Chapter 03 Kinematics

CONTENT

1. KINEMATICS:

Study of motion of the objects without taking into account the factors which cause the motion or change in motion (i.e. nature of force).

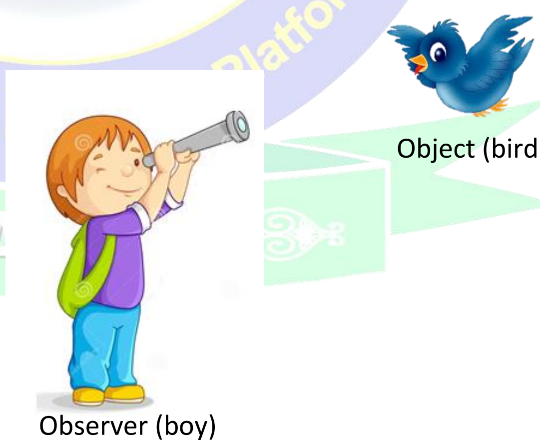
1.1 Object:

Which is studied or simply the body chosen for study / observation is called object.

1.2 Observer:

Who studies (distance, displacement, speed, velocity, acceleration or anything) or simply who notices some change in physical quantities is called observer.

Example: A bird is flying as shown in figure and a boy is observing the motion of bird then bird is said to be object for observer (boy) and boy is said to be observer for the object (bird).

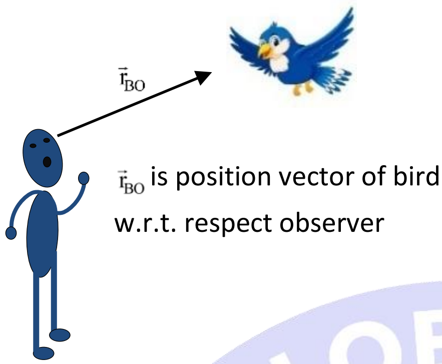


DETECTIVE MIND

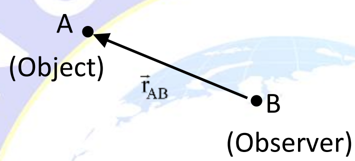
Object and observer both may be living or non-living things.

1.3 Position vector $(\vec{r})$:

It is a vector quantity used to show the position of an object with respect to an observer.



➢ Position of a point A (object) w.r.t. point B (observer) is represented as follows.

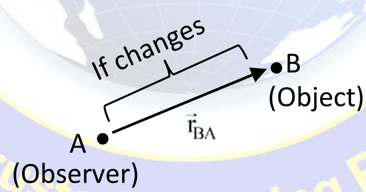


1.4 Concept of Point Object :

An object having its dimensions very small (negligible volume) as compared to the distances in consideration, will be considered as a point object.

1.5 Rest & Motion:

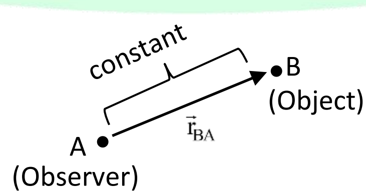
If position of an object w.r.t. an observer changes with time the object is said to be in motion for that observer. Also, if the position of an object w.r.t. the observer does not change with time, the object is said to be at rest for that observer.



For example:

If $\frac{d\vec{r}_{BA}}{dt} = 0$ i.e. if $\vec{r}_{BA}$ doesn't change with time 'B' is said to be at rest for 'A'

and if $\frac{d(\vec{r}_{BA})}{dt} \neq 0$ i.e. $\vec{r}_{BA}$ changes with time 'B' is said to be in motion for A.



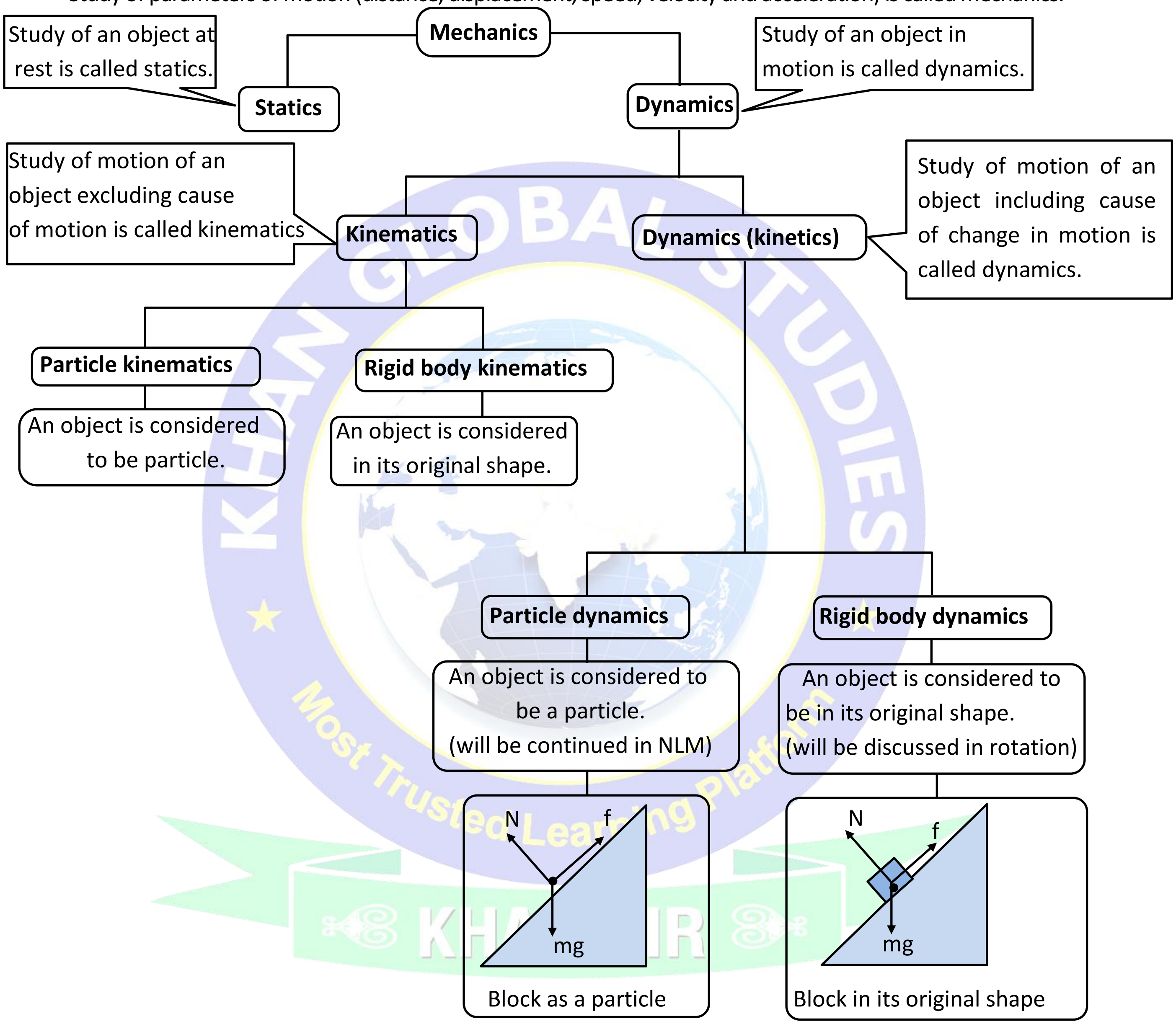
➢ A particle at rest with respect to an observer can be in motion with respect to another observer.

➢ A particle in motion with respect to an observer can be at rest with respect to another observer.

DETECTIVE MIND
Rest and Motion are relative terms, there is nothing like absolute rest or absolute motion in the universe.

1.6 MACHENICS:

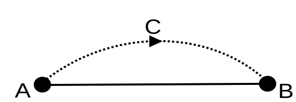
Study of parameters of motion (distance, displacement, speed, velocity and acceleration) is called mechanics.



PARTICLES KINEMATICS:

2. DISTANCE AND DISPLACEMENT :

2.1 Distance Covered :



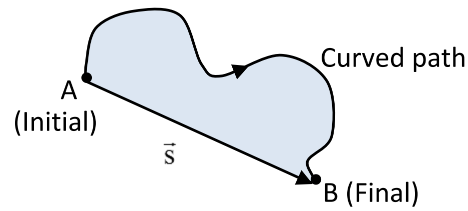
- It is the length of actual path followed by the object. Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.
- Since it is length and negative length has no significance, it is never negative.
- Since length a scalar, it is a scalar.

- During a motion it never decreases but it may be constant.
- SI unit: metre.
- CGS unit: centimeter.
- Dimension: $M^0L T^0$

2.2 Displacement:

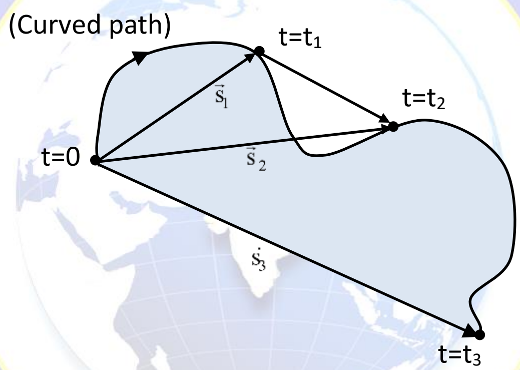
- It is the position vector of final position w.r.t. initial position.

