Practice questions

(1) Reduce the following matrix to a row reduced echelon form and hence find its rank:

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$$

IAS 2021
10M

$\boxed{\text{Ans: } r(A) = 3}$

(2) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then show that $A^2 = A^{-1}$ (without finding A^{-1})

IAS 2021
10M

Hint: show $A^3 = I_3$

$\therefore A^{-1} = A^2$

$$\boxed{\text{Ans: } A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}}$$

(3) Define an $n \times n$ matrix as $A = I - 2uu^T$, where u is a unit column vector.

(i) Examine if A is symmetric.
(ii) Examine if A is orthogonal.
(iii) Show that trace $(A) = n-2$.

(iv) Find $A_{3\times3}$, when $u = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$

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20 M

Ans: (iv) $A = \begin{bmatrix} 7/9 & -4/9 & -4/9 \\ -4/9 & \frac{1}{9} & -8/9 \\ -4/9 & -8/9 & \frac{1}{9} \end{bmatrix}$

(4.) state the cayley-Hamilton theorem. use this theorem to find A^{100}, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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15 M

Hint: A satisfies its ch. eqn.

so $A^3 = A^2 + A - I$

From this set $A^4, A^5, A^6, -- A^{10}$ in terms of A^2 & I. Now you can deduce by seeing pattern,

$A^{100} = 50A^2 - 49I$

Ans: $A^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$

(5) Let A and B be two orthogonal matrices of same order and det A + det B = 0. Show that A+B is a singular matrix.

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Hint: $A \cdot A^T = A^T \cdot A = I$, $B \cdot B^T = B^T \cdot B = I$

$|A \cdot A^T| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$ | $\text{Illy } |B| = \pm 1$

$|A| + |B| = 0 \Rightarrow |A| = -|B|$

$|AB| = |A| \cdot |B| = -|A|^2 = -1$ —— (1)

$A+B = AI + B \cdot I = AI + IB$
$= AB^TB + AA^TB$
$= A(B^T + A^T)B$

$\therefore |A+B| = |A(A^T+B^T)B|$
$= |AB||A+B| = -|A+B|$

using (1)

$\Rightarrow 2|A+B| = 0$

(6) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$ then show that $AB = 6I_3$. Use this result to solve the following system of equations:

$$2x + y + z = 5$$
$$x - y = 0$$
$$2x + y - z = 1$$

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10M

Hint:
$$AB = 6I$$
$$\Rightarrow B^{-1} = \frac{1}{6}A$$

Sol^n. is $B^{-1}\cdot\begin{bmatrix}5\\0\\1\end{bmatrix} = \frac{1}{6}A\cdot\begin{bmatrix}5\\0\\1\end{bmatrix}$

$\boxed{\text{Ans: } x=1,\ y=1,\ z=2}$

(7) For the system of linear equations

$$x+3y-2z=-1$$
$$5y+3z=-8$$
$$x-2y-5z=7$$

Determine which of the following statements are true and which are false.

(i) The system has no sol^n.
(ii) The system has a unique sol^n.
(iii) The system has infinitely many sol^n.

IAS 2018
13 M

Ans: (iii) true
(i) & (ii) False

(8) Let A be a 3×2 matrix and B a 2×3 matrix. Show that $C = AB$ is a singular matrix.

IAS 2017
10 M

Hint: $r(AB) \le \min\{r(A), r(B)\}$.

(9) Consider the following system of equations in x, y, z:

$$x + 2y + 2z = 1$$
$$x + ay + 3z = 3$$
$$x + 11y + az = b$$

(i) For which values of 'a' does the system have a unique solution?

(ii) For which pair of values (a, b) does the system have more than one solution?

IAS 2017
15 M

Ans: (i) $a \in \mathbb{R} \sim \{-1, 5\}$

(ii) $(-1, -5), (5, 7)$.

(10) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the eigenvalues and eigen vectors of A.

IAS 2016
8 M

Ans: $\lambda = 0, 1, 2$

$k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}; k_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; k_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(11) using elementary row operations; find the condition that the linear equations

$$x - 2y + z = a$$
$$2x + 7y - 3z = b$$
$$3x + 5y - 2z = c$$

have a solution.

IAS 2016

7M

$\boxed{\text{Ans: } c = a + b}$

(12) If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then find $A^{14} + 3A - 2I$.

IAS 2016

4M

Ans: $\begin{bmatrix} 1 & 3 & 9 \\ 15 & 4 & 8 \\ -6 & -3 & -11 \end{bmatrix}$

(13) using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

IAS 2016

6 M

Ans: $\begin{bmatrix} \frac{3}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{2} & 1 & \frac{1}{2} \end{bmatrix}$

(14) Prob: Reduce the following matrix to row echelon form and hence find its rank:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$

IAS 2015

10M

$\boxed{\text{Ans: } 2}$

(15) If matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then find A^{30}.

IAS 2015

12 M

$$\boxed{\text{Ans: } \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 15 & 0 & 1 \end{bmatrix}}$$

(16) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

IAS 2015

12 M

$\boxed{\text{Ans: } \lambda = -2, 3, 6}$

$$\boxed{k_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}; k_2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}; k_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}.$$