

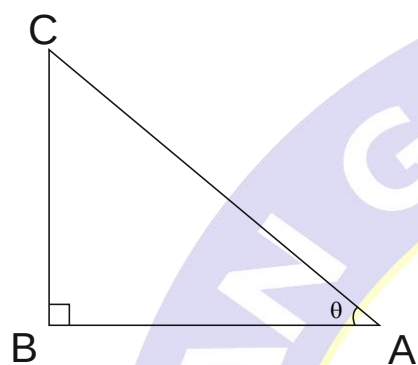
KHAN GLOBAL STUDIES

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Defence (Mathematics)

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Trigonometry formula



$$\sin \theta = \frac{BC}{AC}, \cos \theta = \frac{AB}{AC},$$

$$\tan \theta = \frac{BC}{AB}, \cot \theta = \frac{AB}{BC}$$

$$\sec \theta = \frac{AC}{AB}, \operatorname{cosec} \theta = \frac{AC}{BC}$$

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot A$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\operatorname{cosec} A$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\sec^2 \theta - \tan^2 \theta = 1 \Rightarrow \sec^2 \theta = 1 + \tan^2 \theta$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \Rightarrow \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

$$\sec x = \frac{1}{\cos x}, x \neq (2n+1)\frac{\pi}{2},$$

Where n is any integer

$$\tan x = \frac{\sin x}{\cos x}, x \neq (2n+1)\frac{\pi}{2},$$

where n is any integer

$$\cot x = \frac{\cos x}{\sin x}, x \neq n\pi,$$

Where n is any integer

Note :

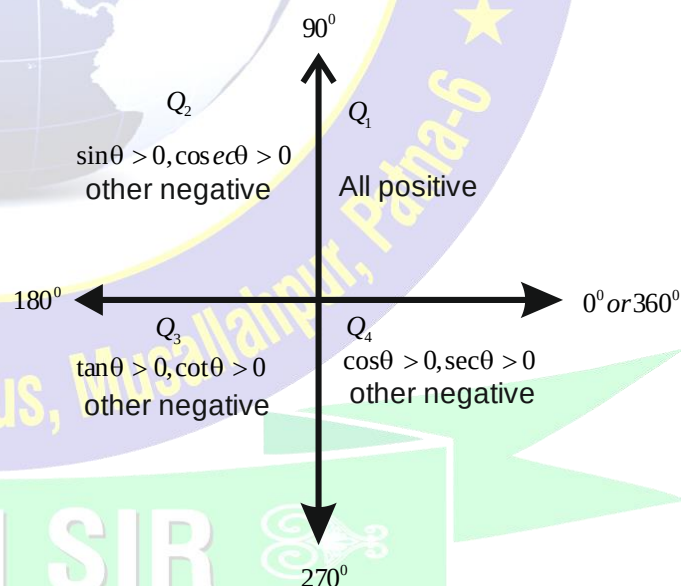
1. If $\theta \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$, then

$$\sec^2 \theta - \tan^2 \theta = 1 \Rightarrow$$

$$(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

$$\Rightarrow \sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

2. If $\theta \neq n\pi, n \in \mathbb{Z}$ then $\operatorname{cosec} \theta - \cot \theta = \frac{1}{\operatorname{cosec} \theta + \cot \theta}$



i. $\sin n\pi = \tan n\pi = \cos(2n+1)\frac{\pi}{2} = 0, \forall n \in \mathbb{Z}$

ii. $\sin(2n+1)\frac{\pi}{2} = (-1)^n$ and $\cos n\pi = (-1)^n, \forall n \in \mathbb{Z}$

➤. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

➤. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

➤. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\Rightarrow \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\Rightarrow \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\Rightarrow \cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\Rightarrow \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A + B) \cdot \sin(A - B)$$

$$\Rightarrow \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A + B) \cdot \cos(A - B)$$

$$\Rightarrow \tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

$$\Rightarrow \sin 2A = 2 \sin A \cos A; \sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\Rightarrow \cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A;$$

$$2\cos^2 \frac{\theta}{2} = 1 + \cos \theta; 2\sin^2 \frac{\theta}{2} = 1 - \cos \theta$$

$$\Rightarrow \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}; \tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$\Rightarrow \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}; \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\Rightarrow \sin 3A = 3 \sin A - 4\sin^3 A$$

$$\Rightarrow \cos 3A = 4\cos^3 A - 3 \cos A$$

$$\Rightarrow \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$\Rightarrow \sin A \cdot \sin(60^\circ - A) \sin(60^\circ + A) = \frac{1}{4} \sin 3A$$

$$\Rightarrow \cos A \cdot \cos(60^\circ - A) \cos(60^\circ + A) = \frac{1}{4} \cos 3A$$

$$\Rightarrow \tan A \cdot \tan(60^\circ - A) \tan(60^\circ + A) = \tan 3A$$

$$\Rightarrow \cos \theta \cdot \cos 2\theta \cdot \cos 2^2 \theta \cdot \cos 2^3 \theta \dots \cos 2^{n-1} \theta = \frac{\sin 2^n \theta}{2^n \sin \theta}$$

$$\Rightarrow 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$\Rightarrow 2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$\Rightarrow 2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$\Rightarrow 2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\Rightarrow \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\Rightarrow \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\Rightarrow \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\Rightarrow \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\Rightarrow \sin n\pi = 0; \quad \cos n\pi = (-1)^n; \quad \tan n\pi = 0 \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow \sin 15^\circ \text{ or } \sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ \text{ or } \cos \frac{5\pi}{12};$$

$$\cos 15^\circ \text{ or } \cos \frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ \text{ or } \sin \frac{5\pi}{12};$$

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3} = \cot 75^\circ;$$

$$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3} = \cot 15^\circ;$$

$$\Rightarrow \sin \frac{\pi}{10} \text{ or } \sin 18^\circ = \frac{\sqrt{5}-1}{4} \&$$

$$\cos 36^\circ \text{ or } \cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$$

$$\Rightarrow \sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots + \sin(\alpha + n - 1\beta)$$

$$= \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \sin \left(\alpha + \frac{n-1}{2} \beta \right)$$

$$\Rightarrow \cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots + \cos(\alpha + n - 1\beta)$$

$$= \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \cos \left(\alpha + \frac{n-1}{2} \beta \right)$$

$$\Rightarrow \sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

$$\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\Rightarrow \text{If } a \cos \theta + b \sin \theta = c \text{ and } a \sin \theta - b \cos \theta = K \text{ then } a^2 + b^2 = c^2 + K^2$$

$$\Rightarrow \text{If } a \sec \theta + b \tan \theta = c \text{ and } a \tan \theta + b \sec \theta = k \text{ then } a^2 - b^2 = c^2 - k^2$$

$$\Rightarrow \text{If } a \operatorname{cosec} \theta + b \cot \theta = c \text{ and } a \cot \theta + b \operatorname{cosec} \theta = k \text{ then } a^2 - b^2 = c^2 - k^2$$

$$\Rightarrow \text{If } 8 \cos \theta + 6 \sin \theta = 5 \text{ then } 8 \sin \theta - 6 \cos \theta =$$

$$\Rightarrow \text{If } a > 0, b > 0 \text{ and } f(x) > 0 \text{ then}$$

$$a f(x) + \frac{b}{f(x)} \geq 2\sqrt{ab}$$