Remainder theorem

→ शेषफल प्रमेय

\# $\dfrac{1\,23456 \ldots\ldots\ldots\ 89\text{ digit}}{125}$

$\dfrac{12345\ldots\ldots 4\boxed{849}}{125}$

$89-9 = \dfrac{\cancel{80}\,D}{\cancel{2}\,D} = \dfrac{9N}{40N}$

$\overline{\quad\quad}$
$49N$

$\boxed{2^n \,|\, 5^n}$

$125)\,849\,($

$R \to 99$

3, 7, 11, 3, 2, 3, 37

54बार +2बार

56बार

$\dfrac{4444\cdots\cdots 56\text{बार}}{13}$

$6)\overline{56}(9$
54
2बार

$R \rightarrow \dfrac{44}{13}$ $\boxed{R=5}$

6बार 6n
3, 7, 11, 13 तथा 37

Factorial (क्रमगुणित)

$\lfloor 0 = 1$

$\lfloor 1 = 1$

$\lfloor 2 = 2\times1 = 2$

$\lfloor 3 = 3\times2\times1 = 6$

$\lfloor 4 = 4\times3\times2\times1 = 24$

1. $\dfrac{\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 + \lfloor 5 + \cdots + \lfloor 19}{8}$

$1 + 2 + 6 \quad | \quad 24$

$\boxed{R=0}$

① $\dfrac{\underline{|1} + \underline{|2} + \underline{|3} + \underline{|4} + \underline{|5} + \cdots\cdots + \underline{|19}}{8} = \dfrac{9}{8} \; \boxed{R=1}$

$1 + 2 + 6 + 24 \quad | \quad 120$

$R=0$

② $\dfrac{1! + 2! + 3! + 4! + 5! + \cdots\cdots\cdots + 306!}{15}$

$\dfrac{33}{15}$ $R \to 3$

$$\frac{\overset{1}{\lfloor 1} + \overset{6}{\lfloor 3} + \overset{120}{\lfloor 5} + \overbrace{\lfloor 7 + \lfloor 9 + \cdots\cdots + \lfloor 729}^{R=0}}{7}$$

$$\# \quad \frac{\overset{1}{1!} + \overset{2}{2!} + \overset{6}{3!} + \overbrace{4! + 5! + \cdots + 30!}^{R=0}}{4! = 24}$$

$$\frac{127}{7} \quad \boxed{\text{Rem} \to 1}$$

$$\frac{9}{24} \quad R=9$$

$$\frac{\overset{1}{0!}+\overset{1}{1!}+\overset{2}{2!}+\overset{6}{3!}+\overset{24}{4!}+\overbrace{5!+\cdots\cdots+39!}^{R=0}}{5!}$$

$$\frac{34}{5!}=\frac{34}{120} \quad \text{Rem}=34$$

± 1

$-1 \quad +1 = 0$

$$\# \quad \frac{\overset{29}{17 + 19}}{18}$$

$\boxed{Rem = 0}$

$$\frac{\overset{-1 \;\; 29}{17}}{18} = -1$$

$$\frac{\overset{19}{}}{18} \; +1$$

$$\# \quad \frac{\overset{-1 \;\; 129}{17 - 3}}{18} = -4$$

$18 - 4 = 14$ Ans.

$$\frac{\overset{129}{(17)}}{18} = -1$$

± 1

$5 \nearrow 4 = 2^2$, $5 \searrow \cancel{6}$

$9 \nearrow \cancel{10}$, $9 \searrow \boxed{8 = 2^3}$

$\frac{2^{32}}{5} = \frac{(2^2)^{16}}{5}$

$= \frac{(\overset{-1}{4})^{16}}{5} = +1$ Ans.

$\frac{2^{33}}{9} = \frac{(2^3)^{11}}{9} = \frac{(\overset{-1}{8})^{11}}{9} = -1$

Rem $\to 9-1$
$= 8$

$\frac{2^{34}}{9} = \frac{(2^3)^{11} \times 2^1}{9} = \frac{(\overset{-1}{8})^{11} \times 2}{9} = -2$

± 1

$9-2 = 7$ Ans.

$8 \Rightarrow 2^3$

$27 = 3^3$

$\dfrac{2^{41}}{9} = \dfrac{(2^3)^{13} \times 2^2}{9}$

$$= \frac{(\overset{-1}{8})^{13} \times 4}{9} = -4$$

Rem $\rightarrow 9-4$

$= 5$

$$\frac{3^{66}}{28} = \frac{(3^3)^{22}}{28}$$

$$= \frac{(\overset{-1}{27})^{22}}{28} = +1$$

Ans.

$\dfrac{5^{69}}{126}$

$$\frac{5^{69}}{126} = \frac{(5^3)^{23}}{126} = \frac{(125)^{23}}{126} = -1$$

$125 \to -1$

Rem $\to 126-1$
$= 125$

$81 \to 3^4$

$$\frac{3^{21}}{5} = \frac{(3^2)^{10} \times 3^1}{5} = \frac{(9)^{10} \times 3}{5}$$

$9 \to -1$

$= +1 \times 3$
$= 3$

$$\frac{3^{45}}{82} = \frac{(3^4)^{11} \times 3^1}{82} = \frac{(81)^{11} \times 3}{82} = -3$$

$81 \to -1$

Rem $\to 82-3 = 79$

Totient method

prime no.
अभाज्य सं०

composite no
भाज्य सं०

Euler's theorem

$\frac{N^{x}}{D}$ R→

$N, D \xrightarrow{HCF} 1$

अभाज्य सं० का Totient

(i) $2 \longrightarrow \phi = 2 \times \left(1 - \frac{1}{2}\right)$
$= \cancel{2} \times \frac{1}{\cancel{2}}$
$= 1$

(ii) $3 \longrightarrow \phi = 3 \times \left(1 - \frac{1}{3}\right)$
$= \cancel{3} \times \frac{2}{\cancel{3}}$
$= 2$

(iv) $5 \longrightarrow \phi = \cancel{5} \times \frac{4}{\cancel{5}} = 4$

(iv) $7 \longrightarrow \phi = \cancel{7} \times \frac{6}{\cancel{7}} = 6$

Note: → अभाज्य सं० का Totient उसी सं० से एक कम होती है।

\# $\dfrac{(17)^{\frac{68}{6}=2}}{7} = \dfrac{17^2}{7} = \dfrac{289}{7}\ \boxed{R=2}$

HCF 1

$7 \rightarrow \phi = 6$

yes|no

(i) Binomial

(ii) Totient

संख्या को छोटा

power को छोटा

$$\frac{17^{68}}{7} = \frac{3^{\frac{68}{6}=2}}{7} = \frac{3^2}{7} = \frac{9}{7}\quad R \to 2$$

$$\boxed{7 \to \phi = 6}$$

B ✓

T ✓

$$\frac{(51)^{103}}{11} = \frac{(7)^{\frac{103}{10}=3}}{11} = \frac{7^3}{11} = \frac{343}{11}\quad R \to 2$$

$$\frac{2^{\frac{32}{4}=0}}{5} = \frac{2^0}{5} = \frac{1}{5} \boxed{Rem = 1}$$

HCF = 1

$5 \rightarrow \phi = 4$

$\frac{3^{\frac{21}{4}=1}}{5} = 3^1 = 3$

Note:→ यदि power, totient से पूर्णतः विभाजित हो तो उस सं० का Remainder हमेशा 1 होती है।

x! B ✓ T ✓