Time and Work

1. If A can finish a work in 't' days, then
   1 day's work by A = 1/t part.
2. Let an employee named M can finish a task in x days, and another employee named N can finish the same task in y days. If both employees start working together then they can complete the task in days
   $$\frac{xy}{x+y} \text{ days.}$$

Here, their combined 1-day work will be (1/x + 1/y) part.

3. Let two employees M and N together can finish a task in x days. If only M works alone, he can finish the task in y days then employee N can finish the same task alone in days-
   $$\frac{xy}{y-x} \text{ days.}$$

The Concept of Negative Work

Suppose, that A and B are working to build a wall while C is working to break the wall. In such a case, the wall is being built by A and B while it is being broken by C. Here, if we consider the work as the building of the wall, we can say that C is doing negative work.

Example: A can build a wall in 10 days and B can build it in 5 days, while C can completely destroy the wall in 20 days. If they start working at the same time, in how many days will the work be completed.

Solution: The net combined work per day here is: A's work + B's work – C's work = 10% + 20% – 5% = 25% work in one day.

Hence, the work will get completed (100% work) in 4 days.

WORK EQUIVALENCE METHOD

The work equivalence method is nothing but an application of the formula:

Work rate x Time = Work done (or work to be done)

Thus, if the work to be done is doubled, the product of work rate x time also has to be doubled. Similarly, if the work to be done increases by 20%, the product of work rate x time also has to be increased by 20% and so on.

Example: A contractor estimates that he will finish the road construction project in 100 days by employing 50 men. However, at the end of the 50th day, when as per his estimation half the work should have been completed, he finds that only 40% of his work is done.

(a) How many more days will be required to complete the work?

(b) How many more men should he employ in order to complete the work in time?

Solution: (a) The contactor has completed 40% of the work in 50 days. If the number of men working on the project remains constant, the rate of work also remains constant. Hence, to complete 100% work, he will have to complete the remaining 60% of the work. For this he would require 75 more days.

(b) In order to complete the work on time, it is obvious that he will have to increase the number of men working on the project.

This can be solved as: 50 men working for 50 days i.e 50 x 50 = 2500 man-days.

2500 man-days have resulted in 40% work completion.

Hence, the total work to be done in terms of the number of man-days is got by using unitary method:

Work left = 60% = 2500 x 1.5 = 3750 man-days

This has to be completed in 50 days. hence, the number of men required per day is 3750/50 = 75 men. Since, 50 men are already working.

The Specific Case of Building a Wall

As already mentioned, in certain cases, the unit of work can also be considered to be in terms of the volume of work. For example, building of a wall of a certain length, breadth and height.

In such cases, the following formula applies:

$$L_1B_1H_1/L_2B_2H_2 = m_1t_1d_1/m_2d_2t_2$$

where L, B and H are respectively, the length, breadth and height of the wall to be built, while m, t and d, are respectively the number of men, the amount of time per day and the number of days. Further, the suffix 1 is for the first work situation, while the suffix 2 is for the second work situation.

The Concept of Efficiency

The concept of efficiency is closely related to the concept of work rate. When we make a statement saying A is twice as efficient as B, we mean to say that A does twice the work as B in the same time. In other words, we can also understand this as A will require half the time required by B to do the same work.

Note

$$\text{Efficiency} \propto \frac{1}{\text{number of time units}}$$

$\therefore$ Efficiency × time = constant work

Hence, Required time = $\frac{\text{work}}{\text{efficiency}}$

Whole work is always considered as 1, in terms of fraction and 100%, in terms of percentage.

In general, number of days or hours = $\frac{100}{\text{efficiency}}$

Solved Examples

Q 1. When they work alone, B needs 25% more time to finish a job than A does. They two finish the job in 13 days in the following manner: A works alone till half the job is done, then A and B work together for four days, and finally B works alone to complete the remaining 5% of the job. In how many days can B alone finish the entire job?

(a). 16
(b). 22
(c). 20
(d). 18

Answer: c

Explanation

It's given that B needs 25% more time than A
So, if Time taken by A = 4x days, Time taken by B = 5x days
A does half the work, So A works alone for 2x days --- (1)
A and B work for 4 days --- (2)
B works alone for 5x/20 = x/4 days --- (3)
Total working days = 13
(1) + (2) + (3) = 13
2x + 4 + x/4 = 13 we get, x = 4
So, B alone can do the work in 5 × 4 = 20 days

Q 2. Two friends A and B simultaneously start running around a circular track. They run in the same direction. A travel at 6 m/s and B runs at b m/s. If they cross each other at exactly two points on the circular track and b is a natural number less than 30, how many values can b take?