For example: A fly is flying from point A to B then its displacement $\vec{s}$ can be represented as follows



DETECTIVE MIND

For a motion initial position i.e. position from where motion starts is one but number of final position may be infinite as shown in below figure.



- A particle is moving on a curved path as shown in the above figure. The observer starts the observation of motion of the object at t = 0 which is initial position but its final position at different time is different.

Final position of particle at $t = t_1$ is at $t = t_1$, (i.e. $\vec{s}_1$)

Final position at $t = t_2$ is at $t = t_2$, (i.e. $\vec{s}_2$)

and final position at $t = t_3$ is at $t = t_3$, (i.e $\vec{s}_3$)

SPOT LIGHT

- The time, t = 0 is not the time from where object starts the motion but it is the time from where observer starts the observation of motion of the object.

2.3 Displacement In Terms Of Position Vector:

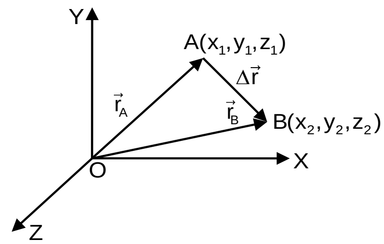
Let a body is displaced from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ then its

displacement is given by vector $\vec{AB} = \Delta\vec{r}$

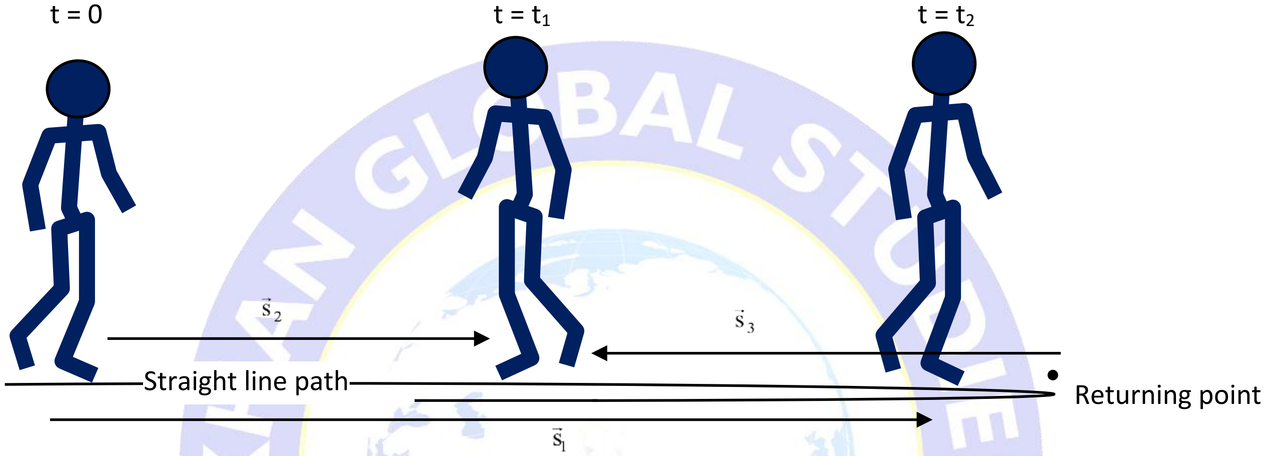
$\because \quad \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\therefore \quad \Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

i.e. displacement is equal to change in position vector.



2.4 How to draw displacement vector :



A man is walking on a straight line path and returning after some time. The observation of motion of man starts at time t = 0. So, it displacement vector at $t = t_1$ is $\vec{s}_2$ but not the $\vec{S}_3$ and displacement at time $t = t_2$ is $\vec{s}_1$

- Magnitude of displacement is equal to distance between initial position and final position.
- If direction of motion of changes then magnitude of displacement can never be greater than distance covered.
- if object move on straight line path and does not reverse the direction of motion. The magnitude of displacement will be same as distance covered.

SPOT LIGHT

- If a vector changes its direction randomly, it cannot be shown on the graph as its direction cannot be distinguished on the graph. However, if the vector is along straight line (unidirectional) it can be shown on the graph as its two different directions can be distinguished on the graph with positive and negative sign.

Distance (s)	Displacement ($\vec{s}$)
It is a scalar quantity	It is a vector quantity
It depends upon path followed	It depends upon only initial and final position
Distance can never be negative and zero	It can be negative, positive and zero
It may increase or remain constant but never decrease with the time	Its magnitude may increase or decrease with time

Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.	Only single value of displacement is possible between two fixed points.

DETECTIVE MIND

Magnitude of displacement may be equal or less than distance but never be greater than distance.

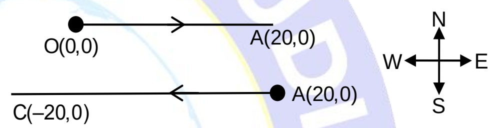
i.e., distance ≥ |displacement| & $\frac{Distance}{|displacement|} \geq 1$

SOLVED EXAMPLES

Example 1: A particle starts from the origin, goes along the X-axis to the point (20 m, 0) and then returns along the same line to the point (–20 m, 0). Find the distance and displacement of the particle during the trip.

(1) $20, -20\hat{i}$ (2) $60, -20\hat{i}$ (3) $40, -10\hat{i}$ (4) $40, -20\hat{i}$

Solution: Distance = $|\overrightarrow{OA}| + |\overrightarrow{OC}|$ = 20 + 40 = 60 m



$\Delta\vec{r}$ = Displacement = $|\overrightarrow{OA}| + |\overrightarrow{OC}|$

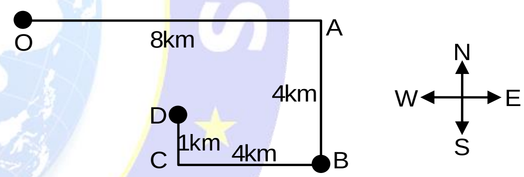
$= 20\hat{i} + (-40\hat{i}) = (-20\hat{i})$

Example 2: A car moves from O to D along the path OABCD shown in fig.

What is distance travelled and net displacement.

(1) 13km, 17km (2) 17km, 5km

(3) 14km, 8km (4) 17km, zero



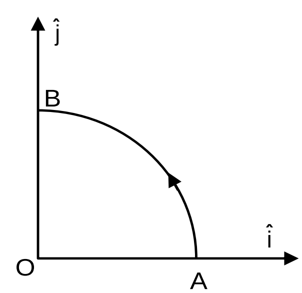
Solution: Distance = $|\overrightarrow{OA}| + |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CD}| = 8 + 4 + 4 + 1 = 17$ km

Displacement = $\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = 8\hat{i} + (-4\hat{j}) + (-4\hat{i}) + \hat{j} = 4\hat{i} - 3\hat{j}$

$\Rightarrow |\Delta\vec{r}|$ = |displacement| = $\sqrt{(4)^2 + (3)^2} = 5$

So, Displacement = 5 km, 37^0 S of E

Example 3: A particle goes along a quadrant from A to B of a circle radius 10 m as shown in fig. Find the magnitude of displacement and distance along path shown AB, and angle between displacement vector $\overrightarrow{AB}$ and x-axis?



(1) $10\sqrt{2}, 5\pi, 45°$ (2) $5\pi, 10\sqrt{2}, 30°$

(3) $10, 10\pi, 60°$ (4) $5\sqrt{2}, \frac{5}{\pi}, 45°$

Solution: Displacement = $\vec{AB} = \vec{OB} - \vec{OA} = 10\hat{j} - 10\hat{i} \Rightarrow |\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m

From $\Delta OBA \Rightarrow \tan\theta = \dfrac{OA}{OB} = \dfrac{10}{10} = 1 \Rightarrow \theta = 45°$

Angle between displacement vector $\vec{AB}$ and x-axis = 90° + 45° = 135°

Distance of path AB = $\dfrac{1}{4}$ (circumference) = $\dfrac{1}{4}$ $(2\pi R)$ m = (5π) m

Example 4: A body initially having position vector $\vec{r}_A = 2\hat{i} + \hat{j} + 4\hat{k}$ moves to $\vec{r}_B = 6\hat{i} + 9\hat{j} - 2\hat{k}$, then find the change in position vector of the body.

