logarithmic DPPS 2 solution

① $\frac{\log_{10} a}{2} = \frac{\log_{10} b}{3} = \frac{\log_{10} c}{5} = K$

$\log_{10} a = 2K \quad \log_{10} b = 3K \quad \log_{10} c = 5K$

$a = 10^{2K} \quad b = 10^{3K} \quad c = 10^{5K}$

$bc = 10^{3K} \cdot 10^{5K} = 10^{8K} = (10^{2K})^4 = a^4$

$\boxed{bc = a^4}$

② $\log_4 5 = x \quad \log_5 6 = y$

$xy = \log_4 5 \times \log_5 6$

$xy = \frac{\cancel{\log 5}}{\log 4} \times \frac{\log 6}{\cancel{\log 5}}$

$xy = \frac{\log 6}{\log 2^2} = \frac{\log 6}{2\log 2}$

$2xy = \cancel{2} \times \frac{\log(2\times 3)}{\cancel{2}\log 2}$

$2xy = \frac{\log 2 + \log 3}{\log 2}$

$2xy = \frac{\log 2}{\log 2} + \frac{\log 3}{\log 2}$

$2xy = 1 + \log_2 3$

$\boxed{2xy - 1 = \log_2 3}$

③ $\log_3 y = x \Rightarrow y = 3^x$

$\log_2 z = x \quad z = 2^x$

$(72)^x = (2^3 \times 3^2)^x$

$= (2^3)^x \times (3^2)^x$

$= (2^x)^3 \times (3^x)^2$

$(72)^x = z^3 y^2 = y^2 z^3$

④ $x = \log_{0.1} 0.001$

$x = \log_{10^{-1}} 10^{-3}$

$x = \frac{\cancel{+3}\,-3}{-1} (\log_{10} 10)$

$x = 3 \times 1 = 3$

$y = \log_9 81 = \log_9 9^2$

$y = 2(\log_9 9) = 2 \times 1 = 2$

$x = 3, y = 2$

$\sqrt{x - 2\sqrt{y}} = \sqrt{3 - 2\sqrt{2}}$

$= \sqrt{(\sqrt{2}-1)^2}$

$= |\sqrt{2}-1| = \sqrt{2}-1$

⑤ $|\log_b a + \log_a b|$

$= \left|\log_b a + \frac{1}{\log_b a}\right| \geq 2\sqrt{\cancel{\log_b a} \times \frac{1}{\cancel{\log_b a}}}$

≥ 2

Min value 2

(we know that $a>0, b>0$ then $a+b \geq 2\sqrt{ab}$

⑥ $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ac} abc}$

$\frac{1}{\frac{\log abc}{\log ab}} + \frac{1}{\frac{\log abc}{\log bc}} + \frac{1}{\frac{\log abc}{\log ac}}$

$\frac{\log ab}{\log abc} + \frac{\log bc}{\log abc} + \frac{\log ac}{\log abc}$

$= \frac{\log a^2b^2c^2}{\log abc} = \frac{2\log(abc)}{\log(abc)} = 2$

(7) $\log_3 a \times \log_a x = 4$

$\frac{\log a}{\log 3} \times \frac{\log x}{\log a} = 4$

$\frac{\log x}{\log 3} = 4$

$\log_3 x = 4$

$x = 3^4 = 81$

(8) $x = 27 \quad y = \log_3 4$

$x^y = (27)^{\log_3 4}$

$= (3^3)^{\log_3 4}$

$= 3^{3\log_3 4}$

$= 3^{\log_3 4^3}$

$= 4^3 \quad (\because a^{\log_a f(x)} = f(x))$

$= 64$

(9) $\log_3 x + \log_9 x^2 + \log_{27} x^3 = 9$

$\log_3 x + \log_{3^2} x^2 + \log_{3^3} x^3 = 9$

$\log_3 x + \frac{\cancel{2}}{\cancel{2}} \log_3 x + \frac{\cancel{3}}{\cancel{3}} \log_3 x = 9$

$3 \log_3 x = 9$

$\log_3 x = 3$

$x = 3^3 = 27$

(10) $2^{\log_3 5} - 5^{\log_3 2}$

$2^{\log_3 5} = 5^{\log_3 2}$ (we know that)

$2^{\log_3 5} - 5^{\log_3 2} = 0$

(11) $x = a^k$

$\boxed{\log_a x = \log_a a^k = k}$ --?

