KHAN GLOBAL STUDIES

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Defence (Mathematics)

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Sequence and Series short note

AP is a sequence whose terms increase or decreased by a fixed number
$a \to$ first term
$d \to$ common difference
AP : $a, a+d, a+2d, a+3d, \ldots$

- nth term of AP - $T_n = a + (n-1)d$

  Where $d = T_n - T_{n-1}, n \geq 2$

- Sum of first n terms

  $S_n = \frac{n}{2}[2a + (n-1)d]$

  $S_n = \frac{n}{2}(a + l)$, *where* $l \to$ last term

- $T_n = S_n - S_{n-1}, n \geq 2$

Series : In a sequence, the sum of the directed terms is called a series.
Ex- 1, 2, 3, 4…. Sequence then
$1 + 2 + 3 + 4 + \cdots$ is called a series
$T_1, T_2, T_3, ,,, T_n \to$ sequence.
then $S_n = T_1 + T_2 + \cdots + T_n$
$S_n = \sum_{r=1}^{n} T_r$
Finite Series : A series having finite no of terms is called a finite series.
Ex- $1 + 2 + 3 + \cdots + 10$

Infinite Series : A series having an infinite number of terms is called an infinite series
$1 + 2 + 3 + \cdots + n$
Note :

1. If $a_1, a_2, a_3, \ldots, a_{n-1}, a_n \to$AP
   $a_1 + k,\ a_2 + k,\ a_3 + k, \ldots.\ a_{n-1} + k,\ a_n + k \to$AP
   $a_1 - k,\ a_2 - k,\ a_3 - k, \ldots.\ a_{n-1} - k,\ a_n - k \to$AP
   $ka_1, ka_2, ka_3, \ldots.\ ka_{n-1}, ka_n \to$AP
   $\frac{a_1}{k}, \frac{a_2}{k}, \ldots \frac{a_{n-1}}{k}, \frac{a_n}{k} \to$AP
   $(k \neq 0)$

2. $a_1, a_2, \ldots.., a_n \to$ AP (1st)
   $b_1, b_2, \ldots\ldots., b_n \to$AP (2nd)
   Then,
   $a_1 + b_1, a_2 + b_2, \ldots.. (a_n + b_n) \to$AP
3. When sum of Numbers of given
   3 terms $a - d, a, a + d$
   4 terms $a - 3d, a - d, a + d, a + 3d$
   5 terms $a - 2d, a - d, a, a + d, a + 2d$
4. If $a_1, a_2, \ldots\ldots., a_{n-1}, a_n$ an are in AP then $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$
5. If sum of first n terms of an AP is of the form $An^2 + Bn$ ie, a Quadratic expression in n then common difference is twice the coefficient of n^2 ie 2A.
6. If $T_n = An + B$ (linear in n) then $d = A$
7. In AP if $T_p = q$ and $T_q = p$ then $T_r = p + q - r$
8. Common difference of an AP may be positive, zero, negative or imaginary
9. Constant AP : $d = 0$
10. Increasing AP : $d > 0$
11. Decreasing AP : $d < 0$
12. nth term from end (In case of AP)
    $T'_n = l - (n-1)d$, where $l =$ last term
13. a, b, c are in AP then $2b = a + c$
14. First negative term = largest negative term.
15. If ratio of the sums of n terms of two AP's is given by
    $\frac{S_n}{S'_n} = \frac{Pn+Q}{Rn+S} (P, R \neq 0)$
    then $\frac{t_n}{t'_n} = \frac{P(2n-1)+Q}{R(2n-1)+S}$
    $\frac{d}{d'} = \frac{P}{R}$
    where
    $t_n, S_n, d \to$ are nth term, sum of first n terms and common difference of first AP respectively
    $t_n', S_n', d' \to$ For 2nd AP
16. In case of two AP
    $\frac{t_n}{t_n'} = \frac{Pn+Q}{Rn+S}$
    Then $\frac{S_n}{S_n'} = \frac{P\frac{(n+1)}{2}+Q}{R\left(\frac{n+1}{2}\right)+S}$

GP (Geometric Progression)

GP is a sequence of non zero numbers whose succeeding terms is equal to the preceding terms multiplied by a constant

$T_1, T_2, T_3, \ldots., T_{n-1}, T_n \rightarrow$GP

If $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \cdots \frac{T_n}{T_{n-1}} = r$

$r \rightarrow$ common ratio

GP are $a, ar, ar^2, \ldots. ar^{n-1}$

$T_n = ar^{n-1} \rightarrow$ nth term

$S_n =$ Sum of first n terms

$= a\frac{(r^n-1)}{r-1}$ if $r \neq 1$

- If $r = 1$
  Then all terms are same
- Infinite GP
  When $|r| < 1$ ie $-1 < r < 1$
  $S_\infty = \frac{a}{1-r}$; $|r| < 1$
- In GP neither $a = 0$ nor $r = 0$
- r may be positive, negative or Imaginary
- Constant GP $r = 1$
  Increasing GP / Decreasing GP

1. If $a > 0$

   (1) $r > 1 \rightarrow$ Increasing GP

   (2) $0 < r < 1 \rightarrow$ Decreasing GP

2. If $a < 0$

   (1) $r > 1 \rightarrow$ Decreasing GP

   (2) $0 < r < 1 \rightarrow$ Increasing GP

- a, b, c are in GP and $a > 0$, $b > 0$, $c > 0$ then $\log a$, $\log b$, $\log c$ are in AP converse also holds.
- If $a_1, a_2, a_3, \ldots, a_n$ are in GP then $\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$ are also in GP
- If $a_1, a_2, \ldots a_n$ are in GP then