(1) $4\hat{i} + 8\hat{j} + 6\hat{k}$ (2) $2\hat{i} + 3\hat{j} + 5\hat{k}$

(3) $4\hat{i} + 8\hat{j} - 6\hat{k}$ (4) $2\hat{i} - 3\hat{j} + 5\hat{k}$

Solution: Since = $\Delta\vec{r} = \vec{r}_B - \vec{r}_A = (6\hat{i} + 9\hat{j} - 2\hat{k}) - (2\hat{i} + \hat{j} + 4\hat{k}) = 4\hat{i} + 8\hat{j} - 6\hat{k}$

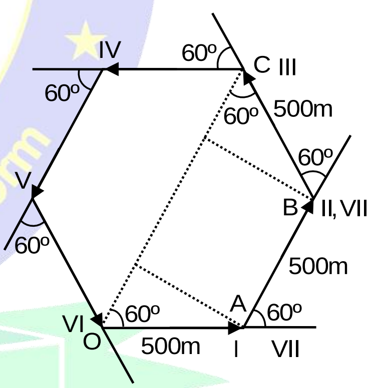
Example 5: On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Solution: At III turn

Displacement = $\vec{OA} + \vec{AB} + \vec{BC} = \vec{OC}$ = 500 cos 60° + 500 + 500 cos 60° = 1000 m from O to C

Distance = 500 + 500 + 500 = 1500 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{1000}{1500} = \dfrac{2}{3}$



At VI turn

$\because$ initial and final positions are same so

displacement = 0 and distance = 500 × 6 = 3000 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{0}{3000} = 0$

At VIII turn

Displacement = $2(500)\cos\left(\dfrac{60°}{2}\right)$ = 1000 × cos 30° = $1000 \times \dfrac{\sqrt{3}}{2} = 500\sqrt{3}$m

Distance = 500 × 8 = 4000 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{500\sqrt{3}}{4000} = \dfrac{\sqrt{3}}{8}$

Example 6: A particle moves on a circular path of radius 'r', It completes one revolution in 40 s. Calculate distance and displacement in 2 min 20 s.

(1) $22r, \sqrt{2}r$ (2) $11r, 2\sqrt{2}r$

(3) 22r , zero (4) zero , zero

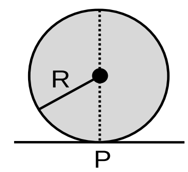
Solution: Radius of path = r

Time period for one revolution = 40 second total number of revolution = $\frac{140}{40} = \frac{7}{2}$

Now distance covered = $2\pi r \times \frac{7}{2} = \frac{44}{7} \times r \times \frac{7}{2} = 22r$

Displacement of particle in one revolution is zero so that displacement of particle = 2r.

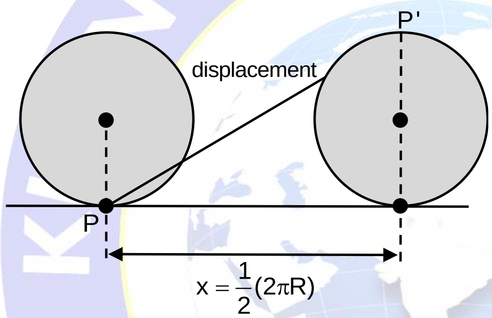
Example 7: A wheel of radius 'R' is placed on ground and its contact point is 'P'. If wheel rolls down without slipping completing half a revolution, find the displacement of point P.



(1) $R\sqrt{\pi^2 + 2}$ (2) $2R$

(3) $R2\sqrt{2}$ (4) $R\sqrt{\pi^2 + 4}$

Solution:



Displacement = $\sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4}$

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3. SPEED AND VELOCITY :

3.1 Speed: It is the rate of change of distance covered w.r.t. to time.

- It is a scalar quantity
- Dimension $[M^0L^1T^{-1}]$
- S.I: metre/second
- C.G.S unit: cm/s
- For a moving particle speed can never be negative or zero it is always positive.

3.2 Uniform Speed:

When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.

- Uniform speed = $\frac{\text{Distance}}{\text{Time}}$

3.3 Non-Uniform (Variable) Speed:

- In non-uniform speed particle covers unequal distances in equal intervals of time.

3.4 Average Speed:

The average speed of a particle for a given "interval of time" is defined as the ratio of total distance travelled to the time taken.

Average speed = $\dfrac{\text{Total Distance travelled}}{\text{Time taken}}$ i.e. $\boxed{v_{av} = \dfrac{\Delta s}{\Delta t}}$

- **When a particle moves with different uniform speeds** v_1, v_2, v_3 **........**v_n, in same time intervals t_1, t_2, t_3 **........**t_n, respectively, its average speed over the total time of journey is given as

  $v_{av} = \dfrac{v_1 + v_2 + v_3 + ...v_n}{n}$ (Arithmetic mean of speeds)

- When a particle describes same distances s_1, s_2, s_3..... s_n with speeds v_1, v_2, v_3.......v_n respectively then the average speed of particle over the total distance will be given as

  $v_{av} = \dfrac{n}{\dfrac{1}{v_1} + \dfrac{1}{v_2} + \dfrac{1}{v_3} + ... + \dfrac{1}{v_n}}$ (Harmonic mean of speeds)

3.5 Instantaneous Speed:

It is the speed of particle at a particular instant of time.

Instantaneous speed v = $\lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t}$ i.e $\boxed{v = \dfrac{ds}{dt}}$

DETECTIVE MIND
Instantaneous speed is simply called speed.

- If s is distance covered and v is instantaneous speed then $v = \dfrac{ds}{dt}$ also, ds = vdt

  $\therefore$ Total distance covered during t = t_1 to t = t_2, $\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt$

DETECTIVE MIND
Since distance covered never decreases so its rate of change w.r.t. time is never negative and so speed is never negative.

- A particle travels with speed v_1 for first half time and with speed v_2 for next half time then find average speed of the particle is arithematic mean of v_1 and v_2 i.e. $\dfrac{v_1 + v_2}{2}$

3.6 Velocity:

It is rate of change of displacement with respect to time is called velocity.

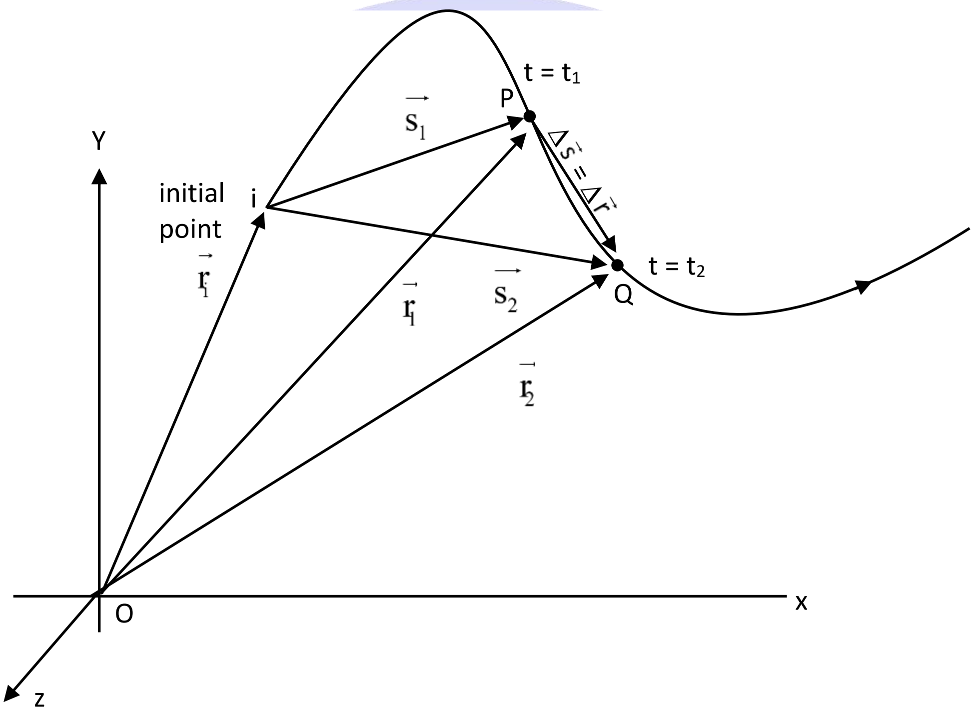
- It is a vector quantity.
- **Dimension:** $[M^0L^1T^{-1}]$
- S.I. unit: metre/second
- C.G.S. unit cm/s
- Motion of an object is reflected by its velocity.

SPOT LIGHT

- Displacement and position are two different vectors but change in position vector and change in displacement vector are always same so their rate (velocity) with respect to time is also same.

