Chapter 02 Unit & Measurement

CONTENT

1. PHYSICAL QUANTITIES

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities.

eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

Physical quantities are of three types

Fundamental Quantities

- Fundamental quantities are chosen such that they should be different, that means independent of each other.

Derived Quantities

- Physical quantities which can be expressed in terms of fundamental quantities (M,L,T....) are called derived quantities.

E.g. speed = $\frac{\text{length}}{\text{time}}$

Supplementary Quantities

- Besides seven fundamental quantities two supplementary quantities are also defined. They are
(i) Plane angle
(ii) Solid angle

1.1 Fundamental quantities :

- These are the elementary quantities which covers the entire span of physics.
- Any other quantities can be derived from these.
- All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance, time and velocity cannot be chosen as basic quantities as $V = \frac{d}{t}$). An International Organization named CGPM : General Conference on weight and Measures, choose seven physical quantities as basic or fundamental.

FUNDAMENTAL QUANTITIES

- Length (L)
- Time (T)
- Mass (M)
- Temperature (K)
- Electrical current (A)
- Luminous Intensity (Cd)
- Amount of Substance (mol)

2. THE INTERNATIONAL SYSTEMS OF UNITS :

In earlier time scientists of different countries were using different system of units for measurement. There such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently. The base units for length, mass and time in these systems were as follows :

- **CGS system**→ centimetre, gram and second.
- **FPS system**→ foot, pound and second.
- **MKS system** → metre, kilogram and second.

2.1 S.I. SYSTEM

- The system of units which is at present internationally accepted for measurement is the system international d' Unites (French for International system of Units), units and abbreviations, was developed and recommended by General Conference on Weights and Measures in 1971 for international usage in scientific, technical, industrial and commercial work. Because SI units used decimal system, conversions within the system are quite simple and convenient.

2.2 S.I. UNIT & BASE QUANTITIES

- **Length (meter)** : The meter is the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second (1983). It is denoted by m.

- **Mass (kilogram)** : The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889). It is denoted by kg.

- **Time (second)** : The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967). It is denoted by s.

- **Electric Current (ampere)** : The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per metre of length. (1948). It is denoted by A.

- **Thermodynamic Temperature (kelvin)** : The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967). It is denoted by K.

- **Amount of Substance (mole)** : The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971). It is denoted by mol.

- **Luminous Intensity (candela)** : The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979). It is denoted by Cd.

SI UNIT FUNDAMENTAL QUANTITIES			
S.NO	**PHYSICAL QUANTITY**	**UNIT**	**SYMBOL**
1.	LENGTH	metre	m
2.	MASS	Kilogram	kg
3.	Time	Second	s
4.	Temperature	Kelvin	K
5.	Electric Current	Ampere	A
6.	Luminous intensity	Candela	cd
7.	Amount of substance	mole	mol

2.3 S.I. UNITS OF DERIVED QUANTITIES :

- Velocity = $\frac{\text{displacement(meter)}}{\text{time(second)}}$

  So unit of velocity will be m/s

- Acceleration = $\frac{\text{change in velocity}}{\text{time}} = \frac{m/s}{s} = \frac{m}{s^2}$
- Momentum = mass × velocity, Unit= (kg) (m/s) = kg m/s
- Force = mass × acceleration Unit = (kg) × (m/s^2) = kg m/s^2 called newton (N)
- Work = Force × displacement Unit = (N) × (m) = N m called joule (J)
- Power = $\frac{\text{work}}{\text{time}}$ Unit = J / s called watt (W).

2.4 SUPPLEMENTARY UNITS:

Two supplementary units were also defined:

- **Plane angle – Unit = radian (rad)**
- **Solid Angle- unit = steradian(sr)**

2.5 SI PREFIXES

Power of 10	prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	**giga**	**G**
10^{6}	**mega**	**M**
10^{3}	**kilo**	**k**
10^{2}	hecto	h
10^{1}	deca	da

Power of 10	prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	**milli**	**m**
10^{-6}	**micro**	**μ**
10^{-9}	**nano**	**n**
10^{-12}	**pico**	**p**
10^{-15}	femto	f
10^{-18}	atto	a

2.5 PRINCIPAL OF MEASUREMENT, NUMERICAL VALUE AND UNIT :

The result of measurement of a physical quantity is expressed as Measurement = *nu*
Were n is numerical value accompanied by a unit 'u'.

3. DIMENSIONS

- The nature of a physical quantity is described by its dimensions.
- All the physical quantities represented by derived units and be expressed in terms of some combination of seven fundamental or base quantities.
- We shall call these base quantities as the seven dimensions of the physical world, which are denoted with square brackets [ ].
- Thus, length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd], and amount of substance [mol].
- The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

  e.g. velocity = $\frac{\text{displacement(length)}}{\text{time}}$

So dimensions of velocity = $\frac{\text{dimension of displacement}}{\text{dimension of time}} = \frac{[L]}{[T]} = [LT^{-1}]$

3.1 DIMENSIONAL FORMULA, DIMENSIONAL EQUATION & CHARACTERISTICS :

The dimensional equations are the equations, which represent the dimensions of a physical quantity in terms of the base quantities.

For example, the dimensional equations of volume [V], speed [v], force [F] and mass density [ρ]expressed as $[V] = [M^0L^3T^0]$ $[v] = [M^0LT^{-1}]$ $[F] = [M\,L\,T^{-2}]$ $[\rho] = [M\,L^{-3}T^0]$

- Pure numbers are dimensionless.
- All trigonometrical ratios, powers, exponential and logarithmic functions are dimensionless.
- All ratio of physical quantities having same dimensional formula are dimensionless.
  e.g. relative density, relative permeability, dielectric constant, angles, refractive index etc.

Note: Plane angle solid angle are dimensionless.

- Dimensions do not depend upon magnitude.
- Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

QUANTITY	SYMBOL	FORMULA	S.I UNIT	DIMENSION
Displacement	s	ℓ	Meter or m	M^0LT^0
Area	A	$\ell \times b$	$(\text{Metre})^2$ or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	$(\text{Metre})^3$or m^3	$M^0L^3T^0$
Velocity	V	$V = \frac{\Delta S}{\Delta T}$	m/s	M^0LT^{-1}
Momentum	**P**	**P = mv**	**Kgm/s**	MLT^{-1}
Acceleration	**A**	$a = \frac{\Delta v}{\Delta t}$	**m/s^2**	M^0LT^{-2}
Force	**F**	**F = ma**	**Newton or N**	MLT^{-2}
Impulse	**–**	**F × t**	**N.sec**	MLT^{-1}
Work	**W**	**F . d**	**N.m**	$ML^2\,T^{-2}$
Energy	**KE or U**	$K.E = \frac{1}{2}mv^2$ **P.E = mgh**	**Joule or j**	ML^2T^{-2}
Power	P	$P = \frac{W}{t}$	Watt or W	ML^2T^{-3}
Density	d	d = mass/ volume	Kg/m^3	$ML^{-3}T^0$
Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m	ML^2T^{-2}
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	No dimension
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^0L^0T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^0L^0T^{-2}$
Moment of Inertia	I	$I = mr^2$	$Kg\text{–}m^2$	ML^2T^0
Frequency	f	$f = \frac{1}{T}$	Hertz or Hz	$M^0L^0T^{-1}$
Stress	–	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	–	$\frac{\Delta\ell}{\ell};\ \frac{\Delta A}{A};\ \frac{\Delta V}{V}$	–	No dimension

