AVERAGE (औसत)

AVERAGE औसत

The result you get by adding two or more amounts together and dividing the total by the number of amounts, is called average.

दो या दो से अधिक राशियों को एक साथ जोड़ कर राशियों की संख्या से विभाजित करने पर जो परिणाम मिलता है, उसे औसत कहा जाता है।

$$Average/औसत = \frac{Sum\ of\ all\ terms}{Number\ of\ terms}$$

Finding Average using Deviation method (विचलन विधि का उपयोग करके औसत ज्ञात करना)

Step I: Assume any term as average in observation.
चरण 1: प्रेक्षण में किसी भी पद को औसत मानें।

Step II: Calculate the difference between the observations and assumed average. (These differences are known as Deviation).
चरण II : प्रेक्षणों और अनुमानित औसत के बीच अंतर की गणना करें। (इन अंतरों को विचलन के रूप में जाना जाता है)।

Step III: Add all deviations calculated in Step-II and divide it by total number of terms given in the observations. (This is known as net deviation in the average).
चरण III: चरण : II में गणना किए गए सभी विचलनों को जोड़ें और इसे अवलोकनों में दिए गए पदों की कुल संख्या से विभाजित करें। (इसे औसत में शुद्ध विचलन के रूप में जाना जाता है)।

Step IV: to get the actual average, add or subtract the net deviation in the assumed average(as per the result of Step-III).
चरण IV: वास्तविक औसत प्राप्त करने के लिए, अनुमानित औसत में शुद्ध विचलन जोड़ें या घटाएं (चरण-III के परिणाम के अनुसार)।

EX: - Find the average of the following observations:
निम्नलिखित प्रेक्षणों का औसत ज्ञात कीजिए:-
48, 52, 40, 60, 53, 54, 49

HINTS:

Step I: Let Average = 50
Step II: Deviations we get when we subtract from the observations- $-2 +2 - 10 + 10 + 3 + 4 - 1$
Step III: Net deviation

$$= \frac{-2 + 2 - 10 + 10 + 3 + 4 - 1}{7} = \frac{6}{7} = +0.857$$

Step IV: Actual average (वास्तविक औसत)
= 50 + 0.857 = 50.857

For consecutive natural numbers (क्रमागत प्राकृतिक संख्याओं के लिए)

(i) Average of first n natural number / प्रथम n प्राकृतिक संख्याओं का औसत $= \frac{(n+1)}{2}$

(ii) Average of square of first n natural number/प्रथम n प्राकृतिक संख्याओं के वर्गों का औसत $= \frac{(n+1)(2n+1)}{6}$

(iii) Average of cube of first n natural number/प्रथम n प्राकृतिक संख्याओं के घनों का औसत $= \frac{n(n+1)^2}{4}$

For consecutive even numbers (क्रमागत सम संख्याओं के लिए)

(i) Average of first n even no. / प्रथम n सम संख्याओं का औसत $= (n+1)$

(ii) Average of square of first n even no. / प्रथम n सम संख्याओं के वर्गों का औसत $= \frac{2(n+1)(2n+1)}{3}$

(iii) Average of cube of first n even no. / प्रथम n सम संख्याओं के घनों का औसत $= 2n(n+1)^2$

For consecutive odd numbers (क्रमागत विषम संख्याओं के लिए)

(i) Average of first n odd no. / प्रथम n विषम संख्याओं का औसत $= n$

(ii) Average of square of first n odd no./प्रथम n विषम संख्याओं के वर्गों का औसत $= \frac{(2n+1)(2n-1)}{3}$

(iii) Average of cube of first n odd no./प्रथम n विषम संख्याओं के घनों का औसत $= n(2n^2-1)$

(iv) Average of consecutive numbers/क्रमागत संख्याओं का औसत $= \frac{First\ number + Last\ number}{2}$