Proof:

Say a particle which has started motion from initial point 'i' on a curve path as shown in figure whose displacement at time $t=t_1$ is $\vec{s}_1$ and displacement at time t_2 is $\vec{s}_2$ and its corresponding position vector of initial position 'i' is $\vec{r}_i$, position vector at time $t = t_1$ is $\vec{r}_1$ and position vector at time $t= t_2$ is $\vec{r}_2$



$\Delta\vec{s}_2$ = change in position vector between the time interval $t = t_1$ to $t = t_2$

$\Delta\vec{r}_2$ = change in position vector between the time interval $t = t_1$ to $t = t_2$

Using triangle law of vector addition in triangle OPQ

$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2 \qquad \Rightarrow \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Again using triangle law of vector addition in triangle iPQ

$\vec{s}_1 + \Delta\vec{s} = \vec{s}_2 \qquad \Rightarrow \Delta\vec{s} = \vec{s}_2 - \vec{s}_1$

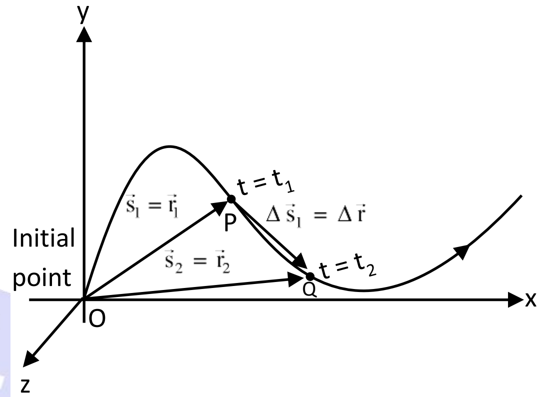
In above diagram $\Delta\vec{s}$ and $\Delta\vec{r}$ same resultant vector in triangle OPQ and iPQ

So we can conclude that position vector and displacement vector are different vector if starting point of motion and origin of position vector are different but change in position vector, $\Delta\vec{r}$ is always equal to change in displacement vector, $\Delta\vec{s}$

i.e. $\Delta\vec{r} = \Delta\vec{s}$

- if the starting point of motion and the origin of position vector coinside (i.e. becomes same) then position vector and displacement vector also become equal. i.e. in above diagram if $\vec{r}_i = 0$ then $\vec{s}_1 = \vec{r}_1$ & $\vec{s}_2 = \vec{r}_2$

Hence defining velocity :



DETECTIVE MIND

It is the rate of change of displacement or position w.r.t. time is called velocity

3.7 Uniform Velocity (Constant Velocity):

A particle is said to have uniform velocity, if magnitude as well as direction of its velocity remain same. This is possible only when it moves in a straight line without reversing its direction.

3.8 Non-Uniform Velocity (Variable Velocity):

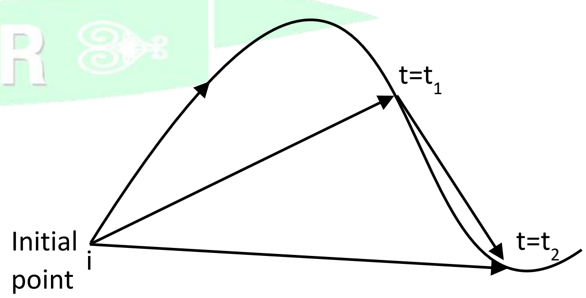
A particle is said to have non-uniform velocity, if both either magnitude or direction of velocity change.

3.9 Average Velocity:

It is defined as the ratio of displacement to time taken by the body

Average velocity = $\frac{\text{Displacement}}{\text{Time taken}}$; $\vec{v}_{av} = \frac{\Delta\vec{s}}{\Delta t}$

Say a particle is moving on the curve path as show in figure which is starting motion from initial point 'i' its displacement at time $t = t_1$ is s_1 and displacement time $t = t_2$ is s_2 then its average velocity during time interval $t = t_1$ to $t = t_2$ will be directed along $\Delta\vec{s}$ or $\Delta\vec{r}$



$$\vec{v}_{av} = \frac{\Delta\vec{s}}{\Delta t} \Rightarrow \vec{v}_{av} \,||\, \Delta\vec{s}$$

3.10 Instantaneous Velocity:

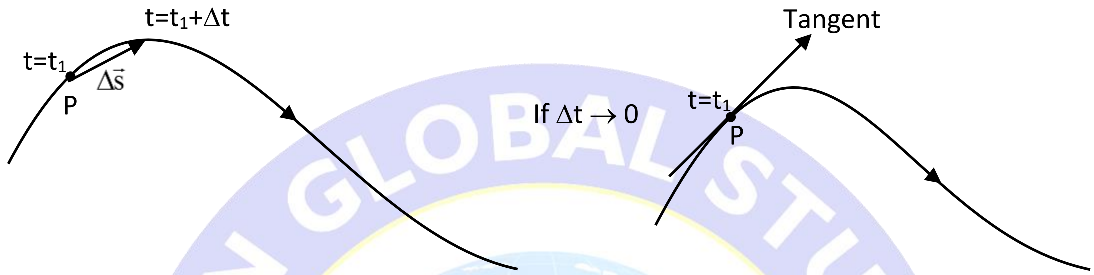
It is the velocity of a particle at a particular instant of time.

DETECTIVE MIND

Instantaneous velocity is simply called velocity.

Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{d\vec{s}}{dt}$

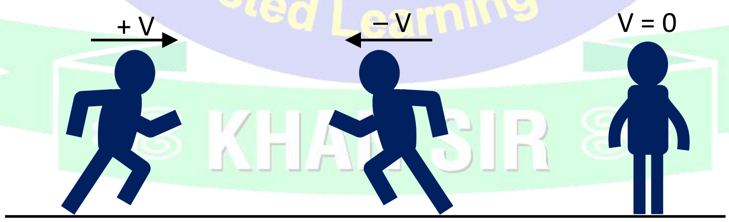
Intanteneous velocity is always directed along tangent to the path followed by the particle.



- Now $\Delta\vec{s}$ for time interval $t = t_1$ to $t = t_1 + \Delta t$ will be so small that it will be along tangent to the path at (point P) position of the particle at time $t = t_1$.
- For 1-D motion the direction can be taken care of positive and negative signs so, $\vec{v} = \frac{d\vec{s}}{dt}$ can be expressed as

$$\boxed{v = \frac{ds}{dt}}$$

- Magnitude of velocity is the speed i.e. Speed $= |\vec{v}|$
- For 1-D motion $v = \frac{ds}{dt} \Rightarrow ds = vdt$. So total displacement from time $t = t_1$ to $t = t_2$, $\boxed{\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} vdt}$.
- For 1-D motion if object reverses its direction of motion velocity changes its sign and it has to be zero at certain Instant.



- If motion is not 1-D (i.e. motion of curve path) then velocity of the particle at reversing point may or may not be zero. For example motion on a circular path.
- In 1-D motion, if velocity is given as function of time, v = f(t) then total displacement as function of time, s = f(t) will be, $v = \frac{ds}{dt} \Rightarrow ds = f(t)dt$

Total displacement during $t = t_1$ to $t = t_2$

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} f(t)dt \Rightarrow \boxed{s_2 - s_1 = \Delta s = \int_{t_1}^{t_2} f(t)dt}$$

- Velocity may be positive, negative or zero.
- If velocity is continuously changing with time i.e. velocity is the function of time then time average velocity

$$<\vec{v}_{av}>_t = \frac{\int \vec{v}\,dt}{\int dt}$$

- If velocity is continuously changing with distance i.e. velocity is the function of space (distance) then space average velocity -

$$<\vec{v}_{av}>_x = \frac{\int \vec{v}\,dx}{\int dx}$$

- Average speed ≥ I Average velocity I & $\frac{\text{average speed}}{|\text{average velocity}|} \geq 1$
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- A particle may have constant speed but variable velocity.
  When a particle is performing uniform circular motion then for every instant of its circular motion its speed
- remains constant but velocity changes at every instant.
- When particle moves with uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.
  $v = |\vec{v}| = v_{av} = |\vec{v}_{av}|$

SOLVED EXAMPLES

Example 8: If a body is moving with velocity $v = t^3 + 2t$. Find the average velocity between 1 and 2 second.

(1) $\frac{9}{2}$ m/s (2) $\frac{27}{4}$ m/s

(3) $\frac{12}{5}$ m/s (4) Zero

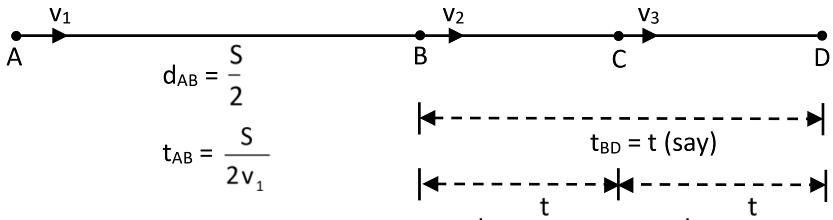
Solution: As $< v_{av} >$ (average velocity) = $\frac{\int_{t_1}^{t_2} v\,dt}{\int_{t1}^{t2} dt} = \frac{\int_1^2 (t^3+2t)dt}{\int_1^2 (dt)} = \frac{27}{4}$ m/s

Example 9: A particle travels first half of the total distance with speed v_1. For the rest journey it travels with speed v_2 for the first half of the time and with speed v_3 for the rest half of the time. Find average speed for the particle during complete journey?

