17. Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix.

IAS 2017

10M

Ans: one of the possible values of P is

$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$ s.t.

$P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

18. Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify Cayley-Hamilton theorem for the matrix. Find the inverse of the matrix A and also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

IAS 2014

10M

Ans: $A^{-1} = \frac{1}{5}\begin{pmatrix} -3 & 4 \\ 2 & -1 \end{pmatrix}$

$A + 5I$

19. Investigate the values of λ and μ so that the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have (i) No solution, (ii) a unique solution, (iii) an infinite no. of sol^n.

IAS 2014
10M

<u>Ans:</u> (i) $\lambda = 3, \mu \neq 10$
(ii) $\lambda \neq 3$, μ any value
(iii) $\lambda = 3, \mu = 10$

(20) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where $(\omega \neq 1)$ is a cube root of unity. If $\lambda_1, \lambda_2, \lambda_3$ denote the eigen values of A^2, show that $|\lambda_1| + |\lambda_2| + |\lambda_3| \leq 9$

IAS 2013
8M

<u>Hint:</u> $1 + \omega + \omega^2 = 0$

$$A^2 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$