(a). 3
(b). 4
(c). 7
(d). 5

Answer: a

Explanation

Let track length be equal to T.
Time taken to meet for the first time = T / relative speed = t/6-b or t/b-6
Time taken for a lap for A = t/6
Time taken for a lap for B = t/b
So, time taken to meet for the first time at the starting point = LCM (t/6, t/b) =
t/ HCF (6, b)
Number of meeting points on the track = Time taken to meet at starting point/Time taken for the first meeting = Relative speed / HCF (6, b).
So, in essence we have to find values for b such that 6-b/ HCF (6, b) = 2 or, b-6 / HCF (6,6) = 2
b = 2, 10, 18 satisfy this equation. So, there are three different values that b can take.
Hence, the answer is 3.

Q 3. A and B together can dive a trench in 12 days, which an alone can dive in 30 days. In how long B alone can burrow it?

(a). 18 days
(b). 19 days
(c). 20 days
(d). 21 days

Answer - c

Explanation

(A+B)'s 1 day work = 1/12, A's 1 day work =1/30
∴B's 1 day work = (1/12-1/30) = 3/60 = 1/20
Henceforth, B alone can dive the trench in 20 days.

Q 4. A can do a bit of work in 25 days which B can complete in 20 days. Both together labour for 5 days and afterward A leaves off. How long will B take to complete the remaining work?

(a). 7 days
(b). 8 days
(c). 9 days
(d). 11 days

Answer - d

Explanation

(A+B)'s 5 days' work = 5(1/25+1/20) = (5*9/100) = 9/20
Remaining work = (1-9/20) = 11/20
1/20 work is finished by B in 1 day
11/20 work is finished by B in (1*20*11/20) = 11 days

Q 5. A and B can do a bit of work in 12 days. B and C can do it in 15 days while C and A can do it in 20 days. In how long will they complete it cooperating? Additionally, in how long can A alone do it?

(a). 10 days, 30 days.
(b). 15 days, 20 days.
(c). 20 days, 40 days.
(d). 10 days, 50 days.

Answer - a

Explanation

(A+B)'s 1 day work = 1/12,
(B+C)'s 1 day work = 1/15,
(C+A)'S 1 day work = 1/20
Including: 2(A+B+C)'s 1 day work = (1/12 + 1/15 + 1/20) = 12/60 = 1/5
∴ (A+B+C) `s 1 day work = (1/2 *1/5) = 1/10
∴ working together they can complete the work in 10 days.
A's 1 day work = (1/10-1/15) = 1/30, B`s 1 day work = (1/10-1/20) = 1/20, C's 1 day work = (1/10-1/12) = 1/60
∴ A alone can take the necessary steps in 30 days.

Q 6. Will Q take more than 8 hours to complete the job alone?

(i). P works faster than Q.
(ii). P and Q can together finish the job in 5 hours.

(a). I alone sufficient while II alone not sufficient to answer the question
(b). II alone sufficient while I alone not sufficient to answer the question
(c). Both I and II are necessary to answer the question
(d). Both I and II are not sufficient to answer the question

Answer: c

Explanation

From II, we can conclude that if P and Q worked with equal efficiency, each of them alone would do the job in 10 hours. But according to I, Q is slower than P. So, Q alone would take more than 10 hours to complete the job.
Thus, both I and II together are necessary to get the answer.

Q 7. In how many days can Mohan alone complete the work?
(i). Mohan and Prakash together can complete the work in 17 days.
(ii). Rakesh works double as fast as Mohan and can alone complete the work in 10 days.
(a). I alone sufficient while II alone not sufficient to answer the question
(b). II alone sufficient while I alone not sufficient to answer the question
(c). Both I and II are necessary to answer the question
(d). Both I and II are not sufficient to answer the question

Answer: b

Explanation

From II, it is clear that Mohan alone takes double the time as taken by Rakesh alone to do the work i.e., 20 days.
I. is insufficient.
Thus, II alone gives the answer.

