Mensuration → क्षेत्रमिति

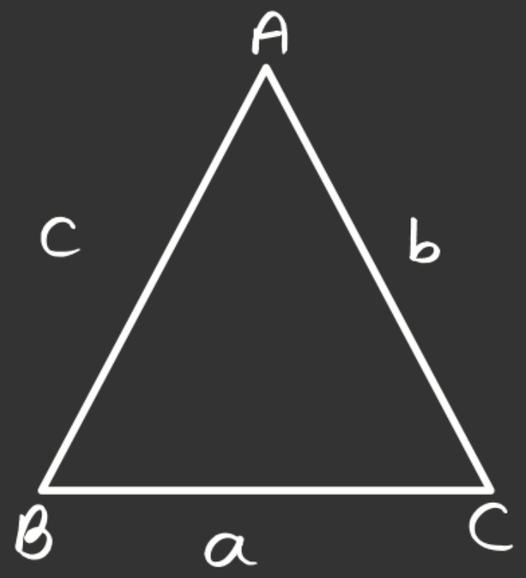
2D(l, b) ⇒ (Area & perimeter)

- त्रिभुज (Triangle)
- चतुर्भुज (Quadrilateral)
- वृत्त (circle)
- बहुभुज (polygon)

3D(l, b, h) ⇒ (Area & volume) → (C.S.A / L.S.A / T.S.A)

- घन (cube)
- घनाभ (cuboid)
- शंकु (cone)
- शंकु छिन्नक (Frustum)
- बेलन (cylinder)
- गोला (sphere)
- प्रिज्म (prism)
- पिरामिड (pyramid)
- समचतुष्फलक (Tetrahedron)

Triangle (त्रिभुज)

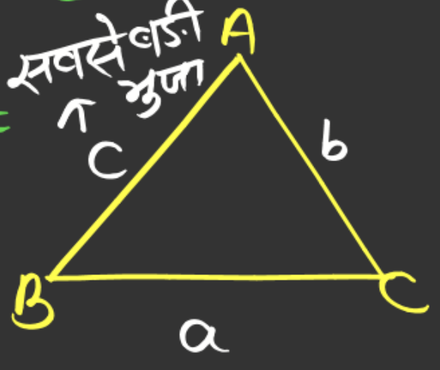


#

(i) $a+b>c$	(i) $\lvert a-b\rvert<c$
(ii) $b+c>a$	(ii) $\lvert b-c\rvert<a$
(iii) $c+a>b$	(iii) $\lvert c-a\rvert<b$

$\angle A+\angle B+\angle C=180^\circ$

सबसे लंबी भुजा



(i) $a^2+b^2>c^2$ ⟶ न्यूनकोण Δ

(ii) $a^2+b^2=c^2$ ⟶ समकोण Δ

(iii) $a^2+b^2<c^2$ ⟶ अधिककोण

Q निम्नलिखित में कौनसी सं० त्रिभुज के भुजा हो सकती हैं?