CONCEPT-01

If there are 3 natural numbers and average of any two number when added with third number gives a, b, c. Then.
यदि 3 प्राकृतिक संख्याएं हो और किसी दो संख्याओं के औसत को यदि तीसरी संख्या में जोड़ा जाए तो प्राप्त संख्याए क्रमशः a, b तथा c हो तो-

$$Sum\ of\ number = \frac{a + b + c}{2} = k$$

First number = 2a − k
Second number = 2b − k
Third number = 2c− k

CONCEPT-02

If the average of n1 observations is a1 the average of n observations is a, and so on, then-

n_2 प्रेक्षणों का औसत a_1, n_2 प्रेक्षणों का औसत a_2, तथा आगे भी इसी प्रकार हों तो-

Average of all the observations/सभी प्रेक्षणों का औसत

$$= \frac{n_2a_1 + n_2a_2 + \cdots}{n_1 + n_2 + \cdots}$$

CONCEPT-03

When data is misread (जब डेटा गलत पढ़ा जाता है)

If average of n numbers is m but later on it was found that a number 'a' was misread as 'b'. The correct average will be.

यदि n संख्याओं का औसत m है लेकिन बाद में यह पाया गया कि एक संख्या 'a' को गलती से 'b' पढ़ लिया गया था। सही औसत होगा।

$$= m + \frac{a - b}{n}$$

EX.– While finding the average of 10 given numbers a student, by mistake, wrote 64 in place of a number 46 and got his average 50. The correct average of the given numbers is:

एक विद्यार्थी जब 10 संख्याओं का औसत ज्ञात करता है, तो त्रुटि के कारण 46 के स्थान पर 64 लिख देता है, तथा औसत 50 प्राप्त करता है। सही औसत ज्ञात कीजिए

HINTS:

Method-01

Correct Average = $50 + \frac{(-18)}{10}. = 48.2$

EX– If the average of n numbers is m but later on it was found that two numbers a and b misread as p and q.

यदि n संख्याओं का औसत m है लेकिन बाद में यह पाया गया कि दो संख्याएँ a और b को गलती से p और q पढ़ लिया गया। The correct average

(सही औसत) = $n + \frac{(a + b - p - q)}{n}$

Ex. The average marks of 40 students was found to be 68. If the marks of two students were incorrectly entered as 48 and 64 instead of 84 and 46 respectively, then what is the correct average?

40 छात्रों के औसत अंक 68 पाए गए। यदि दो छात्रों के अंक भूलवश 84 एवं 46 के बजाय क्रमशः 48 और 64 के रूप में शामिल किये गए हैं, तो सही औसत ज्ञात करें।

HINTS:

CONCEPT-04

Ex. The average of 21 data is 36 out of which the first 12 data are having an average of 15. The average of the rest 9 data is:

21 आंकड़ों का औसत 36 है जिसमें से पहले 12 आंकड़ों का औसत 15 है। शेष 9 आंकड़ों का औसत है:

HINTS:

Average of remaining 9 number

$$= \frac{Total\ sum - sum\ of\ first\ 12\ data}{9}$$

$$= \frac{21 \times 36 - 12 \times 15}{9} = \frac{576}{9} = 64$$

To make it shorter you can solve it using the ratio of number & deviation explained below-

	First	Remaining
Data	12	9
Ratio	4	3
Average	15	x

Here, Overall average = 36

Deviation occurred in 4 = (15 − 36) = 21

OTA

(यहाँ जो − 21 की कमी पहले 12 आंकड़े में आ रही है वो आगे के शेष 9 आकड़े में बढेगी)

Average of remaining, x = $36 + \frac{21 \times 4}{3} = 64$

Ex. In a primary school, there are 60 boys of age 12 years each, 40 of age 13 years each, 50 of age 14 years each and 50 of age 15 years each. The average age (in years) of all the boys of the school is:

एक प्राथमिक विद्यालय में 60 लड़के प्रत्येक 12 वर्ष के 40 लड़के प्रत्येक 13 वर्ष के, 50 लड़के प्रत्येक 14 वर्ष और 50 लड़के प्रत्येक 15 वर्ष के है। विद्यालय के सभी लड़कों का औसत आयु है :

HINTS:

