

09. Find out the unit digit in product of all prime numbers in between 1 and 99999.

1 से 99999 के बीच की सभी अभाज्य संख्याओं के गुणनफल में इकाई का अंक ज्ञात करो -

(a) 9

(b) 7

(c) 0

(d) Not

$$2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 29 \dots$$

$$= 0$$

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(d) Not

$$\textcircled{i} (1, 0, 5, 6)^n \xrightarrow{U.D.} \text{No change}$$

$$\textcircled{ii} (2, 3, 7, 8, 4, 9)^n \xrightarrow{R} \boxed{R=}$$

$$\textcircled{iii} \xrightarrow{R} \boxed{R=0}$$

$$\textcircled{i} 5 \times \text{सम} = 0$$

$$\textcircled{ii} 5 \times \text{विषम} = 5$$

$$\textcircled{i} \text{odd}^4 = 1$$

$$\textcircled{ii} \text{Even}^4 = 6$$

$(2, 3, 7, 8)$   $\sqrt{4, 9}$

Note:  $\rightarrow$  किसी भी संख्या का स्क्वार्ड अंक हमेशा  $(4 \text{ power})$  के बाद Repeat होगा है।

Rem  $\rightarrow$   $\textcircled{1}$   $\textcircled{1}$   $\textcircled{1}$   $\textcircled{1}$   $\textcircled{1}$   $\textcircled{1}$

#  $\frac{45}{14} + \frac{45}{24} + \frac{45}{34} + \frac{45}{44} + \frac{45}{54} + \dots + \frac{45}{34}$  का रिकार्ड का संक

$$1^1 + 2^1 + 3^1 + 4^1 + 5^1 + \dots + 34^1$$

$$1 + 2 + 3 + 4 + 5 + \dots + 34 = \frac{n(n+1)}{2}$$

$$= \frac{17^2 \times 35}{2}$$

$$= 17 \times 35$$

$$= \textcircled{5}$$

$$\# \quad \overset{\textcircled{2}}{1} \frac{62}{4} + \overset{\textcircled{2}}{2} \frac{62}{4} + \overset{\textcircled{2}}{3} \frac{62}{4} + \overset{\textcircled{2}}{4} \frac{62}{4} + \dots + \overset{\textcircled{2}}{21} \frac{62}{4}$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + 21^2 = \frac{n(n+1)(2n+1)}{6}$$

$$7 \times 11 \times 43 \xrightarrow{U.D} \underline{\underline{1 \text{ Ans}}}$$

$\textcircled{7} \quad \textcircled{21}$

$$= \frac{21 \times (21+1) \times (21 \times 2 + 1)}{6}$$

$$= \frac{7 \times 11 \times 6}{\cancel{21} \times \cancel{22} \times 43}$$

$$= \frac{\cancel{62}}{\cancel{62}}$$

$$= 7 \times 11 \times 43$$

R → ②

$$\# \quad \frac{74}{1^4} + 2^{74} + 3^{74} + 4^{74} + \dots + 50^{74}$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + 50^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{50(50+1)(50 \times 2 + 1)}{6}$$

$$= \frac{25 \quad 17 \quad 6}{50 \times 51 \times 101}$$

$$= \frac{25 \times 17 \times 101}{63} \xrightarrow{V.D.} \textcircled{5}$$

③

$$\# \frac{63}{1^4} + 2^{63} + 3^{63} + 4^{63} + 5^{63} + \dots + 52^{63}$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \dots + 52^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

$$= \left[ \frac{52 \times 53}{2} \right]^2$$

$$= 8^2 = 64 \text{ U.D.} = \underline{\underline{4 \text{ Ans.}}}$$

$$\# \quad \boxed{0} \quad \boxed{0} \\ \frac{1^{80}}{4} + 2^{\frac{80}{4}} + 3^{80} + 4^{80} + 5^{80} + 6^{80} + 7^{80} + 8^{80} + 9^{80} + 10^{80}$$

$$1^4 + 2^4 + 3^4 + 4^4 + \textcircled{5^4} + \textcircled{6^4} + 7^4 + 8^4 + 9^4 + \textcircled{10^4}$$

$$1 + 6 + 1 + 6 + 5 + 6 + 1 + 6 + 1 + 0 = 3 \boxed{3}$$

$$1^4 + 2^4 + 3^4 + \dots + 10^4 \xrightarrow{\text{U.D.}} \textcircled{3}$$

$$1^{64} + 2^{64} + 3^{64} + \dots + 50^{64} + 51^{64} + 52^{64}$$

$\underbrace{\hspace{15em}}_{3 \times 5 + 1 + 6}$

$$52^{64} - 2^4$$

$$1^{64} + 2^{64} + 3^{64} + \dots + 10^{64} + 11^{64} + \dots + 20^{64} + 21^{64} + \dots + 30^{64} + 31^{64} + \dots + 40^{64} + \dots + 50^{64} + 51^{64}$$