Q 8. In how many days can B alone complete the work?
(i). B and C together can complete the work in 8 days.
(ii). A and B together can complete the work in 12 days.
(a). I alone sufficient while II alone not sufficient to answer the question
(b). II alone sufficient while I alone not sufficient to answer the question
(c). Both I and II are necessary to answer the question
(d). Both I and II are not sufficient to answer the question

Answer: d

Explanation

I. gives, (B + C)'s 1 day's work = $\frac{1}{8}$ ...(i)

II. gives, (A + B)'s 1 day's work = $\frac{1}{12}$ ...(ii)

We cannot find B's 1 day's work using (i) and (ii). Thus, both I and II together are not sufficient.

Q 9. How long will Machine Y, working alone, take to produce x candles?
(i). I. Machine X produces x candles in 5 minutes.
(ii). II. Machine X and Machine Y working at the same time produce x candles in 2 minutes.

I. gives, Machine X produces x/5 candles in 1 min.

II. gives, Machines X and Y produce x/2 candles in 1 min.

From I and II, Y produces $\left(\frac{x}{2}-\frac{x}{5}\right)$

$= \frac{3x}{10}$ candles in 1 min.

$= \frac{3x}{10}$ candles are produced by Y in 1 min.

X candles will be produced by Y in $\left(\frac{10}{3x}\times x\right)$ min = 10/3 min.

Thus, I and II both are necessary to get the answer.

Q 10. B alone can complete a work in 12 days. How many days will A, B and C together take to complete the work?
(i). A and B together can complete the work in 3 days.
(ii). B and C together can complete the work in 6 days.
(a). I alone sufficient while II alone not sufficient to answer the question
(b). II alone sufficient while I alone not sufficient to answer the question
(c). Both I and II are necessary to answer the question
(d). Both I and II are not sufficient to answer the question

Answer: c

Explanation

Given: B's 1 day's work = $\frac{1}{12}$

I. gives, (A + B)'s 1 day's work = $\frac{1}{3}$

$\Rightarrow$ A's 1 day's work = $\left(\frac{1}{3}-\frac{1}{12}\right)=\frac{3}{12}=\frac{1}{4}$

II. gives, (B+ C)'s 1 day's work = $\frac{1}{6}$

$\Rightarrow$ C's 1 day's work = $\left(\frac{1}{6}-\frac{1}{12}\right)=\frac{1}{12}$

(A+B+C)'s 1 day's work = $\left(\frac{1}{4}+\frac{1}{12}+\frac{1}{12}\right)\frac{5}{12}$

Hence, they all finish the work in $\frac{12}{5}=2\frac{2}{5}$ days.

Thus, I and II both are necessary to get the answer

Practice Questions

Q 1. **Rohit can do a work in 30 days while Mohit can do the same work in 24 days. In how many days, can they complete it by working together?**

(a). $13\frac{1}{3}$

(b). 21

(c). $18\frac{1}{2}$

(d). $17\frac{1}{4}$

Q 2. **A and B alone can do a job in 10 days and 15 days respectively. In how many days will A and B together finish the same job?**

(a). $5\frac{1}{2}$

(b). $7\frac{1}{3}$

(c). 6

(d). 9

Q 3. **Rina and Rupa can finish a piece of work in 3 days, while Rina alone can do it in 5 days. In how many days can Rupa do it alone?**

(a). $2\frac{1}{2}$

(b). $7\frac{1}{2}$

(c). 8

(d). $10\frac{1}{3}$

Q 4. **Ram and Shyam can together do a work in 6 days, while Ram can alone do it in 9 days. How much time will Shyam alone take to finish it?**

(a). 12

(b). 10

(c). 15

(d). 18

Q 5. **Three friends-Mati, Gati and Rati can do a piece of work in 10 days, 12 days and 15 days respectively. In how many days, will they finish it by working together?**

(a). 4

(b). 6

(c). 8

(d). 9

Q 6. **Suresh can do a carpet in 15 days and Ashok can do the same size carpet in 10 days. They work together for 3 days, then Suresh goes away. In how many days will Ashok finish the remaining work?**

(a). 4

(b). 5

(c). 6

(d). 3

Q 7. **A and B can finish a project in 16 days and 12 days respectively. A started the work and worked at it for 2 days. He was then joined by B. The total time to finish the project is**

(a). 6 days

(b). 7 days

(c). 8 days

(d). 9 days

Q 8. **Aditi can do $\frac{2}{3}$ of a certain work in 10 days and Bijeta can do $\frac{1}{4}$ of the same work in 3 days. In how many days can both finish the work together?**