60	40	50	50
12	13	14	15
	+1	+2	+3
		12	

= 1 × 40 + 2 × 50 + 3 × 50

= 40 + 100 + 150 = 290 = $\frac{299}{200} = 1.45$

Average = 12 +1.45 = 13.45

CONCEPT-05

Replacement/Inclusion/Exclusion (प्रतिस्थापन/समावेशन/बहिष्करण)

Ex: There are 50 students in a class. One of them weighing 50 kg goes away and a new student join. By this the average weight of the class is increased by kg. The weight of the new student is:

एक कक्षा में 50 विद्यार्थी हैं। एक विद्यार्थी जिसका वजन 50 किग्रा. है चला जाता है एवं नया विद्यार्थी आता है जिससे कक्षा का औसत वजन किग्रा. बढ़ जाता है। नये विद्यार्थी का वजन ज्ञात करें।

HINTS:

Weight of new student = Old + $\dfrac{1}{2} \times 50$

= 50+25=75 kg

Ex: The mean weight of 34 students of a school is 42 kg. If the weight of the teacher is also included, the mean rises by 400 grams. Find the weight of the teacher (in kg).

एक विद्यालय में 34 छात्रों का औसत भार 42 किग्रा है। यदि अध्यापक का भार भी सम्मिलित कर लिया जाये तो औसत 400 ग्राम बढ़ जाता है। अध्यापक का भार ज्ञात करें

HINTS:

Method-01

Weight of teacher = Old Average +0.4 × 35
= 42 + 14 = 56 kg.

Method-02

Let the weight of the teacher is x kg.
ATQ, 34 × 42 + x = 35 × 42.4
$\Longrightarrow$ x = 56 kg

EX: The average height of 25 boys is 1.4 m. When 5 boys leave the group, then the average height increase by 0.15m. What is the average height of the 5 boys who leave?

25 लड़कों की औसत ऊँचाई 1.4 मी. है। जब 5 लड़के समूह को छोड़ देते हैं तो औसत ऊँचाई 0.15 मी. से बढ़ जाती है। 5 लड़के जिन्होंने समूह छोड़ा उनकी औसत ऊँचाई क्या है?

HINTS:

Method-01
Here Overall Average = 1.4 m When 5 boys left the group,

Average of 20 boys is increased by 0.15 m. (जो लड़कों के औसत वृद्धि होगी वो छोड़ के जाने वाले बच्चों के औसत कमी आएगी)

Deviation = $\dfrac{0.15 \times 20}{5} = 0.6$
Average of Left boys = 1.4 − 0.6 = 0.8 m

Method-02

Let the average height of all 5 boys be x m.
ATQ, 25 × 1.4 − 5 * x = 20 × 1.55
$\Longrightarrow$ x = 0.8 m

CONCEPT-06

Cricket Based (क्रिकेट आधारित)

Ex. A cricket batsman had a certain average of runs for his 11 innings. In the 12th innings, he made a score of 90 runs after that his average run is decreased by 5. Find his average of runs after 12th innings.

एक बल्लेबाज की ।। पारियों का एक निश्चित औसत रन है। 12वीं पारी में, वह 90 रन बनाता है जिसके कारण उसका औसत 5 कम हो जाता है। 12वीं पारी के बाद नया औसत रन ज्ञात करें।

HINTS:

Let the initial average = x

New Average = $\dfrac{(Initial\ Innings) \times Average + Recent\ Run}{Total\ Innings}$

$$\Longrightarrow x - 5 = \dfrac{11 \times x + 90}{12}$$

$$\Longrightarrow x = 150$$

So, New average = 150 − 5 = 145

Method 02

ATQ, Average Run is decreased by 5.
Total in 11th innings = 11 × 5 = 55
So, In the 12th innings he made a score of 90 Runs.
The Average of Runs after 12th innings.
= (90 + 55) = 145

Ex. Average of 64 innings of a player is 62 runs, difference between his maximum and minimum score is 180 runs. If these two innings are excluded then the average of remaining innings is 60 runs. Then his maximum score is:

एक क्रिकेट के खिलाड़ी की 64 पारियों का औसत 62 रन है। उसके अधिकतम और न्यूनतम स्कोर का अंतर 180 रन है। उन दो पारियों को छोड़कर उसकी शेष पारियों का औसत 60 रन आता है। तदानुसार उस खिलाड़ी का अधिकतम स्कोर कितना है? z3

HINTS:

ATQ, $64 × 62 − H − L62 × 60$

$\implies 248 = H + L \longrightarrow (i)$

Given that,

$H − L = 180 \longrightarrow (ii)$

on adding (1) and (2), We get −

$248 = H + L$

$\underline{180 = H - L}$

$428 = 2H$

$\implies H = 214$

Ex. A cricketer whose bowling average is 11.125 runs per wicket. he takes 7 wickets for 38.5 runs and thereby decrease his average by 1.125. The number of wickets taken by him till last matches was:

एक क्रिकेटर जिसकी गेंदबाजी का औसत 11.125 रन प्रति विकेट है। 38.5 रन देकर वह 7 विकेट लेता है, जिससे उसका औसत 1.125 घट जाता है। अंतिम मैच तक उसके द्वारा लिये गये विकेटों की संख्या बताएँ।

HINTS:

Bowling Avg. $= \dfrac{Total\ Rul}{Total\ Wicket} = \dfrac{38.5}{5} = 5.5$

11.125   5.5

4.5   1.125

So, $\dfrac{7}{1.125} × 4.5 = 28$

Number of wicket till the last match

$= 28 + 7 = 35.$

CONCEPT-07

(Age Based/आयु आधारित)

Ex. 5 years ago, the average age of A, B, C and D were 45 years. With the joining of E, the average age of all the five is 48 years. How old is E?

पाँच वर्ष पूर्व, A, B, C तथा D की औसत आयु 45 वर्ष थी। अब E सम्मिलित हो जाता है, तो पाँचों की औसत आयु 48 वर्ष हो जाती है। E की आयु ज्ञात करें।

HINTS:

Method-01

Total present age of (A, B, C, D) = (45 +5) × 4 = 200 years
Total present age of (A, B, C, D, E) = 48 × 5 = 240 years
Present Age of E 240 − 200 = 40 years.

Method-02

Total present Average of (A, B, C, D) = 45 + 5 = 50
Total present Average of (A, B, C, D, E) = 48
Here We Can see that, after joining of E average of is reduced by - 2.
Thus, Age of E = 50 (2 × 5) = 40 years.

Ex: - The average age of Ravi, his wife and their two children is 23 years. His wife is just 4 years younger than Ravi and his wife was 24 years old when his daughter was born. he was 32 years old when his son was born. The average age of Ravi and his daughter is?

रवि, उसकी पत्नी और उसके 2 बच्चों की औसत आयु 23 वर्ष है। उसकी पत्नी उससे आयु में 4 वर्ष छोटी है। जब उसकी पुत्री का जन्म हुआ तब उसकी पत्नी 24 वर्ष की थी और जब उसके पुत्र का जन्म हुआ तब वह 32 वर्ष का था। रवि और उसकी पुत्री की औसत आयु ज्ञात कीजिए।

HINTS:

Total Sum = 23 × 4 = 92

R	W	D	S
32	28	4	0 = 64

| x + | + x | + x | + x

$= 64 + 4x = 92$

$= x = 7$

Ravi = 32 + 7 = 39

Daughter = 4 + 7 = 11

So, the average age of Ravi and his daughter

$= \dfrac{(39+11)}{2} = \dfrac{50}{2} = 25\ years$

CONCEPT-08

(Number Based/संख्या आधारित)

Ex. Average of 25 natural numbers is 67. 1st number is increased by 1, 2nd number increased by 3 and 3rd number is increased by 5, and so on. find the new average.