Youngs modulus (Bulk modulus of rigidity)	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m^2	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m};\frac{j}{m^2}$	ML^0T^{-2}
Force constant (Spring)	K	$F = kx$	N/m	ML^0T^{-2}
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	Kg/ms(poise in C.G.S)	$ML^{-1}T^{-1}$
Gravitational constant	G	$F = \frac{Gm_1m_2}{r^2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^3T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{j}{kg}$	$M^0L^2T^{-2}$
Gravitational field intensity	I	$I = \frac{F}{M}$	$\frac{\text{Newton}}{\text{Kg}}$	$M^0L^1T^{-2}$
Temperature	θ	–	Kelvin or K	$M^0L^0T^0K^{+1}$
Heat	Q	$Q = m \times C \times \Delta t$	Joule or Calorie	ML^2T^{-2}
Specific Heat	C	$C = \frac{Q}{m \times \Delta t}$	$\frac{\text{joule}}{\text{kg.Kelvin}}$	$M^0L^2T^{-2}K^{-1}$
Latent Heat	L	$Q = mL$	$\frac{\text{joule}}{\text{kg}}$	$M^0L^2T^{-2}$
Coefficient of thermal Conductivity	K	$Q = \frac{KA(\theta_1 - \theta_2)}{d}$	$\frac{\text{joule}}{\text{m sec K}}$	$MLT^{-3}K^{-1}$
Universal gas constant	R	$PV = nRT$	$\frac{\text{joule}}{\text{mol K}}$	$ML^2T^{-2}K^{-1}mol^{-1}$
Mechanical equivalent of Heat	J	$W = JH$	–	no dimension
Charge	Q or q	$I = \frac{Q}{t}$, $Q = It$	Coulomb or C	$M^0L^0T^1A^1$
Current	I	–	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$\varepsilon_0 = \frac{1}{4\pi F}\cdot\frac{q_1q_2}{r^2}$	$\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}T^4A^2$
Electric Potential	V	$V = \frac{\Delta W}{q}$	Joule/c	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	$E = \frac{F}{q}$	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	$Q = CV$	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_t	$\varepsilon_t = \frac{\varepsilon}{\varepsilon_0}$	–	no dimension
Resistance	R	$V = IR$	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{+3}A^2$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{\ell}$	Ohm × meter	$ML^3T^{-3}A^{-2}$

Conductivity or Specific conductance	σ	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^{3}A^{2}$
Magnetic induction	B	$F = qvB\sin\theta$ Or $F = BIL$	Tesla or weber/m^2	$MT^{-2}A^{-1}$
Magnetic Flux	ϕ	$e = \frac{d\phi}{dt}$	Weber	$ML^{2}T^{-2}A^{-1}$
Magnetic intensity	H	$B=\mu H$	A/m	$M^{0}L^{-1}T^{0}A$
Magnetic Permeability Or free space or medium	μ_0	$B=\frac{\mu_0}{4\pi}\frac{Id1\sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L\frac{dI}{dt}$	Henry	$ML^{2}T^{-2}A^{-2}$
Electric dipole moment	P	$P = q \times 2\ell$	C.m	$M^{0}LTA$
Magnetic dipole moment	M	M= NIA	amp.m^2	$M^{0}L^{2}AT^{0}$

SPOT LIGHT

Principle of Homogeneity of dimensions :

It states that in a correct equation, the dimensions of each term added or subtracted must be same. Every correct equation must have same dimensions on both sides of the equation.

i.e. (i) dimension of L.H.S = dimension of R.H.S

(ii) if a equation A + B = C– D then A ,B, C, D have same dimension .

4. APPLICATION OF DIMENSIONS :

4.1 To check the Correctness of the formula:

Using Homogeneity of dimension, for a equation-

L.H.S = R.H.S

- If the dimensions of the L.H.S and R.H.S are same, then we can say that this equation is at least dimensionally correct. So, this equation may be correct.
- But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct. So, it cannot be correct.

For example:

Formula for centrifugal force F_e is given by $F_e = \frac{mv^2}{r}$

(where m = mass, v = velocity, r = radius)

we have to check whether it is correct or not.

Dimension of L.H.S is $[F] = [M^1L^1T^{-2}]$

Dimension of R.H.S is $\frac{[m][v^2]}{[r]} = \frac{[M][LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$

$\Rightarrow$ Dimension of L.H.S = Dimension of R.H.S

So this eq^n is at least dimensionally correct.

we can say that this equation **may be** correct.

4.2 Deducting Relation among the physical quantities:

If we know the dependence of the physical quantity to other quantities (upto three physical parameter or linearly independent variables) and relation between them is product type. then we can estimate, to what extent, the quantity depends on the other.

For example:

Time period of a simple pendulum can depend on

Mass of Pendulum (m) | Length of the string (ℓ) | Gravitational acceleration (g)

So we can say that expression of T should be in this form

$T \propto (m)^a (\ell)^b (g)^c$

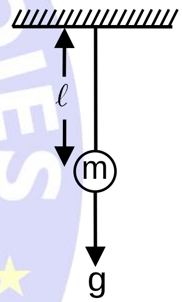
$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$

$\Rightarrow M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$

Comparing the powers of M, L and T, get

$a = 0,\ b + c = 0,\ -2c = 1$ so $a = 0,\ b = \frac{1}{2},\ c = -\frac{1}{2}$

So, $T \propto M^0\,L^{1/2}\,g^{-1/2} \Rightarrow$ $T = (\text{Constant})\sqrt{\frac{\ell}{g}}$



The "Constant" can be found experimentally.

4.3 Conversion of Units :

$n[u]$ = constant

i.e., $n_1[u_1] = n_2[u_2]$

where n is the numerical value and u is the unit.

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where the dimensional formula of the physical quantity is $[M^aL^bT^c]$.

4.4 Limitations of Dimensional Analysis :

- Dimensional analysis doesn't give information about the "constant" (The dimensional constant).
- This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.

  (i.e., $f = x^a\,y^b\,z^c$)

  It fails if a physical quantity depends on sum or difference of two quantities

  (i.e. $f = x + y - z$)

  i.e., we cannot get the relation

  $S = ut + \frac{1}{2}at^2$ from dimensional analysis.

- This method will not work if a quantity depends on another quantity as sine or cosine, logarithmic or exponential relation. The method works only if the dependence is by power functions.
- We equate the powers of M, L and T hence we get only three equations. So we can have only three variable (only three dependent quantities)
  So dimensional analysis will work only if the quantity depends only on three parameters, not more than that.

SOLVED EXAMPLES

Example 1: Which of the following sets cannot enter into the list of fundamental quantities in any newly proposed system of units?

(1) length, mass and velocity (2) pressure, density and velocity

(3) force, velocity and time (4) force, momentum and time

Solution: (2, 4) For (1): Length [L], mass [M] and velocity [LT^{-1}] are independent.

For (2): Pressure [$M^1L^{-1}T^{-2}$], density [$M^1L^{-3}T^0$] and velocity [M^0LT^{-1}] are dependent

For (3): Force [MLT^{-2}], velocity [LT^{-1}] and time [T] are independent

For (4): Force =

$\Rightarrow$ force, momentum and time are dependent.