(a). $7\frac{1}{3}$

(b). $6\frac{1}{3}$

(c). $8\frac{1}{2}$

(d). $7\frac{3}{4}$

Q 9. **Milan and Ria can do a work in 18 days, Ria and Taniya can do it in 24 days and Taniya and Milan can do it in 36 days. If Milan, Riya and Taniya work together, it will be finished in**

(a). 12 days

(b). 15 days

(c). 18 days

(d). 16 days

Q 10. **A is thrice as good a work man as B and together they finish the work in 15 days. In how many days will A alone finish the work?**

(a). 24

(b). 20

(c). 30

(d). 18

Solutions:

1. (a) Required time = $\frac{t_1 \times t_2}{t_1 + t_2} = \frac{30 \times 24}{30 + 24} = \frac{720}{54} = \frac{40}{3}$ days

2. (c) Required time: = $\frac{t_1 \times t_2}{t_1 + t_2} = \frac{10 \times 15}{10 + 15} = \frac{150}{25}$ = 6 days

3. (b) (Rina + Rupa)'s 1 day work = Rina's 1 day work+ Rupa's 1 day work

$\frac{1}{3} = \frac{1}{5}$ + Rupa's 1 day work

⇒Rupa's 1 day work: $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

Rupa can do the work alone in $\frac{15}{2} = 7\frac{1}{2}$ days

4. (d) (Ram and Shyam)'s 1 day work= work+ Shyam's 1 day work

$\frac{1}{6} = \frac{1}{9}$ + Shyam's 1 day work

⇒ Shyam's day work $\frac{1}{6} - \frac{1}{9} = \frac{1}{18}$

Shyam can do it alone in 18 days.

5. (a) Mati's 1 day work = $\frac{1}{10}$

Gati's 1 day work = $\frac{1}{12}$

Rati's 1 day work = $\frac{1}{15}$

(Mati + Gati + Rati)'s 1 day work = $\frac{1}{10} + \frac{1}{12} + \frac{1}{15}$

$= \frac{18 + 15 + 12}{180}$

$= \frac{45}{180} = \frac{1}{4}$

⇒ Mati+ Gati + Rati can finish the work in 4 days.

6. (b) (Suresh + Ashok)'s 1 day work = $\frac{1}{15} + \frac{1}{10} = \frac{1}{6}$

(Suresh + Ashok)'s 3 days' work $3 \times \frac{1}{6} = \frac{1}{2}$

Anaining work = $1 - \frac{1}{2} = \frac{1}{2}$ which is to be done by Ashok.

We know, Ashok does 1 work in 10 days;
⇒ Ashok does $\frac{1}{2}$ work in $\frac{1}{2} \times 10$ = 5 days

7. (c) Let = total time to finish, then
A's 2 days' work + (A + B)'s (1-2) days work =

$$2\left(\frac{1}{16}\right) + (t-2)\left(\frac{1}{16} + \frac{1}{12}\right) = 1$$

$$\frac{1}{8} + (t-2)\left(\frac{3+4}{48}\right) = 1$$

$$\Rightarrow (t-2)\left(\frac{7}{48}\right) = 1 - \frac{1}{8} = \frac{7}{8} \Rightarrow t - 2 = 6$$

⇒ t = 8 days

8. (a) Aditi can do 2/3 of work in 10 days
⇒ Aditi can do 1 work in $\frac{10 \times 3}{2}$ =15 days

Bijeta can do 1/4 of work in 3 days
⇒ Bijeta can do 1 work in $\frac{4 \times 3}{1}$ =12 days

⇒ (Aditi+ Bigeta)'s 1 day work = $\frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{3}{20}$

⇒ Aditi+ Bigeta can finish the work in $1 \div \frac{3}{20} = \frac{20}{3}$

9. (d) (Milan + Ria)'s 1 day work 1/18
(Ria + Taniya)'s 1 day work 1/24
(Taniya + Milan)'s 1 day work = 1/36
2(Milan + Ria + Taniya)'s 1 day work = $\frac{1}{18} + \frac{1}{24} + \frac{1}{36} = \frac{1}{8}$

Milan + Ria + Taniya's 1 day work = $\frac{1}{2 \times 8} = \frac{1}{16}$

Milan + Ria + Taniya can do the work in 16 days.

10. (b) Let A alone do the work in '7' days, then
B will do it alone in '37' days. Then.

⇒ $\frac{1}{t} + \frac{1}{3t} = \frac{1}{15}$ (A + B)'s 1 day work

⇒ $\frac{4}{3t} + \frac{1}{15}$ ⇒ t = 20 days