Sequence and series DPPS-2.

(1) $25, \frac{91}{4}, \frac{41}{2}, \frac{73}{4} \ldots$

Let $T_n < 0$ $\quad a=25 \quad d=\frac{91}{4}-25=\frac{-9}{4}$

$25+(n-1)\frac{-9}{4}<0$

$25<\frac{9}{4}(n-1)$

$\frac{100}{9}<n-1$

$n-1>\frac{100}{9}$

$n>12\ldots$

$T_{13}=-ve \qquad \underline{T_{12}=+}$

$T_{13}=25+12\times\frac{-9}{4}=-2$

$T_{12}=a+11d$

$=25+11\times\frac{-9}{4}=\frac{1}{4}$ $\underline{\text{smallest}}$

(2) $T_1, T_2, T_3, T_4 \ldots \ldots T_{n-3}, T_{n-2}, T_{n-1}, T_n$

$\boxed{a=11}$

$T_1+T_2+T_3+T_4=56$

$a+a+d+a+2d+a+3d=56$

$4a+6d=56$

$2a+3d=28$ —— (1)

$2\times11+3\times d=28$

$3d=28-22=6$

$\boxed{d=2}$

$T_n+T_{n-1}+T_{n-2}+T_{n-3}=112$

$a+(n-1)d+a+(n-2)d+a+(n-3)d+a+(n-4)d=112$

$4a+d(n-1+n-2+n-3+n-4)=112$

$4\times11+d(4n-10)=112$

$44+2(4n-10)=112$

$2(4n-10)=68$

$4n-10=34$

$4n=44$

$\boxed{n=11}$

(3) $2,5,8,11 \ldots \quad T_{60}$

$T_{60}=a+59d$

$=2+59\times3$

$T_{60}=179$.

$3,5,7,9, \ldots \quad T_{50}$

$T_{50}=a+49d$

$=3+49\times2=101$

$2,5,8 \ldots 179 \rightarrow AP(1^{st}) \rightarrow d_1=3$

$3,5,7,9 \ldots 101 \rightarrow AP(2^{nd}AP)\ d_2=2$

Common terms series are also in AP

$D=\text{lcm}(d_1,d_2)=\text{lcm}(2,3)=6$

$\downarrow$

common diff of common term series

$5, 11, 17, \ldots \ldots 101$.

$\therefore$ $101=5+(n-1)6$

$\boxed{n=17}$

(4) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c} \rightarrow AP$

$\frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2$

$\frac{b+c+a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$.

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow AP$

$\Rightarrow a, b, c \rightarrow HP$

(5) Put $a=1, b=2, c=3$

$a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$

$\rightarrow AP$.

(6) $T_{m+n}+T_{m-n}$

$=a+(m+n-1)d+a+(m-n-1)d$

$=2a+d(m+\cancel{n}-1+m-\cancel{n}-1)$

$=2(a+(m-1)d)$

$=2T_m$

(7) $a-d, a, a+d$

$a-d+a+a+d=15$

$\boxed{a=5}$

$100(a-d)+10a+a+d=100(a+d)+10a+a-d+594$

$100[\cancel{a}-d-\cancel{a}-d]+2d=594$

$-200d+2d=594$

$-198d=594$

$\boxed{d=-3}$

(8) $S_1=T_1+T_2+T_3+\dots+T_n \; (n=\text{odd})$

Hence no of odd terms $=\frac{n-1}{2}+1$

no of even terms $=\frac{n-1}{2}$

$S_1=\frac{n}{2}[T_1+T_n]$

$S_2=\frac{n-1}{2(2)}[T_2+T_{n-1}]$

$\frac{S_1}{S_2}=\frac{n}{2}\times\frac{4}{n-1}=\frac{2n}{n-1}$

$\because T_1+T_n=T_2+T_{n-1}$

(9) $(1+x)(1+x^6)(1+x^{11})\dots(1+x^{101})$

degree $1+6+11+\dots+101$

$101=1+(n-1)5$

$\boxed{n=21}$

$S_{21}=\frac{21}{2}[1+101]=21\times51=1071$

(10) $\cancel{\frac{3n}{2}[2a+(3n-1)d]}$

$T_1, T_2, T_3 \dots T_{3n}, T_{3n+1}, T_{3n+2}+\dots$

According to Questions

$\underbrace{T_1+T_2+T_3+\dots T_{3n}}_{3n\text{ terms}}=\underbrace{T_{3n+1}+T_{3n+2}+\dots}_{n\text{ terms}}$