Example 2: What is the solid angle enclosed by a plane of area 6 cm^2 around a point 5 m away from the plane

(1) 1.24×10^{-4} (2) 0.24×10^{-4} (3) 6×10^2 (4) 0.05×10^{-2}

Solution: Since the area is small as compared to the distance of the point, the planar part can be considered as a curved part so solid angle will be $\Omega = \frac{\text{area}}{R^2} = \frac{6\times10^{-4}}{5^2} = 0.24 \times 10^{-4}$

Example 3: Convert all in meters (m) :

(i) 5 μm. (ii) 3 km (iii) 20 mm (iv) 73 pm (v) 7.5 nm

Solution: (i) 5 μm = 5×10^{-6}m (ii) 3 km = 3×10^3 m

(iii) 20 mm = 20×10^{-3}m (iv) 73 pm = 73×10^{-12} m

(v) 7.5 nm = 7.5×10^{-9} m

Example 4: F = 5 N convert it into CGS system

Solution: $F = 5\ \frac{kg\times m}{s^2} = (5)\ \frac{(10^3 g)(100cm)}{s^2} = 5 \times 10^5\ \frac{g\,cm}{s^2}$ (in CGS system).

This unit ($\frac{g\,cm}{s^2}$) is also called dyne

Example 5: Find out the dimensional formula of the following

(i) Density (ii) Velocity (iii) Acceleration

(iv) Momentum (v) Angle (vi) Torque

Solution: (i) $[\text{Density}] = \frac{[\text{mass}]}{[\text{volume}]} = \frac{M}{L^3} = [M^1L^{-3}]$

(ii) Velocity [V] = $\dfrac{[\text{Displacement}]}{[\text{time}]}$ = $[M^0L^1T^{-1}]$

(iii) Acceleration (a) = $[M^0L^1T^{-2}]$

(iv) Momentum (P) = mV = $[M^1L^1T^{-1}]$

(v) Angle (θ) = $\dfrac{[\text{Arc}]}{[\text{radius}]} = \dfrac{L}{L}$ = $[M^0L^0T^0]$ (Dimensionless)

(vi) Torque = Force × Arm length = $[M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

Example 6: To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$

Here s is specific heat capacity. Then find out the dimensional formula of 's'

(1) $[M^1L^2T^{-2}K]$ (2) $[M^0L^2T^{-2}K^{-1}]$

(3) $[M^1L^2T^{-2}K^2]$ (4) $[M^1L^1T^1K^1]$

Solution: $[Q] = [m]\,[s]\,[\Delta T]$ Here Q is heat : A kind of energy so

$[Q] = M^1L^2T^{-2} \Rightarrow [M^1L^2T^{-2}] = [M]\,[S]\,[K]$

$[S] = [M^0L^2T^{-2}K^{-1}]$

Example 7: If any spherical ball of radius r moves with velocity v in a viscous liquid, then viscous force acting on it is given by $F_v = 6\pi\eta rv$

Here η is coefficient of viscosity. Find out the dimensional formula of 'η'

(1) $[M^1L^2T^{-3}]$ (2) $[M^1L^{-2}T^{-1}]$

(3) $[M^1L^{-1}T^{-1}]$ (4) $[M^2L^2T^{-2}]$

Solution: $[F_v] = [6\pi]\,[\eta]\,[r]\,[v] \Rightarrow M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}] \Rightarrow [\eta] = M^1L^{-1}T^{-1}$

Example 8: $\alpha = \dfrac{FV^2}{\beta^2}\log_e\left(\dfrac{2\pi\beta}{V^2}\right)$ where F = force , V = velocity

Find the dimensions of α and β.

(1) $[M^1L^{-1}T^0]$, $[L^2T^{-2}]$ (2) $[M^0L^2T^1]$, $[L^1T^{-1}]$

(3) $[M^0L^0T^0]$, $[M^1L^2]$ (4) $[M^0L^0T^0]$, $[M^0L^0]$

$$\alpha = \frac{FV^2}{\beta^2}\log_e\frac{2\pi\beta}{V^2}$$

dimensionless

Solution: $[\alpha] = \dfrac{[F][V^2]}{[\beta^2]} \Rightarrow \dfrac{[2\pi][\beta]}{[V^2]} = M^0L^0T^0 \Rightarrow \dfrac{[1][\beta]}{L^2T^{-2}} = M^0L^0T^0$

$[\beta] = L^2T^{-2}$ $[\alpha] = M^1L^{-1}T^0$

Example 9: The dimensional formula of product and quotient of two physical quantities A and B are given by $[AB] = [ML^2T^{-2}]$; $\left[\dfrac{A}{B}\right] = [MT^{-2}]$. The quantities A and B respectively are

(1) Force and velocity (2) Force and displacement

(3) Momentum and displacement (4) Work and velocity

Solution: $[A]^2 = [AB].\left[\dfrac{A}{B}\right] = [M^2L^2T^{-4}] \Rightarrow [A] = [MLT^{-2}] \equiv$ Force

$[B] = \dfrac{[AB]}{[A]} = [L] \Rightarrow [B] = [L] \equiv$ Displacement

Example 10 Check whether this equation may be correct or not

Pressure $P_r = \dfrac{3FV^2}{\pi^2 t^2 x}$ (where F = force, V = velocity, t = time, x = distance)

Solution: Dimension of L.H.S = $[P_r] = M^1L^{-1}T^{-2}$

Dimension of R.H.S = $\dfrac{[3][F][v^2]}{[\pi][t^2][x]} = \dfrac{[M^1L^1T^{-2}][L^2T^{-2}]}{[T^2][L]} = M^1L^2T^{-6}$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

Example 11: If velocity (V), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of V, F and T.

(1) $[M] = [V^{-1}F^1T^1]$, $[E] = [V^1][F^1][T^1]$ (2) $[M] = [V^{-2}F^2T^1]$, $[E] = [V^2][F^2][T^2]$

(3) $[M] = [V^0F^1T^2]$, $[E] = [V^3][F^0][T^{-1}]$ (4) $[M] = [V^1F^2T^{-3}]$, $[E] = [V^2][F^1][T^{-2}]$

Solution: Let $M \propto (V)^a (F)^b (T)^c$

Equating dimensions of both the sides

$M^1L^0T^0 = (1)\ [L^1T^{-1}]^a\ [M^1L^1T^{-2}]\ [T^1]^c$

$\Rightarrow\ M^1L^0T^0 = M^b\ L^{a+b}\ T^{-a-2b+c}$

get $a = -1$, $b = 1$, $c = 1$

$\Rightarrow\ M \propto (V^{-1}\ F^1\ T^1)$

$[M] = [V^{-1}\ F^1\ T^1]$

Similarly we can also express energy in terms of V, F, T

Let $[E] \propto [V]^a\ [F]^b\ [T]^c$

$\Rightarrow\ [MLT^{-2}] = [M°L°T°]\ [LT^{-1}]^a\ [MLT^{-2}]^b\ [T]^c$

$\Rightarrow\ [M^1L^1T^{-2}] = [M^b\ L^{a-2b+c}\ T^{-a-2b+c}]$

$\Rightarrow\ 1 = b;\quad 1 = a - 2b + c;\quad -2 = -a - 2b + c$

get $a = 1$; $b = 1$; $c = 1$

$\therefore\ E = (\text{constant}) = [V^1][F^1][T^1]$.

