KGS IAS

Maths Optional

By Dhrub Singh Sir

prob: Solve the equations:

$$\lambda x + 2y - 2z = 1$$
$$4x + 2\lambda y - z = 2$$
$$6x + 6y + \lambda z = 3.$$

Also discuss the case when $\lambda = 2$.

$\lambda^3 + 11\lambda - 30 \neq 0$

$(\lambda - 2)(\lambda^2 + 2\lambda + 15)$
$\neq 0$

$\lambda^3 + 11\lambda - 30 = 0$
only when
$\lambda = 2$

$$\rightarrow \quad \Delta = \begin{vmatrix} \lambda & 2 & -2 \\ 4 & 2\lambda & -1 \\ 6 & 6 & \lambda \end{vmatrix} = 2\lambda^3 + 22\lambda - 60$$

$\Delta \neq 0$ i.e. if $2\lambda^3 + 22\lambda - 60 \neq 0$

i.e. $2(\lambda^3 + 11\lambda - 30) \neq 0$

$$\sim\left[\begin{array}{ccc|c} 2 & 2 & -2 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 \to R_3 - \frac{8}{3} R_2$

$r[A:B] = r(A) = 2$

$<$ No. of var.

Infinitely many sol^n.

writing eq^n. in reduced form with the help of echelon matrix.

For $\lambda = 2$

$$[A:B] = \left[\begin{array}{ccc|c} 2 & 2 & -2 & 1 \\ 4 & 4 & -1 & 2 \\ 6 & 6 & 2 & 3 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 2 & 2 & -2 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$2x + 2y - 2z = 1$$

$$3z = 0 \Rightarrow z = 0$$

$$2x + 2y = 1$$

$$y = k \in \mathbb{R} \checkmark$$

$$2x + 2k = 1$$

$$2x = 1 - 2k$$

$$x = \frac{1}{2}(1 - 2k) \checkmark$$

$AX = 0$ $\quad A_{m\times n}$

$\downarrow$

$r(A) = r$

$\rightarrow n - r$ solⁿ. $\quad$ L.I

Basis $= \{X_1, X_2 \dots X_{n-r}\}$

$\mathrm{Dim} = n - r$

$L\{X_1, X_2, \dots X_{n-r}\}$

$\downarrow$

Solⁿ. space.

$AM = B$ — (i)

$AN = 0$ — (ii)

M particular sol^n. $AX = B$

N General sol^n — $AX = 0$.

What is the general sol^n of $AX = B$.

$\longrightarrow$ $M+N$ — General sol^n. of $AX=B$

As. $A(M+N) = AM + AN$

$= B + 0 = B$

$\therefore$ $M+N$ is a gen. sol^n. of $AX = B$.

Only one soln.
i.e., trivial soln

All unknowns = 0.

Case (II)

$r < n$ (No. of unknowns)

$n - r$ variables arbitrary value.

Working rule for finding the soln. of $AX = 0$

Step I Reduce A to echelon form by E-row operation.

Step II $r(A) = r$

Case (I) $r = n$ (No. of unknowns)

$AX=0$

$$A=\begin{bmatrix}1 & 3 & -2\\ 2 & -1 & 4\\ 1 & -11 & 14\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 3 & -2\\ 0 & -7 & 8\\ 0 & -14 & 16\end{bmatrix} \quad \begin{array}{l} R_2\to R_2-2R_1\\ R_3\to R_3-R_1\end{array}$$

$$\sim\begin{bmatrix}1 & 3 & -2\\ 0 & -7 & 8\\ 0 & 0 & 0\end{bmatrix} \quad R_3\to R_3-2R_2$$

Remaining r variables can be expressed in terms of these $(n-r)$ variables.

prob: Solve the system of equations

$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0$$

Suppose $z = K \in \mathbb{R}$

$-7y + 8K = 0$

$7y = 8K$

$y = \frac{8}{7}K$

$x + 3 \times \frac{8}{7}K - 2K = 0$

$x + \frac{24}{7}K - 2K = 0$

$x = 2K - \frac{24}{7}K = \frac{-10K}{7}$

$\therefore r(A) = 2 <$ No. of unknowns (three)

$3 - 2 = 1$ —— soln. L.I

Writing equations in reduced form with the help of echelon matrix.