$\frac{3n}{2}[2a+(3n-1)d]=\frac{n}{2}[2\times(a+3nd)+(n-1)d]$

$6a+3d(3n-1)=2a+6nd+(n-1)d$

$4a=d[6n+n-1-9n+3]$

$4a=d[-2n+2]$

$\boxed{2a=d[1-n]}$

$\frac{T_1+T_2+\dots T_{2n}}{T_{2n+1}+T_{2n+2}\dots n\text{ terms}}=$

$=\frac{\frac{2n}{2}[2a+(2n-1)d]}{\frac{2n}{2}[2\times[a+2nd]+(2n-1)d]}$

$=\frac{2a-d+2nd}{2a+4nd+(2n-1)d}$

$=\frac{d(1-n)-d+2nd}{(1-n)d+4nd+(2n-1)d}$

$=\frac{\cancel{d}[\cancel{1}-n-\cancel{1}+2n]}{\cancel{d}[\cancel{1}-n+4n+2n-\cancel{1}]}$

$=\frac{n}{5n}=\frac{1}{5}$

(11) $\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}$

$\cancel{ab}-acx+b^2x-\cancel{bcx^2}=\cancel{ab}+acx-b^2x-\cancel{bcx^2}$

$\cancel{2}b^2\cancel{x}=\cancel{2}ac\cancel{x}$

$\boxed{b^2=ac}$

$a, b, c \to GP$ — (1)

$\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}$

$bc+bdx+c^2x-\cancel{cdx^2}$

$=bc+bdx-c^2x-\cancel{cdx^2}$

$\cancel{2}c^2\cancel{x}=\cancel{2}bd\cancel{x}$

$c^2=bd$

$b, c, d \to GP$ — (2)

From (1) and (2) $a, b, c, d \to GP$

(12) $a, b, c, d \to GP$

$a = a \quad b = ar, \ c = ar^2, \ d = ar^3$

$a^n + b^n = a^n + a^n r^n = a^n(1 + r^n)$

$b^n + c^n = (ar)^n + (ar^2)^n = a^n r^n (1 + r^n)$

$c^n + d^n = (ar^2)^n + (ar^3)^n = a^n r^{2n} + a^n r^{3n}$

$= a^n r^{2n}(1 + r^n)$

$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n} = \underline{\underline{r^n}}$

Hence $\underline{GP}$

(13) Let $a, ar, ar^2 \to$ 3 Nos.

$a + ar + ar^2 = 56$ (Given) — (1)

$a-1, \ ar-7, \ ar^2-21 \to AP$

$2ar - 14 = a - 1 + ar^2 - 21$

$2ar = a + ar^2 - 8$ (From (1))

$2ar = 56 - ar - 8$

$3ar = 48$

$\boxed{ar = 16}$

From (1) $a + ar + ar^2 = 56$

$a + 16 + r \cdot ar = 56$

$a + 16 + r \cdot 16 = 56$

$a + 16r = 40$

$\frac{16}{r} + 16r = 40$

$r + \frac{1}{r} = \frac{40}{16} = \frac{5}{2}$

$\Rightarrow r = 2$ or $1/2$

If $r = 2$ $\quad ar = 16$

$a \times 2 = 16$

$\boxed{a = 8}$

$\underline{8, 16, 32}$ Nos

(14) $a, b, c \to AP$

$2b = a + c$ — (1)

$b, c, d \to GP$

$\boxed{c^2 = bd}$ — (2)

$\frac{1}{c}, \frac{1}{d}, \frac{1}{e} \to AP$

$\frac{2}{d} = \frac{1}{c} + \frac{1}{e}$

$\frac{2}{d} = \frac{c+e}{ce}$

$\frac{2b}{c^2} = \frac{c+e}{ce}$ (from (2))