Example 12: The density of a material in CGS system is 2g /cm^3. In a system of units in which unit of length is 2 cm and unit of mass is 4 g, what is the numerical value of the density of the material?

(1) 2 (2) 4 (3) 5 (4) 7

Solution: $n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1 \left[\left[\dfrac{M_1}{M_2}\right]^1 \left[\dfrac{L_1}{L_2}\right]^{-3}\right] = 2\left[\left(\dfrac{1g}{4g}\right)\left(\dfrac{1cm}{2cm}\right)^{-3}\right] = 4$

5. MEASUREMENT

Limitations of dimensional Analysis

(i) Measurement of length

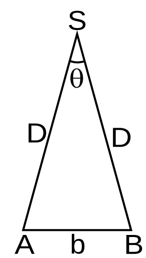
You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^2 m. A vernier callipers is used fro lengths to an accuracy of 10^{-}

^{4} m. A screw gauge and a spherometer can be used to measure lengths as less as to 10^{-5} m. To mesure lengths lengths beyound these ranges, we make use of some special indirect methods.

(ii) Measurement of large Distances :

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method.

When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax. The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes.



To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the earth, separated by distance AB = b at the same time as shown in figure. We measure the angle between the two directions along which the planet is viewed at these two points. The $\angle ASB$ in represented by symbol θ is called the parallax angle or parallactic angle.

As the planet is very far away, $\frac{b}{D} << 1$, and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS = BS so that AB = b =D θ where θ is in radians.

$$D = \frac{b}{\theta}$$

(iii) Measurement of small distance, size of molecules:

To measure a very small size like that of a molecule (10^{-8} m to 10^{-10} m), we have to adopt special methods. Even a microscope has certain limitations.

A simple method for estimating the molecular size of oleic acid is given below. Oleic acid is a soapy liquid with large molecular size of the order of 10^{-9} m.

The idea is to first form mono-molecular layer of oleir acid on water surface.

We dissolve 1 cm^3 of oleic acid in alcohol to make a solution of 20 cm^3. Then we take 1 cm^3 of the solution and dilute is to 20 cm^2, using alcohol. So, the concentration of the solution is equal to $\left(\frac{1}{20\times 20}\right)$cm^3 of oleic acid/cm^3 of solution. Next we lighty sprinkle some lycopodium powder on the surface of water in a large trough and we put one drop of this solution in the water. The oleic acid drop spreads into a thin, large and roughly circular film of molecular thickness on water surface. Then, we quickly measure the diameter of the thin film to get its area A. Suppose we have dropped n drops in the water. Initially, we determine the approximate volume of each drop (V cm^3).

Volume of n drops of solution = nV cm^3

Amount of oleic acid in the solution = $nV\left(\frac{1}{20\times 20}\right)$cm^3

This solution of oleic acid spreads very fast on the surface of water and forms a very thin layer of thickness t. If this spreads to form a film of area A cm^2, then the thickness of the film

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}} \quad \text{or,} \quad t = \frac{nV}{(20\times20)A}\ \text{cm}$$

If we assume that the film has mono-molecular thickness, then this becomes the size or diameter of a molecule of oleic acid. The value of this thickness comes out to be of the order of 10^{-9} m.

5.1 RANGE OF LENGTHS :

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-14} m of the tiny nucleus of an atom to the size of the order of 10^{26} m of the extant of the observable universe.

5.2 MEASUREMENT OF MASS:

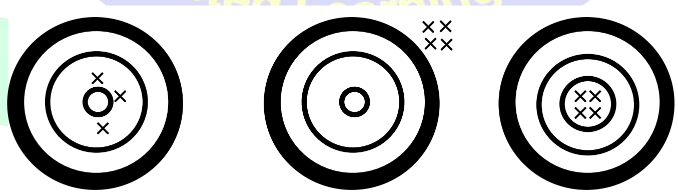
Mass is a basic property of matter. It does not depend on the temperature, pressure of location of the object in space. The SI unit of mass is kilogram (kg).

Mass of commonly available objects can be determined by a common balance like the one used in a grocery shop. Large masses in the universe like planets, stars, etc., based on Newton's law of gravitation can be measured by using gravitational method. For measurement of small masses of atomic/subatomic particles etc., we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

5.3 ACCURACY, PRECISION OF INSTRUMENT :

ACCURACY → The closeness of the measured value to the true value of the physical quantity

PRECISION → the measure of the extent to which successive measurements of a physical quantity differ from one another



Accurate But less precise | Precise But less Accurate | Accurate and Precise

- The accuracy in measurement may depend on several factors, including the limit of the Resolution of the measuring instrument.

  For example, suppose the true value of a certain length is near 3.678 cm.

  1st experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in

2^{nd} experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.88 cm.

The first measurement has more accuracy (because it is closer to the true value) but less precision (its resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

6. ERRORS IN MEASUREMENT :

Errors in measurement
- Systematic Error
  - Instrumental error
  - Imperfection is experimental technique error
  - Personal or observational error
  - environmental error
- Random Errors
- Least count error

The difference between true value and estimated value of a physical quantity is known as error.

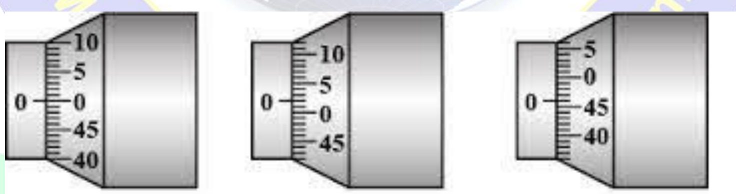
6.1 Systematic Error :

The systematic errors are those errors tend to be in one direction. i.e. They have known sign. the systematic error is removed before begging of calculation. Some of the sources of systematic errors are :

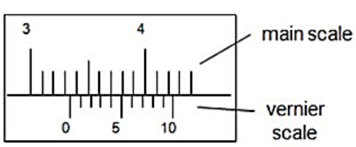
(i) Instrumental error :

Instrumental error that arise from the errors due to imperfect design or calibration of the measuring instrument

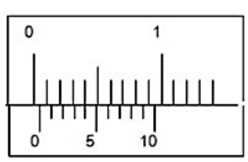
e.g. zero error in the instrument



(a) No zero error (b) Positive zero error (c) Negative zero error

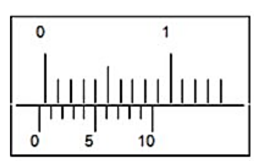


measured reading = 3.3 + 0.04 = 3.34



positive zero error = 0.04

Vernier scale zero is left of main scale zero.
Correct measurement with positive error
= 3.34 – 0.04 = 3.30



negative error = - 0.04

Vernier scale is right of main scale zero.
Correct measurement with negative error
= 3.34-(-0.04)=3.38

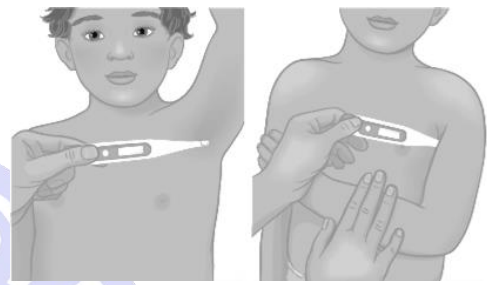
SPOT LIGHT

NOTE: Positive zero error is subtracted in final result but negative zero error is added.