13 = 13

(a) 7, 6, 13 ✗

13·3 < 13·9 ✗

(b) 7·8, 5·5, 13·9

8 < 11

(c) 5, 3, 11 ✗

(d) 9, 8, 10

17 > 10

Q 3, 5, 6 कौन-सी Δ निर्माण होगा

$3^2 + 5^2$ 6^2

$9 + 25$ 36

$34 < 36$

$a^2 + b^2 < c^2$ ⇓ obtuse angled Δ

(a) न्यूनकोण Δ

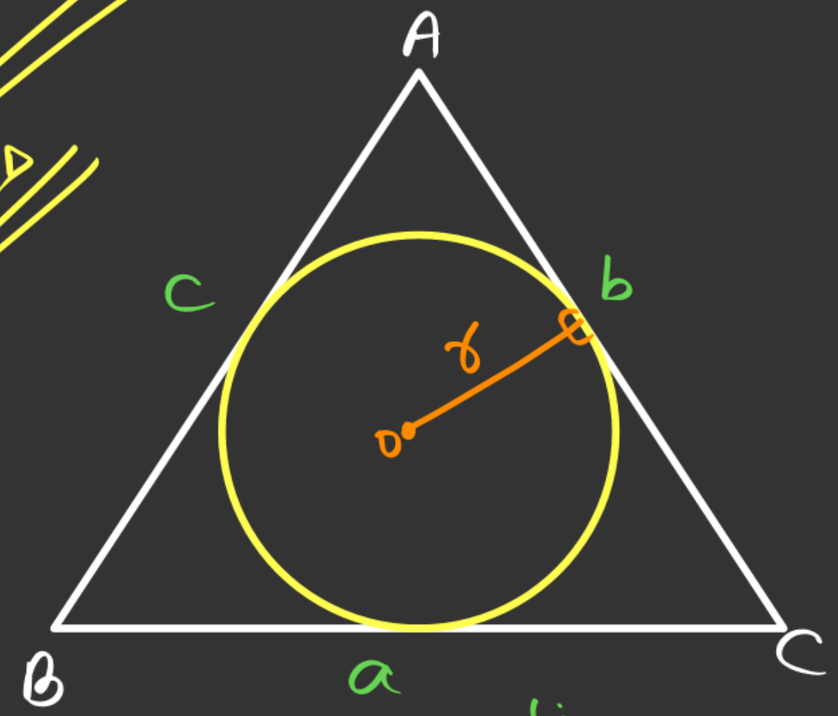
(b) अधिक कोण Δ

(c) समकोण Δ

(d) none of these

त्रिभुज में Δ के

any Δ



$$\gamma = \frac{\Delta}{S}$$

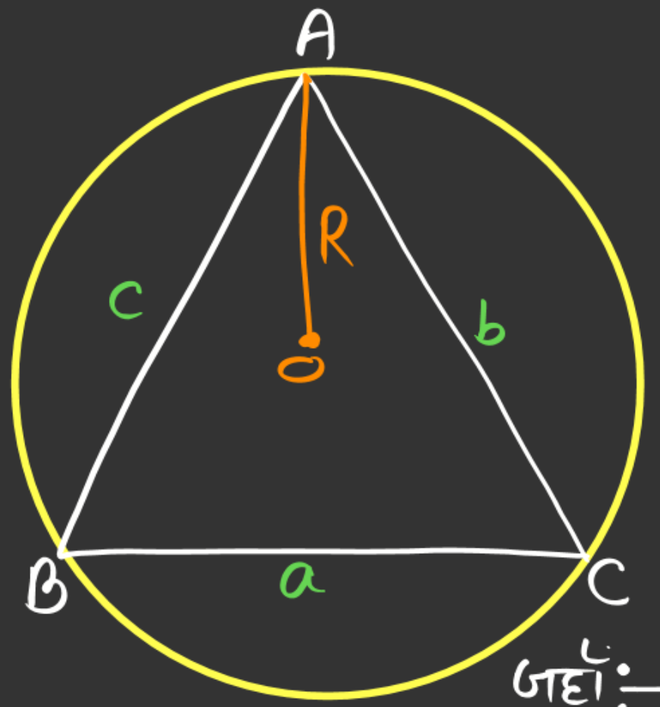
जहाँ :-

$\Delta \rightarrow$ त्रिभुज का क्षेत्रफल है.

$\gamma \rightarrow$ अंत : वृत्त के त्रिज्या (Inradius)

$S \rightarrow$ अर्धपरिमाप (semiperimeter)

