{(x, x) : x \in X\}$  on  $X$  is called the identity relation on  $X$ . i.e. a relation  $I_X$  on  $X$  is an identity relation if every element of  $X$  related to itself only. e.g.  $y = x$

**Example:** - Let  $A = \{1, 2, 3\}$  be a set. Then

$I_A = \{(1, 1), (2, 2), (3, 3)\}$  is the identity relation on  $A$ .



### DETECTIVE MIND

- (1) **An empty relation (or void relation):** if  $A$  and  $B$  are two non-empty sets and  $R$  is a relation from  $A$  to  $B$  then  $R$  is called an empty relation if there is no relation between any elements of a set  $A$  to  $B$  ( $R = \phi$ ).
- (2) **A universal (or full relation):** If  $A$  and  $B$  are two non-empty sets and  $R$  is a relation from  $A$  to  $B$  then  $R$  is said to be a universal relation if  $R = A \times B$ .
- (3) All identity relations are reflexive but all reflexive relations are not identity.
- (4) A relation  $R$  from a set  $X$  to set  $Y$  ( $R : X \rightarrow Y$ ) is said to be a partial order relation if it is reflexive, Anti-symmetric as well as transitive.

### (C) Symmetric Relation

$R : X \rightarrow Y$  is said to be symmetric iff  $(x, y) \in R \Rightarrow (y, x) \in R$  for all  $(x, y) \in R$  i.e.  $x R y \Rightarrow y R x$  for all  $(x, y) \in R$ . e.g. perpendicularity of lines in a plane is symmetric relation.

### (D) Transitive Relation

$R : X \rightarrow Y$  is transitive iff  $(x, y) \in R$  and  $(y, z) \in R \Rightarrow (x, z) \in R$  for all  $(x, y)$  and  $(y, z) \in R$ . i.e.  $x R y$  and  $y R z \Rightarrow x R z$ .

### (E) Anti-symmetric Relation

Let  $A$  be any set. A relation  $R$  on set  $A$  is said to be an antisymmetric relation iff  $(a, b) \in R$  and  $(b, a) \in R \Rightarrow a = b$  for all  $a, b \in A$  e.g. Relations "being subset of"; "is greater than or equal to" and "identity relation on any set  $A$ " are antisymmetric relations.

### (F) Equivalence Relation

A relation  $R$  from a set  $X$  to set  $Y$  ( $R : X \rightarrow Y$ ) is said to be an equivalence relation iff it is reflexive, symmetric as well as transitive. The equivalence relation is denoted by ' $\sim$ '

e.g. Relation "is equal to" Equality, Similarity and congruence of triangles, parallelism of lines are equivalence relation.

## 16. INVERSE OF A RELATION

Let  $A, B$  be two sets and let  $R$  be a relation from a set  $A$  to  $B$ . Then the inverse of  $R$ , denoted by  $R^{-1}$ , is a relation from  $B$  to  $A$  and is defined by  $R^{-1} = \{(b, a) : (a, b) \in R\}$ . Clearly,

$(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$  Also,

Dom of  $R =$  Range of  $R^{-1}$  and

Range of  $R =$  Dom of  $R^{-1}$

## SOLVED EXAMPLES

**Example: 11** Let R be the relation on the set N of natural numbers defined by

$R : \{(x, y) : x + 3y = 12, x \in N, y \in N\}$  Find

(i) R (ii) Domain of R (iii) Range of R

**Solution:**

(i) We have,  $x + 3y = 12 \Rightarrow x = 12 - 3y$

Putting  $y = 1, 2, 3$ , we get  $x = 9, 6, 3$  respectively

For  $y = 4$ , we get  $x = 0 \notin N$ . Also for  $y > 4$ ,  $x \notin N$

$\therefore R = \{(9, 1), (6, 2), (3, 3)\}$

(ii) Domain of  $R = \{9, 6, 3\}$

(iii) Range of  $R = \{1, 2, 3\}$

**Example: 12** If  $X = \{x_1, x_2, x_3\}$  and  $Y = \{x_1, x_2, x_3, x_4, x_5\}$  then find which is a reflexive relation of the following :

(1)  $R_1 : \{(x_1, x_1), (x_2, x_2)\}$

(2)  $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)\}$

(3)  $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_3), (x_2, x_4)\}$

(4)  $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4)\}$

**Solution:**

(1) non-reflexive because  $(x_3, x_3) \notin R_1$

(2) Reflexive

(3) Reflexive

(4) non-reflexive because  $x_4 \notin X$

**Example: 13** If  $x = \{a, b, c\}$  and  $y = \{a, b, c, d, e, f\}$  then find which of the following relation is symmetric relation:

$R_1 : \{\}$  i.e. void relation

$R_2 : \{(a, b)\}$

$R_3 : \{(a, b), (b, a), (a, c), (c, a), (a, a)\}$

**Solution:**

$R_1$  is symmetric relation because it has no element in it.