$y = a^{\frac{1}{1-\log_a x}}$

$\log_a y = \log_a a^{\left(\frac{1}{1-\log_a x}\right)}$

$\log_a y = \frac{1}{1-\log_a x}$

$z = a^{\frac{1}{1-\log_a y}}$

$\log_a z = \log_a a^{\frac{1}{1-\log_a y}}$

$\log_a z = \frac{1}{1-\log_a y}$

$\log_a z = \frac{1}{1-\left[\frac{1}{1-\log_a x}\right]}$

$\log_a z = \frac{1-\log_a x}{\cancel{1}-\log_a x-\cancel{1}}$

$\log_a z = -\frac{1}{\log_a x} + 1$

$\frac{1}{\log_a x} = 1 - \log_a z$

$\boxed{k = \log_a x = \frac{1}{1-\log_a z}}$

(12) $\frac{\log_a x}{\log_{ab} x} = 4 + k + \log_a b$

$\frac{\frac{\log x}{\log a}}{\frac{\log x}{\log ab}} = 4 + k + \log_a b$

$\frac{\log ab}{\log a} = 4 + k + \log_a b$

$\frac{\log a + \log b}{\log a} = 4 + k + \log_a b$

$\frac{\log a}{\log a} + \frac{\log b}{\log a} = 4 + k + \log_a b$

$1 + \cancel{\log_a b} = 4 + k + \cancel{\log_a b}$

$\boxed{k = 1 - 4 = -3}$

(13) $\log_2|4-5x|>2$

$|4-5x|>2^2$

$|4-5x|>4$

$4-5x>4$ or $4-5x<-4$

$\boxed{x<0}$ or $8<5x$

$\boxed{x>\frac{8}{5}}$

$x\in(-\infty\ 0)\cup(\frac{8}{5},\infty)$

(14) $\log_{0.3}(x-1)<\log_{0.09}(x-1)$

(For log valid $x-1>0 \Rightarrow \boxed{x>1}$ — (1)

$\log_{0.3}(x-1)<\log_{(0.3)^2}(x-1)^1$

$\log_{0.3}(x-1)<\frac{1}{2}\log_{0.3}(x-1)$

$2\log_{0.3}(x-1)<\log_{0.3}(x-1)$

$\log_{0.3}(x-1)^2<\log_{0.3}(x-1)$

$\because 0<0.3<1$

$(x-1)^2>(x-1)$

$(x-1)^2-(x-1)>0$

$(x-1)(x-1-1)>0$

$(x-1)(x-2)>0$

$\because x-1>0$ Hence $x-2>0$

$\boxed{x>2}$ — (2)

(1) $\cap$ (2) $\Rightarrow x>2$

$x\in(2,\infty)$

(15)

$\frac{1}{\log_2 a}+\frac{1}{\log_4 a}+\frac{1}{\log_8 a}+\cdots+\frac{1}{\log_{2^n} a}$

$\frac{1}{\frac{\log a}{\log 2}}+\frac{1}{\frac{\log a}{\log 4}}+\frac{1}{\frac{\log a}{\log 8}}+\cdots\frac{1}{\frac{\log a}{\log 2^n}}$

$\frac{\log 2}{\log a}+\frac{\log 4}{\log a}+\frac{\log 8}{\log a}+\cdots\frac{\log 2^n}{\log a}$

$\frac{\log 2}{\log a}+\frac{2\log 2}{\log a}+\frac{\log 2^3}{\log a}+\cdots\frac{n\log 2}{\log a}$

$\frac{\log 2}{\log a}[1+2+3+\cdots+n]$

$\log_a 2\ \frac{n(n+1)}{2}=\frac{n(n+1)}{\lambda}$

$\lambda=\frac{2}{\log_a 2}=2\log_2 a$

$\boxed{\lambda=\log_2 a^2}$

(16) $x^{\log_5 x}>5$

$\log_5 x^{\log_5 x}>\log_5 5$

$(\log_5 x)(\log_5 x)>1$

$(\log_5 x)^2-1>0$

$(\log_5 x+1)(\log_5 x-1)>0$

+ -1 − 1 +

$\log_5 x<-1$ or $\log_5 x>1$

$0<x<5^{-1}$ or $x>5$

$0<x<\frac{1}{5}$ or $x>5$

$x\in(0\ \frac{1}{5})\cup(5,\infty)$

(17) $\log_{0.2}\left(\frac{x+2}{x}\right)\leq 1$

$\frac{x+2}{x}\geq(0.2)^1$ ($\because 0<0.2<1$)

$\frac{x+2}{x}\geq\frac{2}{10}\ \frac{1}{5}$

$\frac{5(x+2)}{x}\geq 1$

$\frac{5x+10}{x}-1\geq 0$

$\frac{5x+10-x}{x}\geq 0$

$\frac{(x+5/2)}{x}\geq 0$

+ $-5/2$ − 0 +

$x\in(-\infty\ -\frac{5}{2}]\cup(0,\infty)$

(18) $\frac{1}{\log_a bc+1}+\frac{1}{\log_b ac+1}+\frac{1}{\log_c ab+1}$

$\frac{1}{\frac{\log bc}{\log a}+1}+\frac{1}{\frac{\log ac}{\log b}+1}+\frac{1}{\frac{\log ab}{\log c}+1}$

$\frac{\log a}{\log bc+\log a}+\frac{\log b}{\log ac+\log b}+\frac{\log c}{\log ab+\log c}$