25 प्राकृत संख्याओं की औसत 67 है। प्रथम संख्या को 1 से बढ़ा दिया गया, दूसरी संख्या को 3 से और तीसरे को 5 से और ऐसे ही आगे तो नई औसत ज्ञात करें।

HINTS:

We know, Average of n-odd number = n
So, Average of 25 odd no. 25
Thus, New average = 67 + 25 = 92

Ex. The average of 21 numbers is 44. The average of first 11 numbers is 48 and that of last 11 numbers is 42. If 11th number is excluded, what is the average of the remaining number?

21 संख्याओं का औसत 44 है। पहली 11 संख्याओं का औसत 48 तथा अंतिम 11 संख्याओं का औसत 42 है। यदि 11 वीं संख्या को हटा दिया जाए, तो शेष संख्याओं का औसत क्या होगा?

HINTS:

No.   21 | 11   11

Average   44 | 48   42

$\Longrightarrow$ + 44 − 22 = + 22

11th Number = 44 + 22 = 66
New average after removing 11th number to A

$$= \frac{21 \times 44 - 66}{20} = 42.9$$

Ex. The average of 15 numbers is 45. The average of the first six numbers is 42 and that of last six numbers is 43. The seventh number is two times the 8th number but 5 more than the 9th number. The average of the seventh and the ninth number is:

15 संख्याओं का औसत 45 है। प्रथम छह संख्याओं का औसत 42 है और अंतिम छह संख्याओं का औसत 43 है। 7वीं संख्या, 8वीं संख्या की दोगुनी है, लेकिन 9वीं संख्या से 5 अधिक है, तो 7वीं और 9वीं संख्याओं का औसत कितना है?

HINTS:

	All	First	Last
Number	15	6	6
Average	45	42	43

Deviation = $(-3 \times 6) + (-2 \times 6) = -30$

Let	7th	8th	9th
	2x	x	2x-5

Now, $5x - 5 = 45 \times 3 + 30$
$\Longrightarrow 5x = 170 \Longrightarrow x = 34$

Average of 7th and 9th number = $\dfrac{68 + 63}{2}$ = 65.5

PRACTICE QUESTIONS

Q 1. Ramesh brought 15 magazines for 30 from one shop and 25 magazines for 38 from another. What is the average price he paid per magazine?

(a) 25

(b) 30

(c) 35

(d) 33

Q 2. In a cricket eleven, the average age of eleven players is 28 years. Out of these, the average ages of three groups of three players each are 25 years, 28 years and 30 years respectively. If in these groups the captain and the youngest player are not included and the captain is eleven years older than the youngest player, what is the age of the captain?

(a) 33 years

(b) 34 years

(c) 35 years

(d) 36 years

Q 3. The average of 27 numbers is zero. Out of them, how many may be greater than zero, at the most?

(a) 0

(b) 15

(c) 26

(d) 20

Q 4. The difference between the average of first ten prime numbers and the first ten prime numbers of two digits is

(a) 14.5

(b) 16.5

(c) 12.5

(d) 13.5

Q 5. A shop of electronic goods is closed on Monday. The average daily sales for remaining six days of a week is 15,640/- and the average sale of Tuesday to Saturday is 14,124/-. The sales on Sunday is

(a) 20,188/-

(b) Data inadequate

(c) 23,220/-

(d) (d) 21,704/-

Q 6. The mean high temperature of the first four days of a week is 25C whereas the mean of the last four days is 25.5C. If the mean of the whole week is 25.2C, then the temperature of the 4th day is

(a) 25.2C
(b) 25.5C
(c) 25.6C
(d) 25C

Q 7. The average marks of a student in 8 subjects is 87. Of these, the highest marks are 2 more than the one next in value. If these two subjects are eliminated, the average marks of the remaining subjects are 85. What are the highest marks now obtained by him?

(a) 89
(b) 91
(c) 94
(d) 96

Q 8. The batting average for 40 innings of a cricket player is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded, the average of the remaining 38 innings is 48 runs. The highest score of the player is

(a) 165 runs
(b) 170 runs
(c) 172 runs
(d) 174 runs

Q 9. Find the average score for all the juniors and seniors combined.

(i) The average of the scores was 85 for juniors and 89 for seniors.
(ii) The groups are of equal size.