(1) $\frac{2v_1(v_2+v_3)}{2v_1+v_2+v_3}$ (2) $\frac{v_1+v_2+v_3}{3}$

(3) $\frac{2v_1v_2v_3}{v_1+v_2+v_3}$ (4) $\frac{3v_1(v_2v_3)}{v_1+v_2+v_3}$

Solution: Say particle travelled S distance during the complete journey, $\frac{S}{2}$ distance travelled with speed v_1 and rest $\frac{S}{2}$ distance travelled for time t_2 and $\frac{t_2}{2}$ time travelled with v_2 and next $\frac{t_2}{2}$ travelled with speed v_3.



$d_{AB} = \frac{S}{2}$

$t_{AB} = \frac{S}{2v_1}$

$t_{BC} = \frac{t}{2}$ $\quad t_{CD} = \frac{t}{2}$

$\Rightarrow d_{BD} = \frac{S}{2} \qquad \Rightarrow v_2t_{BC} + v_3t_{CD} = \frac{S}{2} \qquad \Rightarrow v_2\frac{t}{2} + v_3\frac{t}{2} = \frac{S}{2}$

$\Rightarrow v_2t + v_3t = S \quad \Rightarrow t = \frac{S}{v_2 + v_3}$

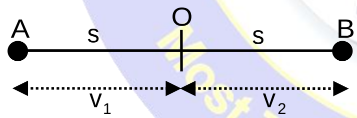
$$\text{Average speed } = \frac{S}{\frac{S}{2v_1} + \frac{S}{v_2 + v_3}} = \frac{2v_1(v_2 + v_3)}{2v_1 + v_2 + v_3}$$

Example 10: If a particle travels the first half distance with speed v_1 and second half distance with speed v_2. Find its average speed during the journey.

(1) $\frac{v_1 + v_2}{2}$ (2) $\frac{2v_1v_2}{v_1 + v_2}$

(3) $\frac{v_1v_2}{v_1 + v_2}$ (4) $\sqrt{v_1v_2}$

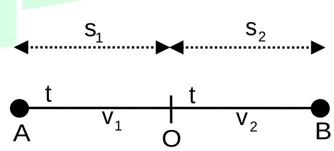
Solution: $v_{avg.} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$ = Harmonic mean of v_1 and v_2



$t_1 = \frac{s}{v_1}$ $\quad t_2 = \frac{s}{v_2}$

(For same distances covered with different speed)

Example 11: If a particle travels with speed v_1 during first half time interval and with v_2 speed during second half time interval. Find its average speed during its journey.



(1) $\frac{v_1 + v_2}{2}$ (2) $\frac{2v_1v_2}{v_1 + v_2}$

(3) $\frac{v_1v_2}{v_1 + v_2}$ (4) $\sqrt{v_1v_2}$

Solution: $v_{avg} = \frac{s_1 + s_2}{t + t} = \frac{(v_1 + v_2)t}{2t} = \frac{v_1 + v_2}{2}$ = Arithmetic mean of v_1, v_2

(For distances covered in equal time)

Example 12: A car moves at a velocity 2.24 km h^{-1} in first minute, at 3.60 km h^{-1} in the second minute and at 5.18 km h^{-1} in the third minute. Calculate the average velocity in these three minutes.

(1) 1.33 km/h (2) 2.25 km/h (3) 3.67 km/h (4) Zero

Solution: Average velocity = $\frac{2.24+3.60+5.18}{3}$ = 3.67 kmh^{-1} (distance covered in equal time)

Example 13: A bird flies due North with velocity 20 ms^{-1} for 15 s it rests for 5 s and then flies due South with velocity 24 ms^{-1} for 10 s. Find the average speed and magnitude of average velocity. For the whole trip.

(1) 18 m/s, 3 m/s (2) 12 m/s, 4 m/s (3) 9 m/s, 2 m/s (4) 18 m/s, 2 m/s

Solution: Average speed = $\frac{\text{Total Distance}}{\text{Total time}} = \frac{20\times15+24\times10}{15+5+10} = \frac{540}{30}$ = 18 ms^{-1}

Average Velocity = $\frac{\text{Displacement}}{\text{Total time}} = \frac{60\hat{j}}{30} = 2\hat{j}$

Magnitude of average velocity = $|2\hat{j}|$ = 2 m/s

Example 14: The displacement of a point moving along a straight line is given by s = $4t^2$ + 5t- 6. Here s is in cm and t is in seconds calculate

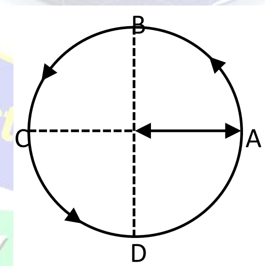
(i) Initial speed of particle (ii) Speed at t = 4s

(1) 5 cm/s, 37 cm/s (2) 8 cm/s, 22 cm/s (3) 5 cm/s, 28 cm/s (4) 8 cm/s, zero

Solution: (i) Speed (v) = $\frac{ds}{dt}$ = 8t + 5, Initial speed (i.e. at t = 0), v = 5 cm/s

(ii) At t = 4s, v = 8(4) + 5 = 37 cm/s

Example 15: A cyclist moves on a circular track from A to D in time t as shown in figure. What is the average speed and averge velocity of the cyclist.



(1) $\frac{2\pi r}{t}, \frac{r\sqrt{2}}{t}$ (2) $\frac{3\pi r}{t}, \frac{r}{\sqrt{2}t}$ (3) $\frac{2\pi r}{t}, \frac{2\sqrt{2}r}{t}$ (4) $\frac{3\pi r}{t}, \frac{r\sqrt{2}}{t}$

Solution: Distance travelled in time, t = ABCD = $\frac{3\pi r}{2}$

Average speed = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{3\pi r/2}{t}$

$= \frac{3\pi r}{t}$

Distancement in time t = AD = $\sqrt{r^2 + r^2} = r\sqrt{2}$

Average velocity = $\frac{r\sqrt{2}}{t}$

Example 16: The boby travels a distance s_1 with velocity v_1 and s_2 with v_2 in the same direction. Calculate the average velocity of the body.

(1) $\frac{(s_1+s_2)v_1 v_2}{s_1\ v_2 + s_2\ v_1}$ (2) $\frac{(s_1+s_2)v_1 v_2}{s_2\ v_2 + s_1\ v_1}$ (3) $\frac{s_2\ v_2 + s_1\ v_1}{(s_1+s_2)v_1 v_2}$ (4) $\frac{s_1\ v_2 + s_2\ v_1}{(s_1+s_2)v_1 v_2}$

Solution: Time taken for distance (s_1) is given by,

$t_1 = s_1/v_1$

Time taken for distance (s_2) is given by,

$t_2 = s_2/v_2$

Total distance covered = $s_1 + s_2$;

Total time taken = $t_1 + t_2$

$\therefore$ Average velocity = $\frac{\text{total distance}}{\text{total time taken}}$

$$= \frac{s_1+s_2}{t_1+t_2} = \frac{s_1+s_2}{(s_1/v_1)+(s_2/s_2)}$$

$$= \frac{(s_1+s_2)v_1 v_2}{s_1\ v_2 + s_2\ v_1}$$

4. ACCELERATION :

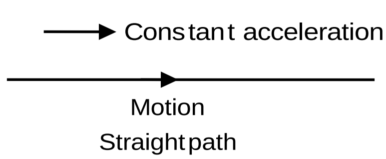
- The rate of change of velocity of an object is called acceleration of the object.
- It is vector quantity. It's direction is same as that of change in velocity (Not in the direction of the velocity)
- Dimension : $[M^0L^1T^{-2}]$
- Unit : metre/second^2(S.I.), cm/s^2(C.G.S.)

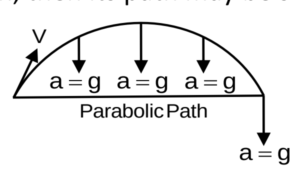
4.1 Uniform acceleration

- A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle.

4.2 Non-uniform acceleration

- A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.
- When a particle moves with constant acceleration, then its path may be straight line or parabolic.





4.3 Average acceleration

- It is ratio of total change in velocity to the total time taken by the particle

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

4.4 Instantaneous acceleration(Simply callled acceleration)

- It is the acceleration of a particle at a particular instant of time.

$$= \vec{a} = \lim_{\Delta t \to o} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

- first derivative of velocity is called instantaneous acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \qquad \left[\text{As } \vec{v} = \frac{d\vec{r}}{dt}\right]$$

- second derivative of position vector is called instantaneous acceleration.

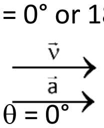
DETECTIVE MIND

- When a particle starts from rest and moves with constant acceleration then its path must be a straight line.
- When a particle moves with variable velocity then acceleration must be present.
- When a particle moves continuously on a same straight line with uniform speed then acceleration of the particle is zero.
- When a particle moves continuously on a curved path with uniform speed then acceleration of the particle is non-zero. For example, uniform circular motion is an accelerated motion.
- For a particle moving with uniform velocity acceleration must be zero.
- When a particle moves with non-uniform speed then acceleration of the particle must be non-zero.
- The direction of average acceleration vector is the direction of the change in velocity vector.
- Acceleration which opposes the motion of body is called retardation.
- Rest is the condition when it's velocity and acceleration both are zero.
- Momentarily rest is the condition when particle reverses it's direction and it's velocity is zero and acceleration is non-zero.