  (a) $a_n, a_{n-1}, \ldots, a_2, a_1$ are also in GP

  (b) $a_1^n, a_2^n, a_3^n \ldots$, are also in GP ($n \in Q$)

- If $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots b_n$ are two GP's then

  (a) $a_1b_1, a_2b_2, \ldots a_nb_n$ are also in GP

  (b) $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots., \frac{a_n}{b_n}$ are also in GP

- If $a_1, a_2, \ldots., a_n$ are in GP then
  $a_1a_n = a_2a_{n-1} = a_3a_{n-2} = \cdots$
- When product is given (GP)

3 terms $\frac{a}{r}, a, ar$

4 terms $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

5 terms $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Harmonic Progression

A harmonic progression (HP) is a sequence if the reciprocals of its terms are in AP

$a_1, a_2, \ldots.$ are in HP if and only if $\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$, in are in HP

- No term of HP can be zero
- If a, b, c are in HP then
  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP
  $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow b = \frac{2ac}{a+c}$

•nth term of HP

let a be the first term and d be the common difference of

An AP $\Rightarrow T_n = a + (n-1)d$ (AP)

Then nth term of HP

$= \frac{1}{a+(n-1)d}$

•Arithmetic Mean

If three terms are in AP then the middle term is called the AM between the other two

So if a, b, c are in AP then b is AM of a and c

•Single $a_1, a_2, \ldots, a_n$ AM of n positive numbers

Let are n positive numbers then

AM of these numbers are

$A = \frac{a_1+a_2+\cdots+a_n}{n}$

Ex AM of $5a, 3b$ and $9c$ is $\frac{5a+3b+9c}{3}$

- n AM's b/w two Nos a and b
  $a, \underbrace{A_1, A_2, \ldots., A_n}_{n\ AM's}, b \rightarrow$ AP
  $A_1 = T_2 = a + d$
  $A_2 = T_3 = a + 2d$
  -
  -
  -
  $A_n = T_{n+1} = a + nd$
  Where $d = \frac{b-a}{n+1}$
  $n \rightarrow$ no of AM's
- $A_1 + A_2 + \cdots + A_n = \sum_{r=1}^{n} A_r$
  $= \frac{n(a+b)}{2}$

Geometric Mean

- <u>Single GM of n positive numbers</u>

GM of n positive numbers

$a_1, a_2, \ldots, a_n$ be

$G = (a_1, a_2, .., a_n)^{\frac{1}{n}}$

- If $a > 0$, $b > 0$ then $G = (ab)^{\frac{1}{2}}$
- If $a < 0$, $b < 0$ then $G = -(ab)^{\frac{1}{2}}$
- If $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$ then $GM \to DNE$
- If a, b, c are in GP then b is GM b/w a and c

<u>n geometric mean b/w two numbers</u>

$a, \underbrace{G_1, G_2, \ldots, G_n}_{n\ GM's}, b \to$ GP

$G_1 = T_2 = ar$

$G_2 = T_3 = ar^2$

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$G_n = T_{n+1} = ar^n$

Where

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$n \to$ no of GM's

- $G_1 . G_2 \ldots G_n = \prod_{r=1}^{n} G_r = \left(\sqrt{ab}\right)^n$

  $n \to$ no of GM's
- a and b are the roots of equation

  $x^2 - 2Ax + G^2 = 0$

  Where

  $A =$ AM b/w a and b

  $G =$ GM b/w a and b
- a and b are given by

  $A \pm \sqrt{A^2 - G^2}$

Harmonic Mean

If a, b, c are in HP then

$b =$ HM b/w a and c

$b = \frac{2ac}{a+c}$

<u>Single HM of n positive numbers</u>

HM of n positive numbers

$a_1, a_2, a_3, \ldots, a_n$

$H = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\right)}$

- 2, 3, 6 are in HP (Let)

<u>Relation b/w A,G,H</u>

(1) If A, G, H are respectively AM, GM, HM between two positive number a and b then

(1) $G^2 = AH$

(2) $A \geq G \geq H$

(3) $A = G = H \Rightarrow a = b$

<u>Arithmetico -Geometric Series</u>

$a, a + d, a + 2d, \ldots . a + (n - 1)d \to$ AP

$1, r, r^2, r^3, \ldots, r^{n-1}$ GP

$a, (a + d)r, (a + 2d)r^2, \ldots, [a + (n - 1)d]r^{n-1} \to$ AGP

If $|r| < 1$ and $n \to \infty$

Then $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Summation

(a) $\sum_{r=1}^{n} (a_r + b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$

(b) $\sum_{r=1}^{n} k a_r = k \sum_{r=1}^{n} a_r$

(c) $\sum_{r=1}^{n} k = nk$, $k =$ constant

Results

(a) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

(b) $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$

(c) $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

(d) $\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2 + 3n - 1)$

(e) $\sum_{r=1}^{n} 2r - 1 = n^2$

(f) $\sum_{r=1}^{n} 2r = n(n+1)$

- If rth term of any series is given $\to T_r$ then $S_n = \sum_{r=1}^{n} T_r$
- If $pT_p = qT_q$ of an AP then $T_{p+q} = 0$
- If $S_p = q$ and $S_q = p$ for an AP

  Then $S_{p+q} = -(p + q)$
- In case of HP

  $T_m = n$ and $T_n = m$

  Then $T_r = \frac{mn}{r}$
- $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \cdots =$

  $\begin{cases} \frac{n(n+1)}{2}, n\ odd \\ -\frac{n(n+1)}{2}, n\ even \end{cases}$