(ii) Imperfection is experimental technique :

This type of error arise due to improper or old measurement technique

For example : To determine the temperature of human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature.



- This type of systematic errors can be minimised by improving experimental techniques, selecting better instrumental and removing personal bias as far as possible

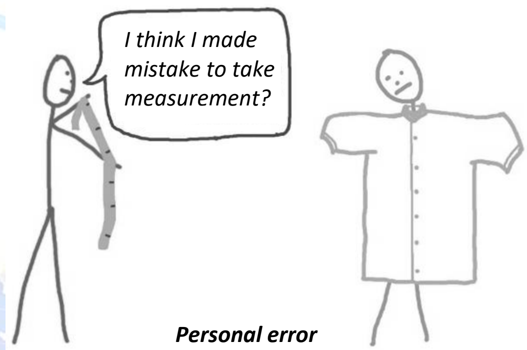
(iii) Personal or observational errors :

Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

e.g.

(i) Wrong estimation of a pointer between two scale point

(ii) Observing colour of solution



Personal error

(iv) Environmental error :–

This type of error appears when some factor in the Environment, such as an uncommon event,(windy, humidity in lab) leads to error.

e.g. We want to measure mass of an apple, but our class room is windy, The wind may cause the scale to read incorrectly.

6.2 Random Errors :

The random errors are those errors, which occur irregularly. These can arise due to random and unpredictable fluctuations in experimental conditions.

i.e. They have known sign & size. They are represented as = A $\pm$ a

where A = measured quantity, a = magnitude of its limit error.

6.3 Least count error :

The smallest value that can be measured by the measuring instrument is called its **least count**. All the readings or measured values are good only up to this value.

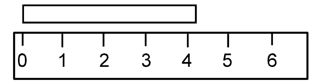
The least count error is the error associated with the resolution of the instrument.

For Example

(i) Lets use a cm scale:

We will measure length = 4 cm

Although the length will be a bit more than 4, but we cannot say its length to be 4.1 cm or 4.2 cm, as the scale can measure upto cm only, not closer than that.



(cm scale)

It (this scale) can measure upto cm accuracy only.

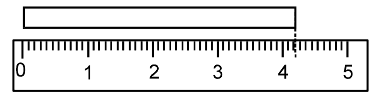
so we'll say that its <u>least count</u> is 1 cm

(ii) Lets use an mm scale :

We will measure length "λ"= 4.2 cm,

Which is a more closer measurement.

Here also if we observe closely, we'll find that the length is a bit more than 4.2, but we cannot say its length to be 4.21, or 4.22, or 4.20 as This scale can measure upto 0.1 cms (1 mm) only, not closer than that.



(mm scale)

To get a closer measurement, W e have to use a more minute scale, that is mm scale

It (this scale) can measure upto 0.1 cm (1 mm) accuracy

Its <u>least count</u> is 0.1 cm (1mm)

DETECTIVE MIND

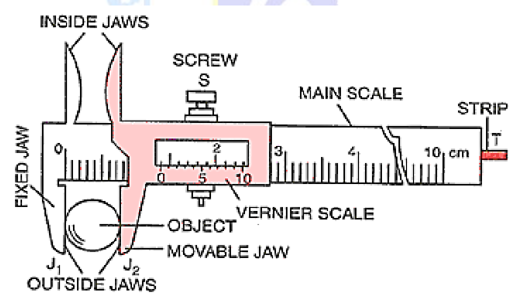
Least Count of Vernier calipers :

Suppose the size of one main scale division (M.S.D.) is M units and that of one vernier scale division (V. S. D.) is V units. Also let the length of 'a' main scale divisions is equal to the length of 'b' vernier scale divisions.

$$aM = bV \Rightarrow V = \frac{a}{b}M$$

$$\therefore \; M - V = M - \frac{a}{b}M \text{ or } M - V = \left(\frac{b-a}{b}\right)M$$

The quantity (M– V) is called *vernier constant* (V. C. or *least count* (L. C.) of the Vernier calipers.



$$\boxed{L.C. = M - V = \left(\frac{b-a}{b}\right)M}$$

Reading = MSR + VSD × LC

SPOT LIGHT

- All the readings or measured value by a instrument are good only up to the least count

6.4 Permissible error :

Error in measurement due to the limitation (least count) of the instrument, is called permissible error.

For example: By mm scale → we can measure upto 1 mm accuracy (least count). From this we will get measurement like ℓ = 34 mm

Max uncertainty can be 1 mm.

Max permissible error ($\Delta\ell$) = 1 mm.

- But if from any other instrument, we get ℓ = 34.5 mm then max permissible error ($\Delta\ell$) = 0.1 mm and if from a more accurate instrument, we get ℓ = 34.527 mm then Max permissible error ($\Delta\ell$) = 0.001 mm = place value of last number

6.5 Maximum Permissible Error in result due to error in each measurable quantity :

Let Result f(x, y) contains two measurable quantity x and y

Let error in x is = $\pm \Delta x$ i.e. $x \in (x - \Delta x, x + \Delta x)$

error in y is = $\pm \Delta y$ i.e. $y \in (y - \Delta y, y + \Delta y)$

Case :(I) If $f(x, y) = x + y \Rightarrow df = dx + dy$

error in f = $\Delta f = \pm \Delta x \pm \Delta y$

max possible error in f = $(\Delta f)_{max}$ = max of $(\pm \Delta x \pm \Delta y)$

$\Rightarrow (\Delta f)_{max} = \Delta x + \Delta y$

Case:(II) If $f = x - y \Rightarrow df = dx - dy$

$\Rightarrow (\Delta f) = \pm \Delta x \mp \Delta y$

max possible error in f = $(\Delta f)_{max}$ = max of $(\pm \Delta x \mp \Delta y)$

$\Rightarrow (\Delta f)_{max} = \Delta x + \Delta y$

- *For getting maximum permissible error, sign should be adjusted, so that errors get added up to give maximum effect*

  i.e. $f = 2x - 3y - z \Rightarrow (\Delta f)_{max} = 2\Delta x + 3\Delta y + \Delta z$

7. ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR :

Suppose to measure some quantity, we take several observations, $a_1, a_2, a_3 .... a_n$. To find the absolute error in each measurement and percentage error , we have to follow these steps

- First of all mean of all the observations is calculated:

$$a_{mean} = \frac{(a_1 + a_2 + a_3 + ... + a_n)}{n}$$

The mean of these values is taken as the **best possible value** of the quantity under the given conditions of measurements.

7.1 Absolute Error :

The magnitude of the difference between the best possible or mean value of the quantity and the individual measurement value is called the **absolute error** of the measurement.

The absolute error in an individual measured value is:

$$\boxed{\Delta a_n = |\, a_{mean} - a_n \,|}$$

The arithmetic mean of all the absolute errors is taken as the **final or mean absolute error**.

$$\Delta a_{mean} = \frac{(|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|)}{n}$$

7.2 Relative Error

Relative error is the ratio of the mean absolute error and arithmetic mean .

$$\boxed{\text{Relative error} =} \frac{\Delta a_{mean}}{a_{mean}}$$

7.3 Percentage Error

When the relative error is expressed in percent, it is called the **percentage error**.