4.5 Velocity Change

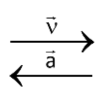
Velocity Change

Only speed changes	Only direction changes	Both speed & direction changes
$\vec{v}$ and $\vec{a}$ are parallel or antiparallel i.e. $\theta = 0°$ or $180°$	$\vec{v} \perp \vec{a}$ i.e. $\theta = 90°$	θ is any value other than $0°, 90°, 180°$ $\theta \in (0°, 90°) \to$ accelerated direction $\theta \in (90°, 180°) \to$ retarded direction changes

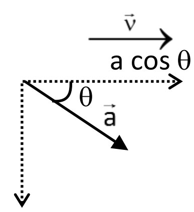


$\theta = 0°$

(accelerated motion) or



$\theta = 180°$



- $\frac{d\vec{v}}{dt} = \vec{a}$ or $\frac{dv}{dt} = a \Rightarrow dv = adt \Rightarrow \boxed{\int_u^v dv = \int_0^t adt} \leftarrow$ always applicable either a is constant or variable

$\Rightarrow v - u = \int_0^t a dt$

$\Rightarrow \int_u^v dv = a\int_0^t dt \Rightarrow v - u = at \Rightarrow \boxed{v = u + at}$ $\leftarrow$ applicable only if a is constant

- $\frac{d\vec{s}}{dt} = \vec{v} \Rightarrow \frac{ds}{dt} = v$ (in case of 1-D motion)

$\Rightarrow \int ds = \int_0^t vdt \qquad \Rightarrow \int_0^s ds = \int_0^t (u + at)dt$

$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$ $\leftarrow$ applicable if a is constant

- A particle is moving on straight line path whose initial velocity at t=0 is u and final velocity at time t=t is v after making displacement s then,

$\Rightarrow \frac{d\vec{v}}{dt} = \vec{a} \Rightarrow \frac{dv}{dt} = a$ (v = velocity in 1-D motion)

$\Rightarrow \frac{dv}{dt} \times \frac{ds}{ds} = a$ (here s is displacement in 1-D motion)

$\Rightarrow \frac{ds}{dt} \times \frac{dv}{ds} = a \Rightarrow \frac{dv}{ds} = a \Rightarrow \int_u^v vdv = \int_0^s ads \Rightarrow \boxed{\frac{v^2}{2} - \frac{u^2}{2} = \int_0^s ads}$ Always applicable either acceleration is constant or variable

$\Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = \int_0^s ads$

If a = constant $\Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = a\int_0^s ds \Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = a_t s \Rightarrow \boxed{v^2 - u^2 = 2as}$

- Displacement making by a particle in n_{th} second In 1-D motion : (if a = constant)

$V = \frac{dx}{dt}$

$\Rightarrow \int dx = \int v\, dt$

$\Rightarrow \int_{s_{n-1}}^{s_n} dx \int_{t=n-1}^{t=n} (u + at)dt$

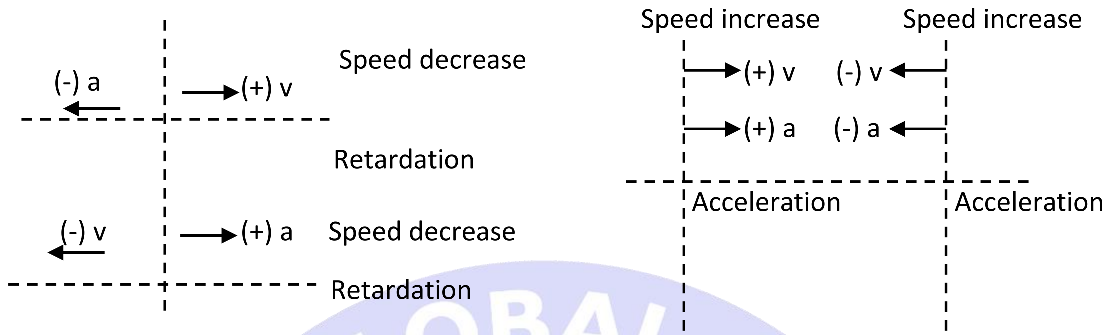
$\Rightarrow S_n - S_{n-1} = u[n - n + 1] + \frac{1}{2} a[n^2 - (n-1)^2]$

$\Rightarrow S_{n^{th}} = u + \frac{1}{2} a\,(n^2 - n^2 - 1 + 2n)$

$\boxed{S_n^{th} = u + \frac{a}{2}(2n - 1)}$ $\leftarrow$ applicable only if 'a' is constant

- To apply the above equation we consider either of the two possible direction along line of motion to be positive and other to be negative. The known parameter are used with proper signs while must not use by guessing the sign of unknown parameter.

- Sign of velocity (+ve or -ve) represents the direction of motion but sign of acceleration indicates the direction of acceleration.
- If velocity and acceleration both are having same sign, then magnitude of velocity (i.e. speed) is increasing and if both have opposite signs, then magnitude of velocity (i.e. speed) is decreasing.



- How to solve variable acceleration problems

(i) If acceleration is given as function of velocity, i.e. a = f(v) then velocity as function time will be

$a=\frac{dv}{dt}$ and a = f(v)

$\Rightarrow \frac{dv}{dt}=f(v)$

$\Rightarrow dv=f(v)dt$

$\Rightarrow \int_u^v dv=\int_{t_1}^{t_2} dt$, where u is velocity at time t_1 and v is velocity is time t_2

Solving this equation, we will get velocity function of time

(ii) If acceleration is given as function of displacement, i.e. a = f(s) then velocity as function of displacement will be

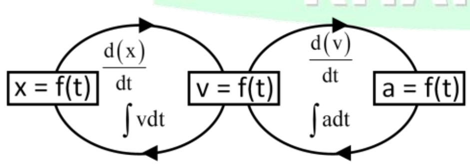
$a=\frac{vdv}{ds}$ and a = f(s)

$\Rightarrow \frac{vdv}{ds}=f(s) \Rightarrow \int_u^v vdv=\int_{s_1}^{s_2} f(s)ds$, where u is velocity at displacement s_1 and v is velocity at displacement s_2

Solving this equation, we will get velocity is function of displacement.

DETECTIVE MIND

- When V = f(x) $\Rightarrow$ $\boxed{a=v\frac{dv}{dx}}$ and V = f(t) $\Rightarrow$ $\boxed{a=\frac{dv}{dt}}$



5. MOTION IN STRAIGHT LINE WITH CONSTANT ACCELERATION :

Equation of motion are valid when acceleration is constant.

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 - u^2 = 2as$

- $s = v_{av}\,t = \dfrac{(u+v)}{2}t$ - $s = vt - \dfrac{1}{2}at^2$ - $s_n - s_{n-1} = s_{nth} = u + \dfrac{1}{2}a(2n-1)$
- All the equation of motion can be used in 2-D motion in vector form.

5.1 Concept of Stopping Distance And Stopping Time:

- A body moving with a velocity *u* is stopped by application of brakes after covering a distance *s*. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $v^2 = u^2 - 2as \quad \Rightarrow \quad 0 = u^2 - 2as \quad \Rightarrow \quad s = \dfrac{u^2}{2a} \quad s \propto u^2$ [since *a* is constant]

So,we can say that if 'u' becomes n time then 's' becomes n^2 times of previous value.

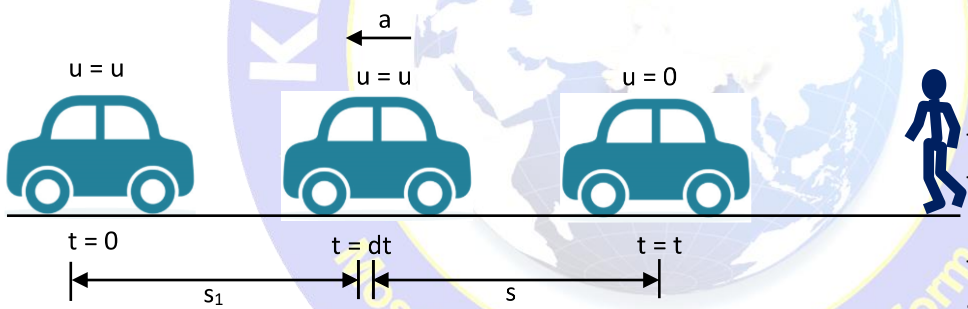
- Stopping time : $v = u - at \Rightarrow 0 = u - at \quad \Rightarrow \quad t = \dfrac{u}{a} \Rightarrow \quad t \propto u$ [since a is constant]

So, we can say that if u becomes n times then t becomes n times that of previous value.