$\underbrace{\hspace{15em}}_{(3)}$ 
 $\underbrace{\hspace{5em}}_{(3)}$ 
 $\underbrace{\hspace{5em}}_{(3)}$ 
 $\underbrace{\hspace{5em}}_{(3)}$ 
 $\underbrace{\hspace{5em}}_{(3)}$ 
 $\underbrace{\hspace{5em}}_{(3)}$ 
 $+ 1$

$$16 \xrightarrow{U.D} (6) + 6$$

$$1 \overset{64}{+} \dots \overset{64}{+} 50 \overset{64}{+} 51$$

$$5 \times 3 + 1$$
$$15 + 1 = 16$$

□

#  $1^2 + 2^2 + 3^2 + \dots + 73^2$

$1^4 + 2^4 + \dots + 70^4 + 71^4 + 72^4 + 73^4$

$7 \times 3 + 1 + 6 + 1$   
 $21 + 8 = 29$

$1^4 + 2^4 + \dots + 10^4$   
 V.D = 3

$$\boxed{0} \quad \frac{124}{1^4} + 2^{124} + 3^{124} + \dots + 88^{124} \quad 9^4$$

$$1^4 + 2^4 + 3^4 + \dots + 88^4 + 89^4 + 90^4 - 89^4 - 90^4$$

-1 -0

सिवाय = 1

$$9 \times 3 = \textcircled{7}$$

$$7 - 1 - 0 = \textcircled{6} \quad \text{U.D}$$

$$\boxed{0} \\ \frac{8}{1^4} + 2^8 + 3^8 + \dots + 97^8$$

$$1^4 + 2^4 + 3^4 + \dots + 97^4 + 98^4 + 99^4 + 100^4 - 98^4 - 99^4 - 100^4$$

$$10 \times 3 = \boxed{30}$$

$$30 - 6 - 1 - 0 \\ = 23 \\ \text{U.D.} = 3$$

$$1^4 + \dots \times 10^4 \\ \textcircled{3}$$

10. What is the unit digit of  $1^5 + 2^5 + 3^5 + \dots + 30^5$ —

$1^5 + 2^5 + 3^5 + \dots + 30^5$  इकाई अंक ज्ञात करो—

(a) 5  
(c) 2

(b) 1  
(d) 4

$$1 + 2 + 3 + \dots + 30 = \frac{n(n+1)}{2} = \frac{30 \times 31}{2} = 465$$

11. What is the unit digit of the sum of first 111 whole numbers—

प्रथम 111 पूर्ण संख्याओं के योग का इकाई अंक है—

(a) 4

(b) 6

(c) 5

(d) 0

0

0, 1, 2, 3, ..., 110

$$\frac{n(n+1)}{2} = \frac{111 \times 112}{2} = 5$$

12. If  $x = (164)^{169} + (333)^{337} - (727)^{726}$ , then what is the unit digit of  $x$ —

यदि  $x = (164)^{169} + (333)^{337} - (727)^{726}$  है, तो  $x$  का इकाई अंक क्या है—

(a) 5

(b) 7

(c) 8

(d) 9

 $R=1$  $169$  $164^4$  $R=1$  $337$  $333^4$  $R=2$  $726$  $727^4$ 

$$4^1 + 3^1 - 7^2$$

$$7 - 9 = -2$$

$$10 - 2$$

$$= 8$$

13. Let  $x = (633)^{24} - (277)^{38} + (266)^{54}$  What is the unit digit of  $x$ ? 0

यदि  $x = (633)^{\frac{24}{4}} - (277)^{\frac{38}{4}} + (266)^{54}$  है, तो का इकाई अंक क्या है ? 2

(a) 7

(b) 6

(c) 4

(d) 8

$$3^4 - 7^2 + 6$$

$$1 - 9 + 6 = -2$$

$$10 - 2 = 8$$

14.  $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 8^{9^{10}} = \text{Unit digit/ इकाई अंक}$ —

(a) 4      (b) 0      (c) 2      (d) 9

$$5 \times 2 = 0$$

# Factorial (क्रमगुणित)

$n$ ,  $n!$

$$\underline{0} = 1$$

$$\underline{1} = 1$$

$$\underline{2} = 2 \times 1 = 2$$

$$\underline{3} = 3 \times 2 \times 1 = 6$$

$$\underline{4} = 4 \times 3 \times 2 \times 1 = 24$$

$$\underline{5} = 5 \times 4 \times 3 \times 2 \times 1 = 120 \checkmark$$

$$\underline{6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \checkmark$$

$$\underline{7} = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 \checkmark$$

$$\underline{4} (+) \xrightarrow{V.D.} 0$$

15. Find out the unit digit in  $\underline{12}$ —

$\underline{12}$  में इकाई का अंक ज्ञात करो—

(a) 0

(b) 2

(c) 4

(d) 5

$$25! \xrightarrow{U.D.} 0$$

$$4! \oplus \boxed{U.D.=0}$$