Thus, $\boxed{\text{Percentage error} = \dfrac{\Delta a_{mean}}{a_{mean}} \times 100\%}$

8. COMBINATION OF ERRORS :

8.1 Error of sum and difference :

- **In Sum :** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$, maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B} = \frac{\Delta A + \Delta B}{A+B}$$

- **In Difference :** If $Z = A - B$, then maximum absolute error is

$\Delta Z = \Delta A + \Delta B$

and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A-B} + \frac{\Delta B}{A-B} = \frac{\Delta A + \Delta B}{A-B}$$

- **In Product :** If $Z = AB$, then the maximum fractional error

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

- **In Division :** If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

- **In Power :** If $Z = A^n$ then $\dfrac{\Delta Z}{Z} = n\dfrac{\Delta A}{A}$

SPOT LIGHT

- In more general form if $Z = \dfrac{A^x B^y}{C^q}$

  then the maximum fractional error in Z is $\dfrac{\Delta Z}{Z} = x\dfrac{\Delta A}{A} + y\dfrac{\Delta B}{B} + q\dfrac{\Delta C}{C}$

9. SIGNIFICANT FIGURES

Every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement.

- Normally, the reported result of measurement is a number that includes all digits in the number at are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as **significant digits or significant figures**.
- Significant figures indicate, as already mentioned, the precision of measurement which depends on the least count of the measuring instrument.

9.1 Common rules of counting significant figures:

Rule 1: All non-zero digits are significant
i.e. 123.56 has five S.F.

Rule 2: All zeros occurring between two non-zero digits are significant (obviously) i.e. 1230.05 has six S.F.

Rule 3:

$\ell = 3.5\text{cm}$	$\ell = 3.50\text{cm}$	$\ell = 3.500\text{cm}$
$\downarrow$	$\downarrow$ closer!	$\downarrow$ more closer!
0.1cm accuracy	0.01cm accuracy	0.001cm accuracy
$\downarrow$	$\downarrow$	$\downarrow$
ℓ lies between $(3.4-3.6)$	ℓ lies between $(3.49-3.51)$	ℓ lies between $(3.499-3.501)$

So trailing zeroes after decimal place are significant (Shows the further accuracy)

3.5cm	3.50cm	3.500cm
$\downarrow$	$\downarrow$	$\downarrow$
Two S.F	Three S.F	Four S.F

Once a measurement is done, significant figures will be decided according to closeness of measurement.

- Now if we want to display the measurement in some different units, the S.F. shouldn't change (S.F. depends only on accuracy of measurement)

SPOT LIGHT

A choice of change of different units does not change the number of significant digits or figures in a measurement.

Suppose measurement was done using mm scale, and we get ℓ = 85 mm (Two S. F.)

If we want to display it in other units.

$85\text{mm} \rightarrow 8.5\text{cm} \rightarrow 0.085$

$85\text{mm} \downarrow 8500\text{mm} = 8.5\times10^{4}\text{mm}$

$0.085 \downarrow 0.000085\text{km} = 8.5\times10^{-5}\text{km}$

All should have two S.F.

The following rules support the conservation of S.F.

Rule 4: From the previous example, we have seen that,

0.000085km $\longrightarrow$ also should has two S.F 8 and 5

Not significant

So leading Zeros are not significant.

- *In the number less than one, all zeros after decimal point and to the left of first non-zero digit are insignificant (arises only due to change of unit)*

0.000305 has three S.F. $\Rightarrow 3.05 \times 10^{-4}$ has three S.F.

Rule 5: From the previous example, we have also seen that

85000μm $\longrightarrow$ Should also has two S.F 8 and 5.

Not significant

So the trailing zeros are also not significant.

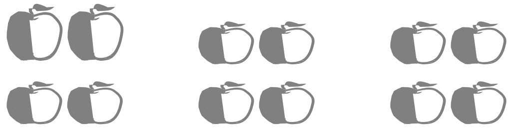
- *The terminal or trailing zeros in a number without a decimal point are not significant.*

(Also arises only due to change of unit)

154 m = 15400 cm = 15400 mm = 154×10^{9} nm

all has only three S.F. all trailing zeros are insignificant

Rule 6: There are certain measurement, which are exact i.e.



Number of apples are = 12 (exactly) = 12.000000.......... ∞

This type of measurement is infinitely accurate so, it has ∞ S.F.

Numbers of students in class = 125 (exact)

Speed of light in the vacuum = 299,792,458 m/s (exact)

9.2 Rules for arithmetic operation with significant figures :

➢ In multiplication or division, the final result should retain as may significant figures as are there in the original number with the least significant figures.

E.g. $1.2 \times 1.11 = 1.3$

➢ In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

E.g. $1.235 - 1.1 = 0.135 = 0.1$

($\because$ 2nd term contain lesser that means one decimal place)

10. ROUNDING OFF THE UNCERTAIN DIGITS :

The result of computation with approximate numbers, which contain more than one uncertain digit, should be rounded of. The rules of rounding off numbers to the appropriate significant figures are obvious in most cases.

Rules of Rounding off :

➢ If removable digit is less than 5 (50%) ; drop it.

For Example:- **47.833** $\xrightarrow[\text{till one decimal place}]{\text{Round off}}$ 47.8

➢ If removable digit is greater than 5(50%), increase the last digit by 1.

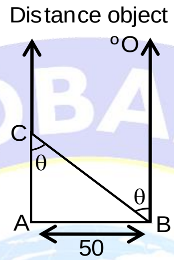
For Example:- 47.8**62** $\xrightarrow[\text{till one decimal place}]{\text{Round off}}$ 47.9

➢ If removable number is exactly 5(50%)

- If last number is even drop 5: $\underline{20.6\mathbf{5}} \rightarrow 20.6$
- If last number is odd, increase the last digit by 1: $\underline{20.7\mathbf{5}} \rightarrow 20.8$

SOLVED EXAMPLES

Example 13: A man wishes to estimate the distance of nearby tower from him he stands at a point A in front of the tower C and spots a very distance object O in the with AC. He then walks perpendicular to AC up to B, a distance of 50 m, and looks at O and C again. Since O is very distant the direction BO is practically the same as AO, but he finds the line of sight of C shifted from the original line of sight by an angle θ = 30° (θ = is known as parallax' estimate the distance of the tower C from his original position A.



(1) $\sqrt{3}$ m (2) 50 $\sqrt{3}$ m (3) 20 $\sqrt{3}$ m (4) 27 $\sqrt{3}$ m

Solution: We have parallax angle θ = 30° from figure AB = AC tan θ

AC = AB / tan θ = 50 m / tan 30° = $50\sqrt{3}$ m

Example 14: The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is 2°, Given the diameter of the Earth to be about 1.276×10^7 m, compute the distance of the moon from the Earth.