5.2 Reaction Time:

It is the time gap between observation & execution

- During the reaction time the body travels with constant velocity.



Here
$t = 0 \rightarrow$ time of observation
$t = dt \rightarrow$ time of brake applied (Reaction time)
$t = t \rightarrow$ car comes at rest
$s \rightarrow$ s topping distance

SOLVED EXAMPLES

Example 17: Given $\vec{v} = t\hat{i} + t^2\hat{j}$. Find rate of change of speed.

(1) $\dfrac{1+2t^2}{\sqrt{1+t^2}}$ (2) $\dfrac{1+t^2}{\sqrt{1+t^2}}$ (3) $\dfrac{t^2}{\sqrt{1+t^2}}$ (4) $\dfrac{\sqrt{1+t^2}}{1+2t^2}$

Solution: $\vec{v} = t\hat{i} + t^2\hat{j}$ = $|\vec{v}| = \sqrt{t^2+t^4} = t\sqrt{1+t^2}$

Rate of change of speed = $\dfrac{d(\vec{v})}{dt}$ = $\sqrt{1+t^2} \times 1 + \dfrac{1}{2}(1+t^2)^{-1/2} \times 2t^2 = \sqrt{1+t^2} + \dfrac{t^2}{\sqrt{1+t^2}} = \dfrac{1+2t^2}{\sqrt{1+t^2}}$

Example 18: The velocity of a particle is given by $v = (2t^2 - 4t + 3)$ m/s where t is time in seconds. Find its acceleration at t = 2 second.

(1) 8 m/s^2 (2) 4 m/s^2 (3) 2 m/s^2 (4) Zero

Solution: Acceleration (a) = $\dfrac{dv}{dt} = \dfrac{d}{dt}(2t^2 - 4t + 3) = 4t - 4$

Therefore, acceleration at t = 2s is equal to a = (4 x 2) - 4 = 4 m/s^2

Example 19: (i) If $s = 2t^3 + 3t^2 + 2t + 8$ then find time at which acceleration is zero.

(1) t = 1 s (2) t = 0.5 s

(3) t = 4 s (4) Acceleration can't be zero

(ii) Velocity of a particle (starting at t = 0) varies with time as v = 4t. Calculate the displacement of particle between t = 2 to t = 4 s

(1) 12 m (2) 24 m (3) 36 m (4) Zero

Solution: (i) $V = \frac{ds}{dt} = 6t^2 + 6t + 2 \Rightarrow a = \frac{dv}{dt} = 12t + 6 = 0 \quad \Rightarrow t = -\frac{1}{2}$ which is impossible.

Therefore, acceleration can never be zero.

(ii) $\therefore \frac{ds}{dt} = v \therefore s = \int v\,dt = \int_2^4 4t\,dt = \left[2t^2\right]_2^4 = 2(4)^2 - 2(2)^2 = 32 - 8 = 24$ m

Example 20: The velocity of particle moving in the positive direction of x axis varies as $v = a\sqrt{x}$, where a is a positive constant. Assuming that at moment t = 0, the particle was located at the point x = 0. Find

(i) The time dependence of the velocity and the acceleration of the particle.

(1) $v \propto t^2 ; a \propto t$ (2) $v \propto t ; a \propto t^0$ (3) $v \propto t^0 ; a \propto t^0$ (4) $v \propto t^{1/2} ; a \propto t^{-1/2}$

(ii) The mean velocity of the particle averaged over the time that the particle takes to cover first s meters of the path.

(1) $a\sqrt{s}$ (2) $a\frac{\sqrt{s}}{2}$ (3) $a\frac{\sqrt{s}}{4}$ (4) $a\frac{\sqrt{s}}{3}$

Solution: (i) $V = a\sqrt{x} \Rightarrow \frac{dx}{dt} = a\sqrt{x} \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = a\int_0^t dt \quad \Rightarrow x = \frac{a^2t^2}{4}$

Velocity $v = \frac{dx}{dt} = \frac{2a^2t}{4} = \frac{1}{2}a^2t$ Acceleration (a) $= \frac{dv}{dt} = \frac{a^2}{2}$

(ii) Time taken to cover the first s distance

$x = \frac{a^2t^2}{4} \Rightarrow s = \frac{a^2t^2}{4} \Rightarrow t = \sqrt{\frac{4s}{a^2}} \quad \Rightarrow v_{av} = \frac{s}{t} = \frac{s}{\sqrt{\frac{4s}{a^2}}} = \frac{a\sqrt{s}}{2}$

Example 21: A body is moving with velocity $2\hat{i} + 3t^2\hat{j}$, find its average acceleration after 2 seconds.

(1) $6\hat{i} + 6\hat{j}$ (2) $3\hat{i}$ (3) $6\hat{j}$ (4) $-6\hat{i} + 6\hat{j}$

Solution: As $<\vec{a}>$ (average acceleration) $= \frac{\Delta v(\text{Change in velocity})}{\Delta t(\text{Total time taken})}$

$\vec{v}_{initial} = 2\hat{i} + 0\hat{j}$, $\vec{v}_{final} = 2\hat{i} + 12\hat{j}$ therefore, $= \Delta\vec{v} = \vec{v}_{final} - \vec{v}_{initial} = 0\hat{i} + 12\hat{j} \quad \Rightarrow <\vec{a}> = 0\hat{i} + 6\hat{j}$

Example 22: A particle is moving along a straight line OX. At a time t (in seconds) the distance x (in meters) of particle from point O is given by $x = 10 + 6t^2 - 2t^3$. How long would the particle travel before its acceleration becomes zero.

(1) 4 m (2) 8 m (3) 16 m (4) 24 m

Solution: Initial value of x at t = 0 x_1 = 10m Velocity $v = \frac{dx}{dt} = 12t - 6t^2$

$a = 12 - 12t$ when $a = 0$, $t = 1$

x at t = 1, $x_2 = 10 + 6 - 2 = 14$ Distance travelled = $x_2 - x_1 = 14 - 10 = 4m$

Example 23: Given $\vec{v} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = \hat{i} + \hat{j}$. Find rate of change of speed?

(1) $\frac{2}{3}\ m/s^2$ (2) $\frac{4}{3}\ m/s^2$ (3) $\frac{7}{5}\ m/s^2$ (4) Zero

Solution: Projection of $\vec{a}$ in the direction of $\vec{v}$ = rate of change of speed = $\vec{a}\cdot\hat{v} = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|} = \frac{3+4}{5} = \frac{7}{5}$ m/s^2.

Example 24: A particle moves on circular path of radius 5 m with constant speed 5 m/s. Find the magnitude of its average acceleration when it completes half revolution.

(1) $\frac{\pi}{5}\ m/s^2$ (2) $\frac{10}{\pi}\ m/s^2$ (3) $\frac{\pi}{10}\ m/s^2$ (4) Zero

Solution: Change in velocity = $2v \sin\frac{\theta}{2} = 2v \sin\frac{180}{2}$ = 10 m/sec

Time taken by it = $\frac{\pi r}{v} = \frac{\pi \times 5}{5} = \pi$; $a_{avg} = \frac{10}{\pi}$

Example 25: Two cars start off a race with velocities 2m/s and 4m/s travel in straight line with uniform accelerations 2 m/s^2 and 1 m/s^2 respectively. What is the length of the path if they reach the final point at the same time?

(1) 4 m (2) 8 m (3) 16 m (4) 24 m

Solution: Since both particle reach at same position in same time t then from $s = ut + \frac{1}{2}at^2$

For 1st particle : $s = 2(t) + \frac{1}{2}(2)^{t^2} = 2t + t^2$

For 2nd particle : $s = 4t + \frac{1}{2}(1)t^2$ Equating above equation we get 4t + = 2t + t^2 $\Rightarrow$ t = 4 s

Substituting value of t in above equation s = 4(4)+ (1)(4)^2 = 16 + 8 = 24 m

Example 26: A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first 's' distance.

(1) $\frac{as}{4}$ (2) $\sqrt{as}$ (3) $\sqrt{\frac{as}{2}}$ (4) $\sqrt{\frac{a}{2s}}$

Solution: $\because s = \frac{1}{2}at^2$ $\therefore \frac{s^2}{t^2} = \frac{1}{2}as$ Average velocity = $\frac{s}{t} = \sqrt{\frac{as}{2}}$

Example 27: A train, travelling at 20km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance the bird covers before the train stops ?

(1) 9 km (2) 12 km (3) 24 km (4) 64 km

Solution: For retardation of train $v^2 = u^2 + 2as$ $\Rightarrow 0 = (20)^2 + 2(a)(2) \Rightarrow a = -100$ km/hr^2

Time required to stop the train v = u + at $\Rightarrow 0 = 20 - 100t \Rightarrow t = \frac{1}{5}$ hr

For Bird, speed = $\frac{\text{Distance}}{\text{time}}$ $\Rightarrow s_B = v_B \times t = 60 \times \frac{1}{5} = 12$ km.