$R_2$  is not symmetric because  $(b, a) \notin R_2$

$R_3$  is symmetric.

**Example: 14** If  $x = \{a, b, c\}$  and  $y = \{a, b, c, d, e\}$  then which of the following are transitive relation.

(1)  $R_1 = \{\}$

(2)  $R_2 = \{(a, a)\}$

(3)  $R_3 = \{(a, a), (c, d)\}$

(4)  $R_4 = \{(a, b), (b, c), (a, c), (a, a), (c, a)\}$

**Solution:**

(1)  $R_1$  is transitive relation because it is null relation.

(2)  $R_2$  is transitive relation because all singleton relations are transitive.

(3)  $R_3$  is transitive relation

(4)  $R_4$  is no transitive relation

**Example: 15** Prove that the relation R on the set Z of all integers numbers defined by  $(x, y) \in R \Leftrightarrow x - y$  is divisible by n is an equivalence relation on Z.

**Solution:** We observe the following properties

**Reflexivity :**

For any  $a \in Z$ , we have

$a - a = 0 \times n \Rightarrow a - a$  is divisible by  $n \Rightarrow (a, a) \in R$

Thus  $(a, a) \in R$  for all  $Z$ . so, R is reflexive on Z.

**Symmetry :**

Let  $(a, b) \in R$ . Then  $(a, b) \in R \Rightarrow (a - b)$  is divisible by  $n$

$\Rightarrow (a - b) = np$  for some  $p \in Z \Rightarrow b - a = n(-p)$

$\Rightarrow b - a$  is divisible by  $n \Rightarrow (b, a) \in R$

Thus  $(a, b) \in R \Rightarrow (b, a) \in R$  for all  $a, b, \in \mathbb{Z}$ .

So  $R$  is symmetric on  $\mathbb{Z}$ .

**Transitivity :**

Let  $a, b, c \in \mathbb{Z}$  such that  $(a, b) \in R$  and  $(b, c) \in R$ .

Then  $(a, b) \in R \Rightarrow (a - b)$  is divisible by  $n$

$\Rightarrow a - b = np$  for some  $p \in \mathbb{Z}$

$(b, c) \in R \Rightarrow (b - c)$  is divisible by  $n$

$\Rightarrow b - c = nq$  for some  $q \in \mathbb{Z}$

$\therefore (a, b) \in R$  and  $(b, c) \in R$

$\Rightarrow a - b = np$  and  $b - c = nq$

$\Rightarrow (a - b) + (b - c) = np + nq$

$\Rightarrow a - c = n(p + q)$

$\Rightarrow a - c$  is divisible by  $n \Rightarrow (a, c) \in R$

Thus  $(a, b) \in R$  and  $(b, c) \in R \Rightarrow (a, c) \in R$  for all  $a, b, c \in \mathbb{Z}$ . So  $R$  is transitive relation on  $\mathbb{Z}$ .

Thus,  $R$  being reflexive, symmetric and transitive is an equivalence relation on  $\mathbb{Z}$ .

**Example: 16** For real numbers  $x$  and  $y$ , we define  $xRy$  iff  $x - y + \sqrt{5}$  is an irrational number. The relation  $R$  is  
 (1) reflexive (2) symmetric (3) transitive (4) none of these

**Solution:**  $x \in \mathbb{R} \Rightarrow x - x + \sqrt{5} = \sqrt{5}$  is an irrational number,

$\therefore (x, x) \in R$

$R$  is reflexive

$(\sqrt{5}, 1) \in R$  because  $\sqrt{5} - 1 + \sqrt{5} = 2\sqrt{5} - 1$ , which is an irrational number.

Also,  $(1, \sqrt{5}) \in R$  if  $1 - \sqrt{5} + \sqrt{5} = 1$ , which is not an irrational number.

$\therefore (1, \sqrt{5}) \notin R$ .

$\therefore R$  is not symmetric.

We have  $(\sqrt{5}, 1), (1, 2\sqrt{5}) \in R$  because  $\sqrt{5} - 1 + \sqrt{5} = 2\sqrt{5} - 1$  and  $1 - 2\sqrt{5} + \sqrt{5} = 1 - \sqrt{5}$  are irrational numbers.

Also  $(\sqrt{5}, 2\sqrt{5}) \notin R$  Because  $\sqrt{5} - 2\sqrt{5} + \sqrt{5} = 0$ , which is not an irrational number.

$\therefore (\sqrt{5}, 2\sqrt{5}) \notin R$

$\therefore R$  is not transitive.

**Example: 17** Two points  $A$  and  $B$  in a plane are related if  $OA = OB$ , where  $O$  is a fixed point. This relation is  
 (1) reflexive but not symmetric (2) symmetric but not transitive  
 (3) an equivalence relation (4) none of these

**Solution:** Let this relation be denoted by  $R$ .

For any point  $A$ , we have  $OA = OA$ .