(1) 36.55×10^7 m (2) 38.66×10^{12} m

(3) 36.55×10^9 m (4) 39.66×10^5 m

Solution: We have $\theta = 2° = 2 \times \frac{\pi}{180} = \frac{\pi}{90}$ Also b = AB = 1.276×10^7 m

Hence from Eq. (2, 1) we have the earth moon distance, $D = b/\theta = \dfrac{1.276\times10^7}{\frac{\pi}{90}} = 36.55 \times 10^7$ m

Example 15: Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are:

	Clock 1	Clock 2
Monday	12:00:05	10:15:06
Tuesday	12:01:15	10:14:59
Wednesday	11:59:08	10:15:18
Thursday	12:01:50	10:15:07
Friday	11:59:15	10:14:53
Saturday	12:01:30	10:15:24
Sunday	12:01:19	10:15:11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer?

Solution: The range of variation over the seven days of observations is 162 s for clock1, and 31 s for clock 2. The average reading of clock 1 average reading of clock 2. The important point for precision work as its variation, because a 'Zero-error' can always be easily corrected. hence clock 2 is to be preferred to clock 1.

Example 16: Following observations were taken with a vernier calipers while measuring the length of a cylinder: 3.29 cm, 3.28 cm, 3.29 cm, 3.31 cm, 3.28 cm, 3.27 cm, 3.29 cm, 3.30cm Then find:

(1) Most accurate length of the cylinder.

(2) Absolute error in each observation.
(3) Mean absolute error
(4) Relative error
(5) Percentage error
Express the result in terms of absolute error and percentage error.

Solution:
(1) Most accurate length of the cylinder will be the mean length = 3.28875 cm = 3.29 cm
(2) Absolute error in the first reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the second reading = 3.29 – 3.28 = 0.01 cm
Absolute error in the third reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the fourth reading = 3.39 – 3.31 = –0.02 cm
Absolute error in the fifth reading = 3.29 – 3.28 = 0.01 cm
Absolute error in the sixth reading = 3.29 – 3.27 = 0.02 cm
Absolute error in the seventh reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the last reading = 3.29 – 3.30 = –0.01 cm

(3) Mean absolute error $\overline{\Delta\ell} = \frac{0.00+0.01+0.00+0.02+0.01+0.02+0.00+0.01}{8}$ = 0.01 cm

(4) Relative error in length = $\frac{\overline{\Delta\ell}}{\ell}$ = $\frac{0.01}{3.29}$ = 0.0030395 = 0.003

(5) Percentage error = $\frac{\overline{\Delta\ell}}{\bar{\ell}}$ × 100 = 0.003 × 100 = 0.3%

So, length ℓ = 3.29 cm ± 0.01 cm (in terms of absolute error)

$\Rightarrow \ell$ = 3.29 cm ± 0.30% (in terms of percentage error)

Example 17: In some observations, value of 'g' are coming as 9.81, 9.80, 9.82, 9.79, 9.78, 9.84, 9.79, 9.78, 9.79 and 9.80 m/s^2. Calculate absolute errors and percentage error in g.
(1) 0.014, 0.14%
(2) 0.024, 0.12%
(3) 0.016, 0.10%
(4) 0.121, 0.2%

Solution:

S.No.	Value of g	Absolute error $\Delta g = \lvert g_i - \bar{g} \rvert$
1	9.81	0.01
2	9.80	0.00
3	9.82	0.02
4	9.79	0.01
5	9.78	0.02
6	9.84	0.04
7	9.79	0.01
8	9.78	0.02
9	9.79	0.01
10	9.80	0.00

g_{mean} = 9.80, $\Delta g_{mean} = \frac{\Sigma \Delta g_i}{10} = \frac{0.14}{10} = 0.014$

Percentage error = $\frac{\Delta \bar{g}}{g_{max}} \times 100 = 0.14\%$

Example 18: We increase the period of oscillation of simple pendulum. In successive measurements the readings turn out to be 2.63s, 2.56s, 2.42s, 2.71s and 2.80s. Calculate the absolute errors relative error or percentage error.

Solution: The mean period of oscillation of the pendulum

$$T = \frac{(2.63+2.56+2.42+2.71+2.80)s}{5} = \frac{13.12}{5} = 2.624s = 2.62s$$

As the periods are measured to a resolution of 0.01 s all times are to the second decimal, it is proper to put this mean period also to the second decimal.

The errors in the measurements are

2.63 s – 2.62 s = 0.01 s   2.56s – 2.62 s = –0.06 s

2.42s – 2.62 s = –0.20 s   2.71 s – 2.62 s = 0.09 s

2.80 s – 2.62 s = 0.18 s

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is

ΔT_{mean} = [0.01 + 0.06 + 0.0 + 0.09 + 0.18)s] / 5 = 0.54 s / 5 = 0.11 s

That means, the period of oscillation of the simple pendulum is (2.62 ± 0.11)s or between 2.73 s and 2.51s. As the arithmetic mean of all the absolute errors is 0.11 s, there is already an error in the tenth of a second. Hence there is no point in giving the period to a hundredth. A more correct way will be to write T = 2.6 ± 0.1 s

Note that the last numeral 6 is unreliable, since it may be anything between 5 and 7. We indicate this by saying that the measurement has two significant figures. In this case, the two significant figures are 2, which is reliable and 6, which has an error associated with it. You will learn more about the significant figures in section 2.7.

For the example, the relative error or the percentage error is

$$= \frac{0.01}{2.6} \times 100 = 4\%$$

Example 19: Two resistors of resistances $R_1 = 100 \pm 3$ ohm and $R_2 = 200 \pm 4$ ohm are connected (a) in series (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for

(a) the relation $R = R_1 + R_2$ and for (b) $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$ and $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

Solution:

(a) The equivalent resistance of series combination

$R = R_1 + R_2$ = (100 ± 3) ohm + (200 ± 4) ohm = 300 ± 7 ohm.

(b) The equivalent resistance of parallel combination

$$R' = \frac{R_1R_2}{R_1R_2} = \frac{200}{3} = 66.7 \text{ ohm}$$

Then, from $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

We get,

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \Rightarrow \Delta R' = (R'^2)\left[\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right]$$

$$\Rightarrow \Delta R' = \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 = 1.8 \text{Then, } R' = 66.7 \pm 1.8 \text{ ohm}$$

(Here, ΔR is expressed as 1.8 instead of 2 to keep in conformity with the rules of significant figures)

Example 20: If measured value of resistance R = 1.05 Ω, wire diameter d = 0.60 mm, and length ℓ = 75.3 cm, then find max. permissible error in resistivity. Given $\rho = \frac{R\left(\frac{\pi d^2}{4}\right)}{\ell}$.