$$S = \frac{a+b+c}{2}$$



$$R = \frac{abc}{4\Delta}$$

जहाँ :—

$R \rightarrow$ परिवृत के त्रिज्या

$\Delta \rightarrow$ त्रिभुज का क्षे०

$a, b, c \rightarrow$ त्रिभुज के भुजा

समकोण Δ में
Right Angled triangle

(i) $r = \frac{p+b-h}{2}$

(ii) $R = \frac{h}{2}$

(iii) $\boxed{r+R = \frac{p+b}{2}}$

समबाहु Δ में
Equilateral Triangle

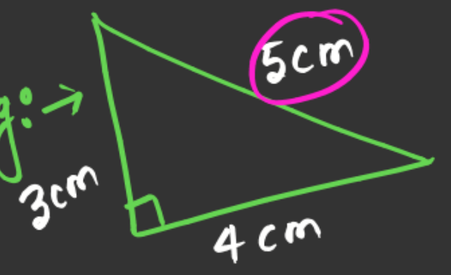
(i) $r = \frac{a}{2\sqrt{3}}$

(ii) $R = \frac{a}{\sqrt{3}}$

(iii) $r : R$
$1 : 2$

Area → $1 : 4$

eg:→



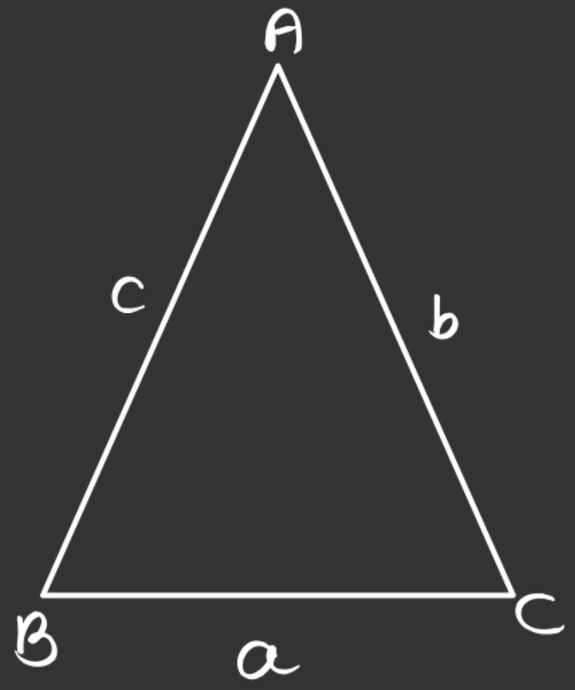
(i) $r = \frac{p+b-h}{2} = \frac{3+4-5}{2} = \frac{2}{2} = 1\,cm$

(ii) $R = \frac{h}{2} = \frac{5}{2} = 2.5\,cm$

(iii) $r+R = 1+2.5 = 3.5\,cm$

$r+R = \frac{p+b}{2} = \frac{3+4}{2} = \frac{7}{2} = 3.5\,cm$

①

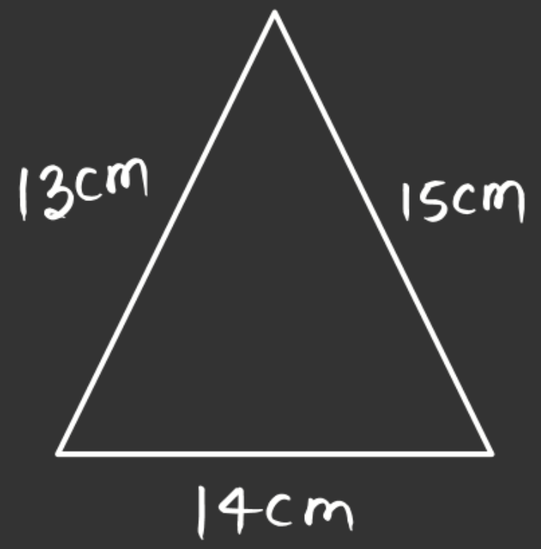


(i) perimeter (परिमाप) = $a+b+c$

(ii) Semiperimeter (अर्धपरिमाप)

$$s = \frac{a+b+c}{2}$$

(iii) Δ का क्षे० = $\sqrt{s(s-a)(s-b)(s-c)}$



$$S = \frac{13+14+15}{2} = \frac{42}{2} = 21\text{cm}$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

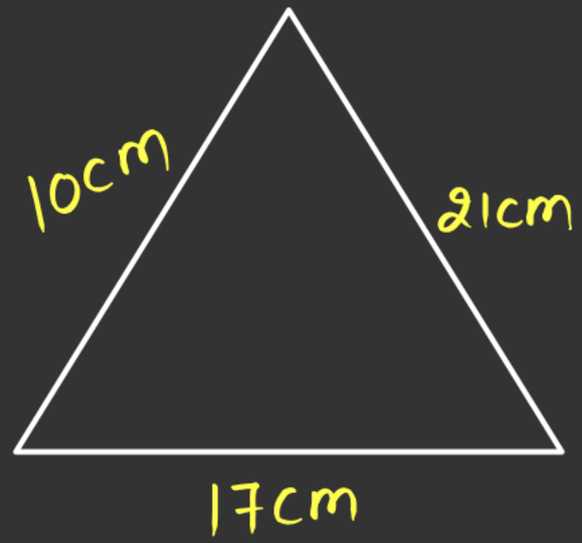
$$= \sqrt{21 \times (21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{3 \times 7 \times 8 \times 7 \times 3 \times 2}$$