$2be = c^2 + ce$

$\Rightarrow (a+c)e = c^2 + ce$ (From 1)

$ae + \cancel{ce} = c^2 + \cancel{ce}$

$c^2 = ae$

$\underline{a, c, e \to GP}$

(15) $11 + 103 + 1005 + \ldots$

$(10+1) + (100+3) + (1000+5) + \ldots$

$(10 + 10^2 + 10^3 + \ldots$ n terms$) + (1 + 3 + 5 + \ldots$ n terms$)$

$\frac{10(10^n - 1)}{10 - 1} + n^2$

$\frac{10(10^n - 1)}{9} + n^2$

(16) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} + \frac{1}{3^6} + \ldots \infty$

$\left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \ldots \infty\right) + \left(\frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \ldots \infty\right)$

$\frac{1/2}{1 - 1/2^2} + \frac{1/3^2}{1 - 1/3^2}$

$\frac{1}{2} \times \frac{4}{3} + \frac{1}{9} \times \frac{9}{8}$

$\frac{2}{3} + \frac{1}{8} = \frac{16+3}{24} = \frac{19}{24}$

(17) $T_8 = 2, \ T_{14} = 3 \to$ (for AP)

$a + 7d = 2$

$a + 13d = 3$

$-6d = -1$

$\boxed{d = 1/6}$

$a + \frac{7}{6} = 2$

$a = 2 - \frac{7}{6} = \frac{5}{6}$

$T_{20} = a + 19d \to AP$

$= \frac{5}{6} + \frac{19}{6} = 4$

Hence HP का $T_{20} = \frac{1}{4}$

(18) $a, b, c \to GP \Rightarrow b^2 = ac$

$a+x,\ \cancel{b+x},\ c+x \to HP$

$$(b+x) = \frac{2\times(a+x)(c+x)}{a+c+2x}$$

$$ab + ax + cb + cx + 2bx + 2x^2 = 2(ac + x^2 + ax + cx)$$

$$ab + ax + bc + cx + 2bx + \cancel{2x^2} = 2ac + \cancel{2x^2} + 2ax + 2cx$$

$$= ax + cx - ab - bc - 2bx + 2ac = 0$$

$$= ax + cx - ab - bc - 2bx + 2b^2 = 0 \quad (\because b^2 = ac)$$

$$= ax - ab + cx - cb + 2b^2 - 2bx = 0$$

$$= a(x-b) + c(x-b) - 2b(x-b) = 0$$

$$(x-b)(a+c-2b) = 0$$

$$\boxed{x = b}$$

(19) $$\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz} = K$$

$$\frac{a-x}{Kx} = p \quad \frac{a-y}{Ky} = q \quad \frac{a-z}{Kz} = r$$

$p, q, r \to AP \Rightarrow \frac{a-x}{Kx}, \frac{a-y}{Ky}, \frac{a-z}{Kz} \to AP$

$$\frac{1}{K}\left(\frac{a}{x}-1\right), \frac{1}{K}\left(\frac{a}{y}-1\right), \frac{1}{K}\left(\frac{a}{z}-1\right) \to AP$$

$$\frac{a}{x}-1,\ \frac{a}{y}-1,\ \frac{a}{z}-1 \to AP$$

$$\frac{a}{x}, \frac{a}{y}, \frac{a}{z} \to AP$$

$$\frac{1}{x}, \frac{1}{y}, \frac{1}{z} \to AP \Rightarrow \boxed{x, y, z \to HP}$$

(20) $$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \cdots + \frac{1}{n(n+1)}$$

$$\frac{2-1}{1\times2} + \frac{3-2}{2\times3} + \frac{4-3}{3\times4} + \cdots + \frac{n+1-n}{n(n+1)}$$

$$1 - \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}} + \cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}} + \cdots + \cancel{\tfrac{1}{n}} - \frac{1}{n+1}$$

$$1 - \frac{1}{n+1} = \frac{n}{n+1}$$