$=\frac{\log a}{\log abc}+\frac{\log b}{\log abc}+\frac{\log c}{\log abc}$

$=\frac{1}{\log abc}[\log a+\log b+\log c]$

$=\frac{\log abc}{\log abc}=1$

(19) $\frac{\log_e x}{2a+3b-5c}=\frac{\log_e y}{2b+3c-5a}=\frac{\log z}{2c+3a-5b}=K$

$\log_e x=K(2a+3b-5c)$

$\boxed{x=e^{K(2a+3b-5c)}}$

$\log_e y=K(2b+3c-5a)$

$\boxed{y=e^{K(2b+3c-5a)}}$

$\log_e z=K(2c+3a-5b)$

$\boxed{z=e^{K(2c+3a-5b)}}$

$xyz=e^{K(2a+3b-5c)}.e^{K(2b+3c-5a)}.e^{K(2c+3a-5b)}$

$xyz=e^{K[2a+3b-5c+2b+3c-5a+2c+3a-5b]}$

$=e^0$

$\boxed{xyz=1}$

(20) $\frac{\log_2 a}{2}=\frac{\log_3 b}{3}=\frac{\log_4 c}{4}=K$

$\frac{\log_2 a}{2}=K$

$\log_2 a=2K$

$a=2^{2K}$

$a^{1/2}=(2^{2K})^{1/2}$

$\boxed{a^{1/2}=2^K}$

$\log_3 b=3K$

$b=3^{3K}$

$(b^{1/3})=(3^{3K})^{1/3}$

$\boxed{b^{1/3}=3^K}$

$\log_4 c=4K$

$c=4^{4K}$

$c^{1/4}=(4^{4K})^{1/4}$

$\boxed{c^{1/4}=4^K}$

$a^{1/2}b^{1/3}c^{1/4}=2^K.3^K.4^K=24$

$2^K.3^K.4^K=2\times3\times4$

$\boxed{K=1}$

$a^{1/2}=2$

$\boxed{a=4}$

$b^{1/3}=3^1$

$\boxed{b=27}$

$c^{1/4}=4$

$\boxed{c=256}$ $\boxed{c=256}$

(21) $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+ \cdots +\frac{1}{\log_{43} n}$

$\frac{1}{\frac{\log n}{\log 2}}+\frac{1}{\frac{\log n}{\log 3}}+\frac{1}{\frac{\log n}{\log 4}}+\cdots+\frac{1}{\frac{\log n}{\log 43}}$

$\frac{\log 2}{\log n}+\frac{\log 3}{\log n}+\frac{\log 4}{\log n}+\cdots+\frac{\log 43}{\log n}$

$\frac{1}{\log n}[\log 2+\log 3+\log 4+ \cdots +\log 43]$

$\frac{1}{\log n}[\log 2\times 3\times 4\times \cdots 43]$

$\frac{1}{\log n}[\log 1\times 2\times 3\times 4 \cdots \times 43]$

$\frac{\log 43!}{\log n} = \log_n 43!$

(22) $\log_4 5 = x \quad \log_5 6 = y$

$xy = \log_4 5 \times \log_5 6$

$xy = \frac{\cancel{\log 5}}{\log 4}\times\frac{\log 6}{\cancel{\log 5}}$

$xy = \frac{\log 2\times 3}{\log 2^2} = \frac{\log 2+\log 3}{2\log 2}$

$2xy = \frac{\log 2}{\log 2}+\frac{\log 3}{\log 2}$

$2xy = 1 + \log_2 3$

$2xy - 1 = \log_2 3$

$\frac{1}{2xy-1} = \log_3 2 \quad [\because \log_2 3\times\log_3 2 = 1]$

(23) $e^{\log_e x+\log_{e^{1/2}} x^1+\log_{e^{1/3}} x^1+ \cdots +\log_{e^{1/10}} x}$

$e^{\log_e x+\frac{1}{1/2}\log_e x+\frac{1}{1/3}\log_e x+ \cdots +\frac{1}{1/10}\log_e x} \quad [\because \log_{a^p} b^q = \frac{q}{p}\log_a b]$

$e^{\log_e x+2\log_e x+3\log_e x+ \cdots +10\log_e x}$

$e^{\log_e x(1+2+3 \cdots 10)} \quad (\because 1+2+3+ \cdots +n = \frac{n(n+1)}{2})$

$e^{\log_e x\left(\frac{10\times 11}{2}\right)}$

$e^{(\log_e x)55} = e^{55\log_e x} = e^{\log_e x^{55}} = x^{55}$

(24) log same as 14 → try it.

(25) $3\log_3|x| = \log_3 x^2$ $(\underline{x \neq 0})$

$3\log_3|\pm x| = \log_3 x^2$

$\log_3|\pm x|^3 = \log_3|x|^2$

$|\pm x|^3 = |x|^2$

$|x|^3 - |x|^2 = 0$

$(|x|)^2(|x|-1) = 0$

$\boxed{x \neq 0}$ $|x| = \pm 1$

$\boxed{x = \pm 1}$

↓

<u>no of soln</u> 2