$$= 3 \times 7 \times 4 = 84\text{cm}^2$$

त्रिभुज का क्षेत्रफल $= 84\text{cm}^2$



$$S = \frac{10+17+21}{2} = \frac{48}{2} = 24$$

$$\Delta\text{का क्षेत्रफल} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{24 \times 14 \times 7 \times 3}$$

$$= \sqrt{8 \times 3 \times 2 \times 7 \times 7 \times 3}$$

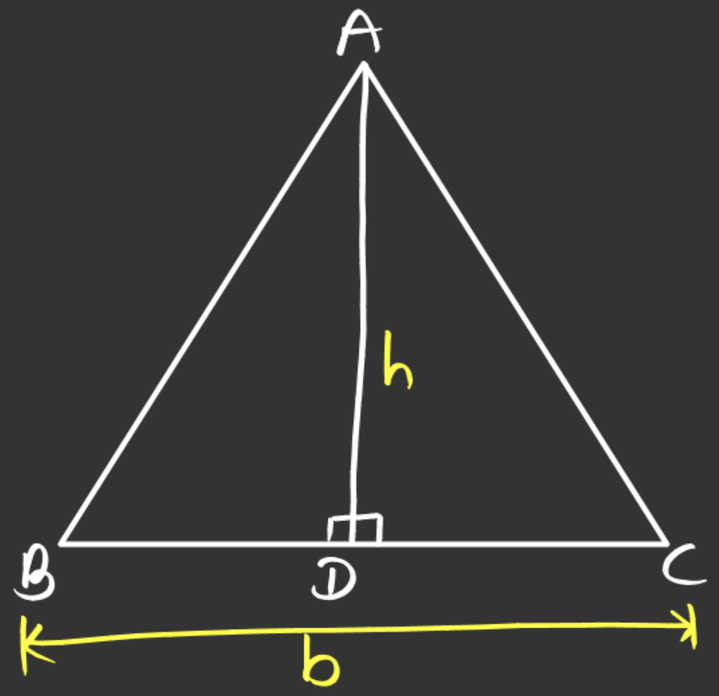
$$= 7 \times 3 \times 4 = 84 cm^2$$

13cm, 14cm, 15cm

10cm, 17cm, 21cm

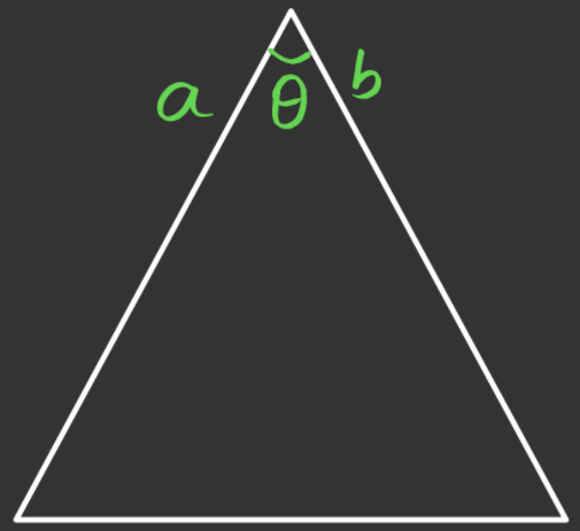
$84 cm^2$

(ii)

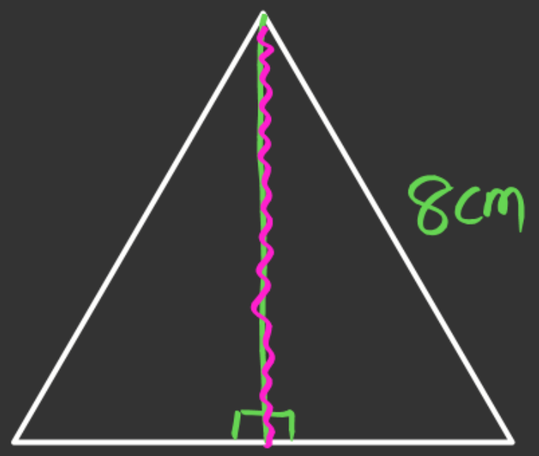


$$\text{Area} = \frac{1}{2} \times b \times h$$

(iii)