$x + 3y - 2z = 0$

$-7y + 8z = 0$

$\text{Sol}^n \text{ space} = \{ K \cdot X_1 : K \in \mathbb{R} \}$

$\text{Bases} = \{ X_1 \}$.

$\text{Dimension} = 1$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{10K}{7} \\ \frac{8}{7}K \\ K \end{bmatrix} = K \begin{bmatrix} -\frac{10}{7} \\ \frac{8}{7} \\ 1 \end{bmatrix}$$

$= K \cdot X_1$,

where $X_1 = \begin{bmatrix} -10/7 \\ 8/7 \\ 1 \end{bmatrix}$

$X_1 \neq 0$ and $\therefore$ L.I.

$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in \mathbb{R}^2$

$S = \{x_1, x_2\}$

$L(S) = \{\alpha x_1 + \beta x_2 : \alpha, \beta \in \mathbb{R}\}$

$$\alpha x_1 + \beta x_2 = 0$$

$$\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\alpha = \beta = 0$$

Prob: Solve the system of equations

$$x + y - 3z + 2w = 0$$

$$2x - y + 2z - 3w = 0$$

$$3x - 2y + z - 4w = 0$$

$$-4x + y - 3z + w = 0.$$

$\longrightarrow$ $r(A) = 4 =$ No. of unknowns.

Trivial soln.

$x = y = z = w = 0$

prob: Find all solutions to the following system of equations by row reduced method (पंक्ति समानीत विधि).

$$x_1 + 2x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = 5$$
$$-x_1 - 3x_2 + 8x_3 = -1$$

IAS 2022
15M

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & -1 & 2 \\ 2 & 3 & 5 & 5 \\ -1 & -3 & 8 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 2 \\ 0 & -1 & 7 & 1 \\ 0 & -1 & 7 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 2 \\ 0 & -1 & 7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to R_3 - R_2$$

$r(A:B) = r(A) = 2 <$ No. of unknowns.

Infinitely many soln.

Writing eqn. in reduced form with the help of echelon matrix.

$x_1 + 2x_2 - x_3 = 2$

$-x_2 + 7x_3 = 1$

$x_3 = K$

$K \in \mathbb{R}$

$-x_2 + 7K = 1$

$-x_2 = 1 - 7K$

$x_2 = 7K - 1$

$x_1 + 2(7K-1) - K = 2$

$\Rightarrow x_1 + 14K - 2 - K = 2$

$\Rightarrow x_1 + 13K = 4$

$\Rightarrow x_1 = 4 - 13K$

(i) $\underline{AB = I_3}$

(ii) $\det(A) = 1$

By (i)

$$|AB| = |I|$$

$$|A| \cdot |B| = 1$$

$$1 \cdot |B| = 1$$

$$\therefore \underline{|B| = 1}$$

<u>prob:</u> Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$

(i) Find AB.

(ii) Find $\det(A)$ and $\det(B)$

(iii) Solve the following system of Linear equations:

$$x + 2z = 3$$
$$2x - y + 3z = 3$$
$$4x + y + 8z = 14$$

IAS 2020
15 M

$$CX = D$$

$$\Rightarrow AX = D$$

$$\Rightarrow X = A^{-1}D$$

$$= BD$$

$$= \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}_{3\times 3} \begin{bmatrix} 3 \\ 3 \\ 14 \end{bmatrix}_{3\times 1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -33+6+28 \\ -12+0+14 \\ 18-3-14 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

(iii) $C \cdot X = D$

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad D = \begin{bmatrix} 3 \\ 3 \\ 14 \end{bmatrix}$$

Here $\underline{C = A}$

From (i), $AB = I$

$$B = A^{-1}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$\rho(A) = 2 <$ No. of unknown.

$3 - 2 = 1$ <u>L.I. soln</u>

writing eqn. in reduced form with the help of echelon matrix.

<u>prob:</u> Solve the following system of equations:

$$x + y + z = 0$$

$$x + 2y + 4z = 0$$

$$x + 4y + 10z = 0.$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2k \\ -3k \\ k \end{bmatrix}$$

$$= k \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$= k \cdot X_1$$

$$X_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \neq 0$$

Solⁿ. Space $= \{ kX_1 : k \in \mathbb{R} \}$.

L.I

$x + y + z = 0$

$y + 3z = 0$

Let $z = k$, $k \in \mathbb{R}$

$y = -3k$

$\Rightarrow x - 3k + k = 0$

$x - 2k = 0$

$x = 2k$.