(a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;
(b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;
(c) if the data even in both Statements I and II together are not sufficient to answer the question;
(d) if the data in both Statements I and II together are necessary to answer the question.

Q 10. What is the average monthly income per family member?

(i) Each male earns 1250 a month and each female earn 1050 a month.
(ii) Ratio of males to females in the family is 2:1.

(a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;
(b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;
(c) if the data even in both Statements I and II together are not sufficient to answer the question;
(d) if the data in both Statements I and II together are necessary to answer the question.

SOLUTIONS

1. c

Total cost of 15 magazines = 15 × 30 = ₹ 450
Total cost of 25 magazines = 25 × 38 = ₹ 950
Total cost of (15 + 25) = 40 magazines
= 450 + 950 = ₹ 1400

∴ Average cost of 40 magazines = $\dfrac{1400}{40}$ = ₹ 35

2. c

Sum of ages of the captain and the youngest player
= [(28 × 11) − [(25 × 3) + (28 × 3) + (30 × 3)] years
= (308 − 249) years = 59 years.
Let the age of the youngest player be x years. Then, age of the captain = (x + 11) years.

∴ x + (x + 11) = 59 ⟹ 2x = 48 ⟹ x = 24.
So, age of captain = 24 + 11 = 35 years.

3. c

As average = $\dfrac{Sum}{number\ of\ observations}$

∴ In order to have average 0, sum should be 0.

So, there can take 26 number at greater than zero and 27 number should be equal to sum of all 26 numbers but with a negative sign.

4. d

Ist ten prime numbers are

= 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

:- Average = $\dfrac{\text{Sum}}{10} \dfrac{129}{10}$

First ten two-digit prime numbers

= 11, 13, 17, 19, 23, 29, 31, 37, 41, 43

Average = $\dfrac{\text{Sum}}{10} \dfrac{264}{10}$ = 26.4

Difference = 26.4 − 12.9 = 13.5

5. c

Average sales per day for six days of the week = ₹ 15,640/-
Total sales of six days of the week = 15640 × 6 = ₹ 93,840/-
Average sales of Tuesday to Saturday = ₹ 14,124/-
Total sales from Tuesday to Saturday = ₹ 14,124 × 5 = ₹ 70,620/-
:- Sales on Sunday = (₹ 93,840 - 70,620) = ₹ 23,220/-

6. c

Average temperature of first four days = 25^{0}C
Total temperature of first four days = 25^0 × 4 = 100^{0}C
Average temperature last four days = 25.5^0
Total temperature of four days = 25.5^0 × 4 = 102^{0}C
Total temperature of whole week = 25.2 × 7 = 176.4^{0}C
:- Temperature of the 4th day = 100^0 + 102^0 − 176.4^0 = 25.6^{0}C

7. c

Let the highest marks obtained by the student be x.

Then, second highest marks = x − 2.
Sum of marks of these 2 subjects
= (87 × 8) − (85 × 6)
= 696 − 510 = 186.
:- x + (x − 2) = 186 $\Longrightarrow$ 2x = 188 $\Longrightarrow$ x = 94.

8. d

Let the highest score be x.
Then, lowest score = (x - 172).
Then, (50 × 40) − [x + (x − 172)] = 38 × 48
$\Longrightarrow$ 2x = 2000 + 172 − 1824
$\Longrightarrow$ 2x = 348
$\Longrightarrow$ x = 174.

9. d

From both I and II, we have:
Let each group have x students.

Then, the average score for all = $\left(\dfrac{85x + 89x}{2x} \right) = \dfrac{174}{2}$ = 87

So, both I and II together give the answer.
:-Correct answer is (d).

10. d

From both I and II, we have: Let the number of males and females in the family be 2x and x respectively.
Then, the average monthly income per member

= ₹ $\left(\dfrac{1250 \times 2x + 1050 \times x}{2x + x} \right)$ + ₹ $\left(\dfrac{3550}{3} \right)$ = ₹ 1183.33

So, both I and II together are needed.
:- Correct answer is (d).