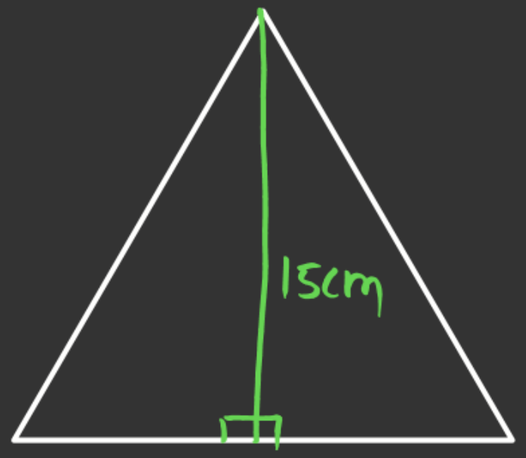


$$\text{Area} = \frac{1}{2} ab \sin\theta$$



$$\text{Area} = \frac{1}{2} \times 10 \times 8^{4} = 40\text{cm}^2$$ → 500% জানন

CND



$$\text{Area} = \frac{1}{2} \times {}^{6}12 \times 15$$

$$= 90\text{cm}^2$$

$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$\text{Area} = \frac{1}{2} \times 16 \times 24 \times \sin 60$$

$$= \frac{1}{2} \times {}^{4}16 \times 24 \times \frac{\sqrt{3}}{2}$$

$$= 96\sqrt{3}\text{cm}^2$$

किसी △ का क्षे०

① Area = $\sqrt{s(s-a)(s-b)(s-c)}$

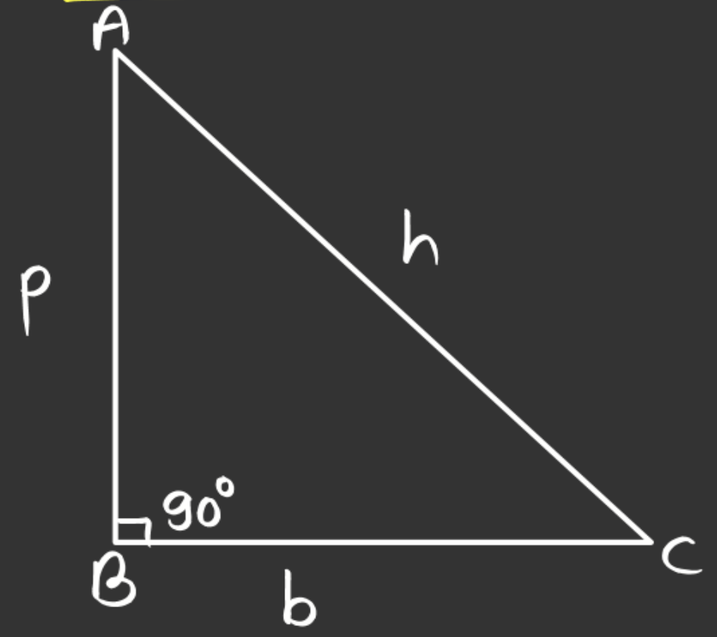
13, 14, 15 → Area = 84 cm²

10, 17, 21

② Area = $\frac{1}{2} \times b \times h$

③ Area = $\frac{1}{2} \times ab\sin\theta$

Right Angled Triangle (समकोण Δ)