(1) 2% (2) 4 % (3) 0.2 % (4) 0.8%

Solution: $\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta R}{R} + 2\frac{\Delta d}{d} + \frac{\Delta \ell}{\ell}$

R = 1.05 Ω → ΔR = 0.01 Ω (least count)

d = 0.60 mm → Δd = 0.01 mm (least count)

ℓ = 75.3 → $\Delta\ell$ = 0.1 cm (least count)

$$\frac{\Delta\rho}{\rho} = \left(\frac{0.01\Omega}{1.05\Omega} + 2\frac{0.01mm}{0.60mm} + \frac{0.1cm}{75.3cm}\right) \times 100 = 4\ \%.$$

Example 21: To find the value of 'g' using simple pendulum. T = 2.00 sec; ℓ = 1.00 m was measured. Estimate maximum permissible error in 'g'. Also find the value of 'g'. (use = 10)

Solution: $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2\ell}{T^2} \Rightarrow \left(\frac{\Delta g}{g}\right)_{max} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta T}{T} = \left(\frac{0.01}{1.00} + 2\frac{0.01}{2.00}\right) \times 100\ \%. = 2\ \%$

value of $g = \frac{4\pi^2\ell}{T^2} = \frac{4\times10\times1.00}{(2.00)^2} = 10.0\ m/s^2 \Rightarrow \left(\frac{\Delta g}{g}\right)_{max} = 2/100$ so $\frac{\Delta g_{max}}{10.0} = \frac{2}{100}$

so $(\Delta g)_{max} = 0.2$ = max error in 'g' so 'g' = $(10.0 \pm 0.2)\ m/s^2$

Example 22: Find the relative error in Z, If $Z = \frac{A^4B^{1/3}}{CD^{3/2}}$.

Solution: The relative error in Z is $\frac{\Delta Z}{Z} = 4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{3}{2}\left(\frac{\Delta D}{D}\right)$

Example 23: The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch. of 1s resolution. What is the accuracy in the determination of g?

(1) 2 % (2) 3 % (3) 1% (4) 5%

Solution: $g = \frac{4\pi^2 L}{T^2}$ Here, $T = \frac{t}{n}$ and $\frac{\Delta T}{T} = \frac{\Delta t}{t}$.

The errors in both L and t are the least count errors. Therefore.

$\left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right) = \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right) = 0.027$ Thus, the percentage error in g is

$100\left(\frac{\Delta g}{g}\right) = \left[100\left(\frac{\Delta L}{L}\right) + 2\times100\left(\frac{\Delta T}{T}\right)\right] \Rightarrow 100\ (\Delta g/g) = [100\ (\Delta L/L) + 2 \times 100\ (\Delta T/T)] = 3\%$

Example 24: The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits.

(1) 19.38 ± 1.48 cm^2 (2) 18.28 ± 2.4 cm^2

(3) 20.20 ± 2.01 cm^2 (4) 21.21 ± 0.3 cm^2

Solution: Given ℓ = (5.7 ± 0.1) cm and b = (3.4 ± 0.2) cm

Area $A = \ell \times b = 5.7 \times 3.4 = 19.38\ cm^2$

$$\frac{\Delta A}{A} = \left(\frac{\Delta\ell}{\ell} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) = \pm\left(\frac{0.34+1.14}{5.7\times3.4}\right) = \pm\frac{1.48}{19.38}$$

$$\Rightarrow \Delta A = \pm\frac{1.48}{19.38} \times A = \pm\frac{1.48}{19.38} \times 19.38 = 1.48$$

Area = (19.38 ± 1.48) cm^2

Example 25: A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity?

(1) ± 8% (2) ± 2% (3) ± 9 % (4) ± 12 %

Solution: Given distance s = (13.8 ± 0.2) m and time t = (4.0 ± 0.3) s,

velocity $v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45\ ms^{-1} = 3.5\ ms^{-1}$

$$\frac{\Delta v}{v} = \pm\left(\frac{\Delta s}{s}+\frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8}+\frac{0.3}{4.0}\right) = \pm\left(\frac{0.8+4.14}{13.8\times4.0}\right) = \pm\frac{4.49}{13.8\times4.0} = \pm 0.0895$$

$\Rightarrow$ $\Delta v = \pm 0.0895 \times v = \pm 0.0895 \times 3.45 = \pm 0.3087 = \pm 0.31$

$\therefore$ $v = (3.5 \pm 0.31)\ ms^{-1}$

Percentage error in velocity $= \frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100 = \pm 8.95\% = \pm 9\%$

Example 26: In ohm's law exp., reading of voltmeter across the resistor is 12.5 V and reading of current i = 0.20 Amp. Estimate the resistance in correct S.F.

(1) 62 (2) 62.5 (3) 63 (4) 62.50

Solution: $R = \frac{V}{i} = \frac{12.5 \to 3SF}{0.20 \to 2SF} = 62.5\Omega \xrightarrow[\text{to2S.F.}]{\text{round off}} 62\Omega$

Example 27: Write down the number of significant figures in the following:

(a) 165 (b) 2.05 (c) 34.000 m (d) 0.005 (e) 0.02340 N m^{-1}

Solution: (a) 3SF (b) 3 SF (c) 5 SF (d) 1 SF (e) 4 SF

Example 28: One cm on the main scale of vernier callipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 8 small divisions of the main scale. What will be the least count of calliper?

Solution: 20 div. of vernier scale = 8 div. of main scale $\Rightarrow$ 1 V. S .D. = $\left(\frac{8}{20}\right)$ M. S. D. = $\left(\frac{2}{5}\right)$M. S. D.

Least count = 1 M. S. D. – 1 V. S . D. = 1 M. S. D. – $\left(\frac{2}{5}\right)$ M. S. D. = $\left(1-\frac{2}{5}\right)$M. S. D. = $\frac{3}{5}$

M.S.D. = $\frac{3}{5} \times 0.1$ cm = 0.06 cm

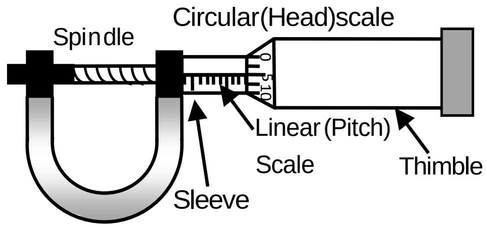
($\because$ 1 M. S. D. = $\frac{1}{10}$ cm = 0.1 cm)

Note: for objective questions

$$L.\ C. = M - V\left(\frac{b-a}{b}\right) = M = \left(\frac{20-8}{20}\right)\left(\frac{1}{10}\right)cm = \frac{3}{50}\ cm = 0.06\ cm$$

Least Count of screw gauge:

$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total no. of division on the circular scale}}$$



where pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation. i.e.

$$\text{Pitch} = \frac{\text{Distance move by the screw on the linear scale}}{\text{No. of full rotations given}}$$

Note: With the decrease in the least count of the measuring instrument, the accuracy of the measurement increases and the error in the measurement decreases.

Example 29: The n^{th} division of main scale coincides with $(n + 1)^{th}$ division of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

(1) $\frac{a}{n+1}$ (2) $\frac{n}{n+1}$ (3) $\frac{1}{n+1}$ (4) Zero

Solution: (n + 1) divisions of vernier scale = n divisions of main scale

$\therefore$ 1 vernier division = $\frac{n}{n+1}$ main scale division

Least count = 1 M. S. D. – 1V. S. D. = $\left(1-\frac{n}{n+1}\right)$M.S.D. = $\left(\frac{n}{n+1}\right)$ M. S. D. = $\frac{a}{n+1}$

Example 30: The least count of a stop watch is $\frac{1}{5}$ second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. How much will be the percentage error in the measurement of time ?

(1) 0.4 % (2) 0.6 % (3) 0.8 % (4) 12 %

Solution: Error in measuring 25 sec.= $\frac{1}{5}$ sec. = 0.2 sec.

$\therefore$ percentage error = $\frac{0.2}{25}$ = 0.8%

Note: The final absolute error in this type of questions is taken to be equal to the least count of the measuring instrument.