$\therefore (A, A) \in R$ .

$\therefore R$  is reflexive

Let  $(A, B) \in R \Rightarrow OA = OB$

$\Rightarrow OB = OA \Rightarrow (B, A) \in R$

$\therefore R$  is symmetric

Let  $(A, B), (B, C) \in R \Rightarrow OA = OB, OB = OC$

$\Rightarrow OA = OC \Rightarrow (A, C) \in R$

$\therefore R$  is transitive

$\therefore R$  is an equivalence relation

**Example: 18** Let  $S$  be a non-empty set. In  $P(S)$ , let  $R$  be a relation defined as  $ARB \Leftrightarrow A \cap B \neq \phi$ . The relation  $R$  is  
 (1) reflexive (2) symmetric (3) transitive (4) none of these

**Solution:** The given relation is only symmetric because  $A \cap B \neq \phi$  implies  $B \cap A \neq \phi$ .

**Example: 19** Let  $R$  be an equivalence relation in a finite set  $A$  having  $n$  elements. The number of ordered pairs in  $R$  is

- (1) less than  $n$  (2) less than or equal to  $n$   
 (3) greater than  $n$  (4) greater than or equal to  $n$

**Solution:** We have  $o(A) = n$ .

$\therefore R$  is an equivalence relation in  $A$ .

$\therefore R$  is reflexive.

$\therefore (a, a) \in R \forall a \in A$

The elements of the type  $(a, a)$  are  $n$  in number, because  $o(A) = n$ .

$\therefore R$  must have at least  $n$  elements.

**Example: 20**  $R$  is a relation defined in  $R \times R$  by  $(a, b) R (c, d)$  iff  $a - c$  is an integer and  $b = d$ . The relation  $R$  is

- (1) the identity relation (2) the universal relation  
 (3) an equivalence relation (4) none of these

**Solution:** We have  $R = \{(a, b), (c, d) : a - c \in \mathbb{Z} \text{ and } b = d; a, b, c, d \in \mathbb{R}\}$ .

Let  $(a, b) \in R \times R$

$\therefore (a, b) R (a, b)$ , because  $a - a = 0 \in \mathbb{Z}$  and  $b = b$ .

$\therefore R$  is reflexive.

Let  $(a, b) R (c, d)$ .

$\Rightarrow a - c \in \mathbb{Z}$  and  $b = d$

$\Rightarrow c - a \in \mathbb{Z}$  and  $d = b$

$\Rightarrow (c, d) R (a, b)$

$\Rightarrow R$  is symmetric.

Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$ .

$\Rightarrow a - c \in \mathbb{Z}, b = d, c - e \in \mathbb{Z}, d = f$

$\Rightarrow (a - c) + (c - e) \in \mathbb{Z}, b = f$

$\Rightarrow a - e \in \mathbb{Z}, b = f$

$\Rightarrow (a, b) R (e, f)$

$\Rightarrow R$  is transitive.

$\therefore R$  is an equivalence relation.



# ANSWER KEY

## PRACTICE SECTION-01

Que.	1	2	3	4	5	6	7	8	9	10	11	12	
Ans:	3	3	2	3	3	4	2	3	1,2	2	3	1	

## PRACTICE SECTION-02

Que.	1	2	3	4	5	6	7	8	
Ans:	3	4	2	3	2	4	2	3	

## TOPIC WISE QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	4	1	3	3	1	4	3	2	3	3	3	1	2	3	2
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	2	2	1	2	2	3	2	2	1	1	2	1	1	3	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	2	1	4	1	1	3	3	3	3	3	4	1	1	2	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	3	3	3	3	1	4	4	1	2	4	2	4	1	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	2	2	2	2	2	1	2	1	2	3	4	3	3	1	4
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	4	3	1	2	2	1	3	2	3	2	1	4	3	2	3
Que.	91	92	93												
Ans.	2	4	3												

## RANKER'S STUFF

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	3	3	2	2	4	4	2	3	4	3	3	2	1	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	4	1	4	1	3	1	4	3	3	160	512	10	16	64	50
Que.	31	32	33	34	35	36	37	38	39	40(i)	40(ii)	40(iii)	41(i)	41(ii)	41(iii)
Ans.	832	96	5	80	3	2	1,2,4	1,2	1,3	4	2	3	2	3	1

Q.42 (1) → (Q), (2) → (R), (3) → (S), (4) → (P)

Q.43 (1) → (P), (2) → (R), (3) → (S), (4) → (Q)

## JEE-FLASHBACK

### JEE-MAINS QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	1	1	3	2	2	2	2	2	4	4	4	2	2	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	2	4	1	3	1	4	4	3	107	112	4	4	4	2	1
Que.	31	32	33	34	35	36	37								
Ans.	1	3	19	2	6	1	2								

### JEE-ADVANCED QUESTIONS

Que.	1	2	3
Ans.	7	4	119