(i) $p^2 + b^2 = h^2$

(ii) $r = \frac{p + b - h}{2}$

(iii) $R = \frac{h}{2}$

(iv) $r + R = \frac{p + b}{2}$

(v) $\text{Area} = \frac{1}{2} \times p \times b$

Triplet's

3, 4, 5

5, 12, 13

6, 8, 10

7, 24, 25

8, 15, 17

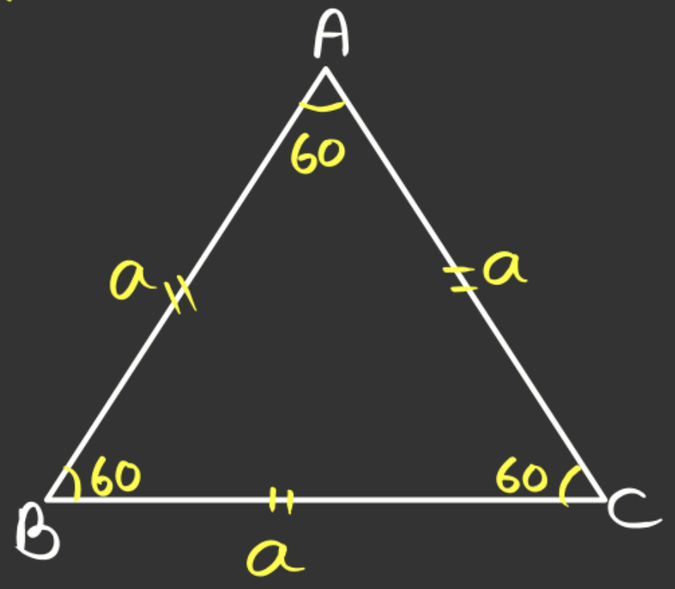
9, 40, 41

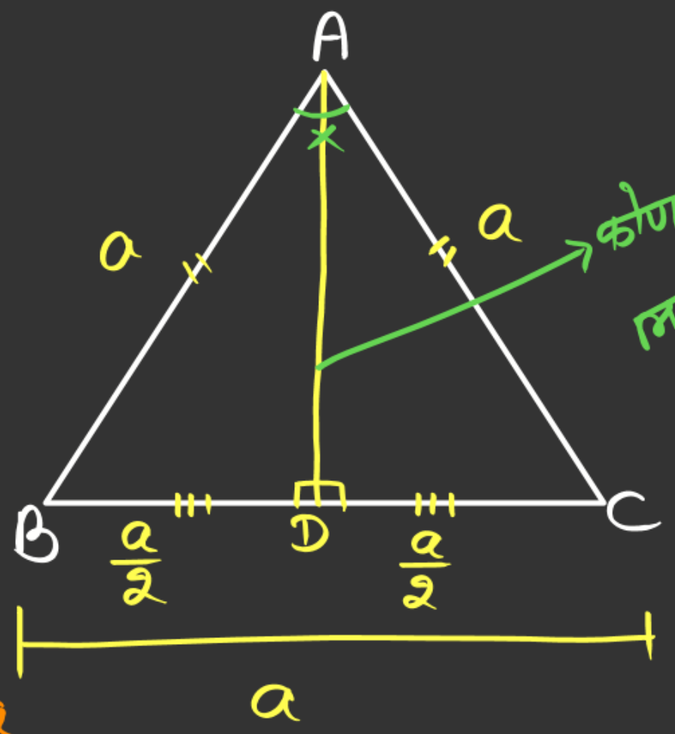
10, 24, 26

11, 60, 61

12, 35, 37

Equilateral Triangle (समबाहु Δ)





कोण समद्विभाजक | माध्यिका | ऊँचाई |

लम्ब समद्विभाजक

(i) perimeter = $3a$

(ii) height (ऊँचाई) = $\frac{\sqrt{3}}{2} \times a$

(iii) Area (क्षेत्रफल) = $\frac{\sqrt{3}}{4} \times a^2$

$$\text{Area} = \frac{h^2}{\sqrt{3}}$$

(iv) $r = \frac{a}{2\sqrt{3}}$

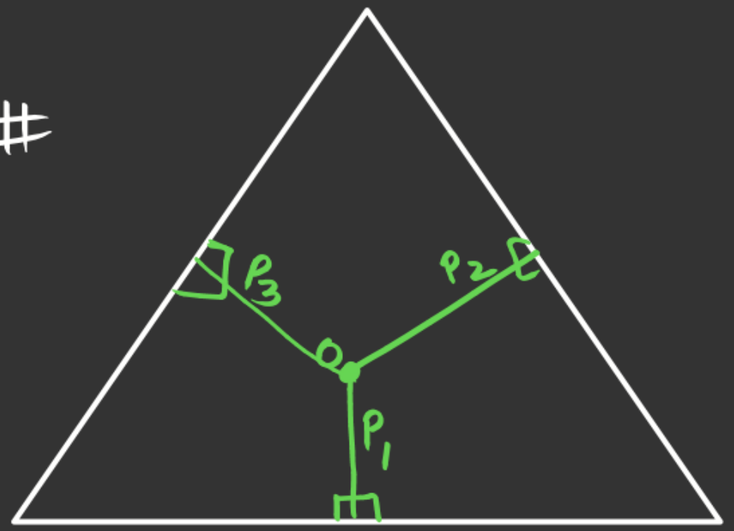
(v) $R = \frac{a}{\sqrt{3}}$

(vi) $r : R$

$1 : 2$

Area $1 : 4$

#



समबाहु Δ के अंदर किसी भी बिन्दु से तिनो भुजा पर लम्बडाल दिया जाए :-

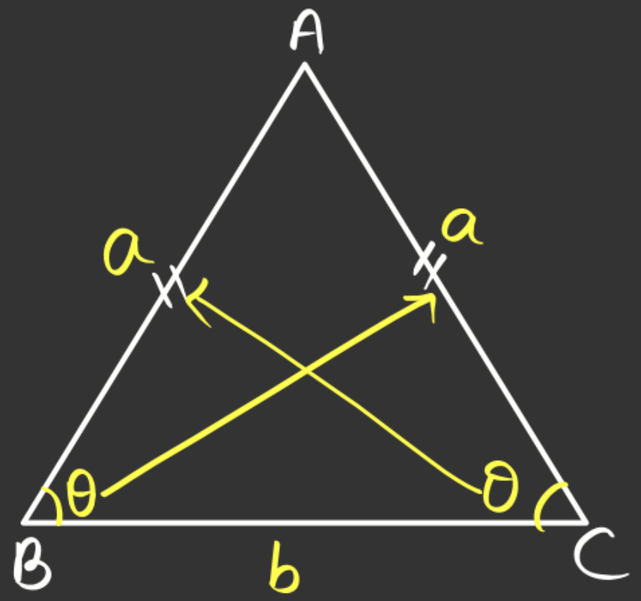
(i) $P_1 + P_2 + P_3 = h$

तिनों लम्बो का योग ऊँचाई के बराबर होती हैं।

(ii) $\text{Side}(a) = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$

(iii) क्षेत्र (Area) $= \frac{(P_1 + P_2 + P_3)^2}{\sqrt{3}} = \frac{h^2}{\sqrt{3}}$

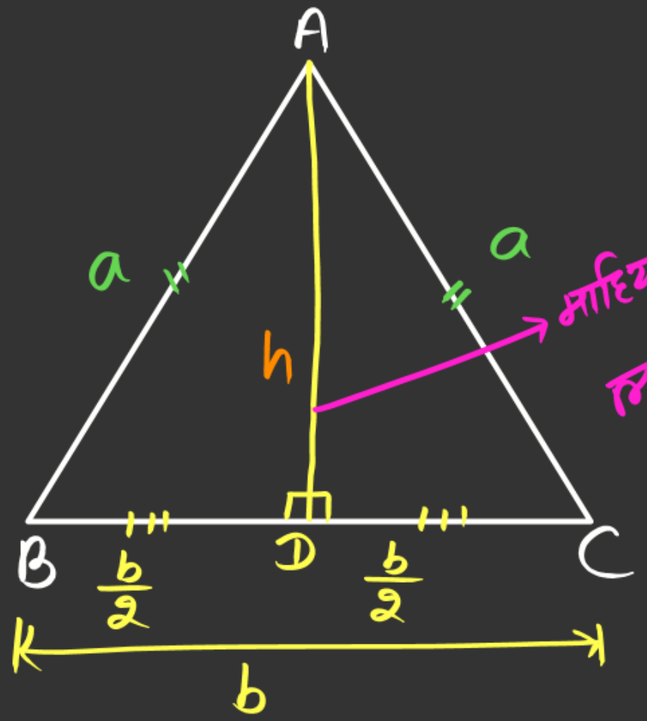
Isoceles Δ (समद्विबाहु Δ)



(i) perimeter (परिमाप) = $2a+b$

(ii) height (ऊँचाई) = $\frac{1}{2}\sqrt{4a^2-b^2}$

(iii) Area = $\frac{b}{4}\sqrt{4a^2-b^2}$



माध्यिका | ऊँचाई | कोण समद्विभाजक | लम्ब समद्विभाजक

$h = \sqrt{a^2-\left(\frac{b}{2}\right)^2}$

$h = \sqrt{a^2-\frac{b^2}{4}}$

$h = \sqrt{\frac{4a^2-b^2}{4}}$

$h = \frac{1}{2}\sqrt{4a^2-b^2}$