Example 28: A train starts from rest and moves with a constant acceleration of 2.0 m/s^2 for half a minute. The brakes are then applied and the train comes to rest in one minute.

(i) Maximum speed attained by the train is :

(1) 24 m/s (2) 32 m/s (3) 60 m/s (4) 90 m/s

(ii) Total distance moved by the train is :

(1) 600 m (2) 900 m (3) 1200 m (4) 2700 m

(iii) The position(s) of the train at half of the maximum speed are :

(1) 1250 m (2) 2250 m (3) 960 m (4) 3600 m

Solution: (i) maximum speed attained $\Rightarrow v = u + at \Rightarrow v = 2 \times 30 = 60$ m/sec

(ii) distance covered in this time $\Rightarrow s_1 = \frac{1}{2} \times 2 \times (30)^2 = 900$ m

Deceleration $\Rightarrow v = u - at \Rightarrow 60 = 60a \Rightarrow a = 1$

Distance covered in deceleration time $\Rightarrow s_2 = 60 \times 60 - \frac{1}{2} 1 \times (60)^2 \Rightarrow s_2 = 1800$ m

Total distance = 900 + 1800 = 2700m = 2.7 km

(iii) Position of train when speed is half

$\frac{v^2}{4} = 0 + 2 \times 2 \times x_1 \Rightarrow x_1 = 225$m when deceleration $\Rightarrow \frac{3v^2}{4} = 2 \times 1 \times x_2 \Rightarrow x_2 = 1350$

So, distance at which speed is half = 1350 + 900 = 2250 m = 2.25 km

Example 29: A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the **reaction time** of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a declaration of 6.0 m/s^2. Find the distance travelled by the car after he sees the need to put the brakes on.

(1) 5.4 m (2) 18.6 m (3) 21.75 m (4) 32.5 m

Solution: Reaction time = 0.20 sec $\Rightarrow$ velocity = 54 km/hr = 15 m/sec $\Rightarrow$ Acceleration = 6 m/sec^2

Now distance travelled in 0.2 sec $= 15 \times 0.2 = 3$ m

Distance travelled during deceleration $\Rightarrow 15^2 = 2 \times 6 \times x \Rightarrow x = 18.75$ m Total distance = 21.75 m

Example 30: A bullet losses (1– 1/n) of its velocity in passing through a plank. What is the least number of planks required to stop the bullet ? (Assuming constant retardation)

(1) $\frac{1}{1+\left(\frac{n-1}{n}\right)^2}$ (2) $\frac{1}{1-\left(\frac{n-1}{n}\right)^2}$ (3) $\frac{n}{1-\left(\frac{n+1}{n}\right)^2}$ (4) $\frac{1}{1+\left(\frac{n+1}{n}\right)^2}$

Solution: Thickness of plank = t. Now deceleration of bullet $\Rightarrow \left(\frac{n-1}{n}\right)^2 v^2 = v^2 - 2at \Rightarrow a = \frac{v^2}{2t}\left(1-\left(\frac{n-1}{n}\right)^2\right)$

Number of planks required = α then $\Rightarrow 0 = v^2 - 2a(\alpha t) \Rightarrow \alpha = \frac{1}{1-\left(\frac{n-1}{n}\right)^2}$

Example 31: A car start for rest and moving with constant acceleration 2 m/sec^2. It covers first 100 meter distance in time t_1 and second 100 meter in time t_2 then find ratio of time $t_1 : t_2$.

(1) 1 : 3 (2) $1 : \sqrt{2}$ (3) $1 : \sqrt{2}-1$ (4) $1 : 2\sqrt{2}-1$

Solution: a = 2 m/sec^2 $\Rightarrow$ For 1st 100 m $\Rightarrow$ $t_1 = \sqrt{\frac{2\times100}{2}} = 10$

$\Rightarrow$ $t_1 + t_2 = \sqrt{\frac{2\times100}{2}} = 10\sqrt{2}$ $\Rightarrow$ $t_2 = 10(\sqrt{2}-1)$ $\Rightarrow$ $t_1 : t_2 = 1 : (\sqrt{2}-1)$

Example 32: A car start from rest and moving with constant acceleration 5 m/sec^2. The distance travelled in the first 5 sec is x_1 next 5 sec is x_2 and last 5 sec is x_3. Then $x_1 : x_2 : x_3$ is

(1) 1 : 4 : 9 (2) 1 : 1 : 1 (3) 1 : 3 : 5 (4) 2 : 3 : 5

Solution: a = 5 m/sec^2

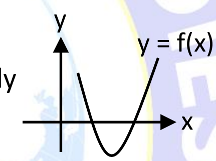
$x_1 = \frac{1}{2} \times 5 \times (5)^2 = \frac{125}{2}$, $x_2 = \frac{1}{2} \times 5 \times (10)^2 - x_1 = \frac{500}{2} - \frac{125}{2} = \frac{375}{2}$

$x_3 = \frac{1}{2} \times 5 \times (15)^2 - x_1 - x_2 = \frac{625}{2}$ $x_1 : x_2 : x_3 = 1 : 3 : 5$

6. GRAPHICAL SECTION :

6.1 Graph: Relation between two variables can be expressed into two ways

(i) Algecbrically or by equation y = f(x) (ii) Graphically

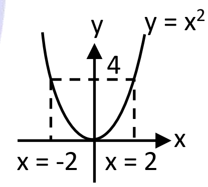


Example: $y = x^2$ find value of y of at x = ± 2.

Solution: Algebraic method: $y_{x=\pm 2} = (\pm 2)^2 = 4$

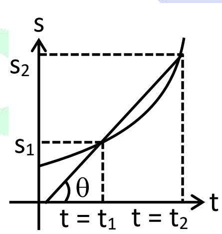
Graphical method: By observing graph at x = ± 2 we get y = 4

Graph: Geometrical representation of relation between two variables.

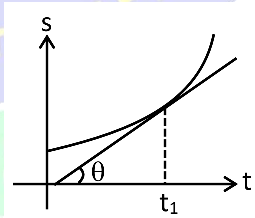


6.2 Distance Time-Graph:

➢ Slope of straight line joining the distance - time graph at $t = t_1$ and $t = t_2$ gives the average rate of change distance with respect to time i.e. speed for time interval $t = t_1$ to $t = t_2$ and slope of tangent of any point on distance covered versus time graph gives instantaneous speed of a particle at any point $t = t_1$.



average speed = $\frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$ = tan θ

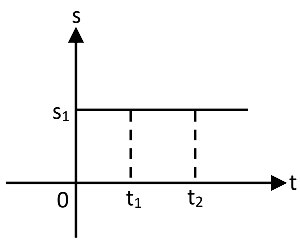


Intantaneous speed = $\frac{ds}{dt}$ = tan θ

➢ Area of distance-time graph has no physical significance.

➢ Time can never be negative.

(i)



As t→increases

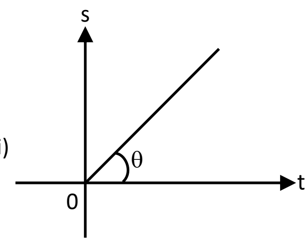
s → constant

Slope of s-t graph = 0

i.e. speed = 0

i.e. Particle is at rest

(ii)



t → increases

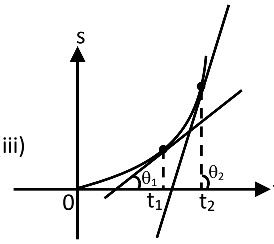
s → increases

Slope is constant

i.e. $\tan\theta =$ constant

∴ speed is constant

(iii)



As t → increases, s→increases

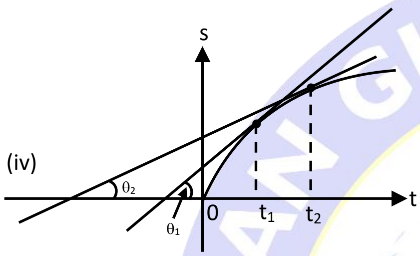
$\text{slope})_{\text{at } t_1} < \text{slope})_{\text{at } t_2}$

$\tan\theta_1)_{\text{at } t_1} < \tan\theta_2)_{\text{at } t_2}$

$\therefore$ $\text{speed})_{\text{at } t_1} < \text{speed})_{\text{at } t_2}$

i.e. speed increases

(iv)



As t → increases, s→increases

$\text{slope})_{\text{at } t_1} > \text{slope})_{\text{at } t_1}$

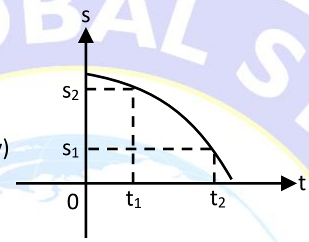
$\tan\theta_1)_{\text{at } t_1} > \tan\theta_2)_{\text{at } t_2}$

$\therefore$ $\text{speed})_{\text{at } t_1} > \text{speed})_{\text{at } t_1}$

i.e. slope of tangent decreases

i.e. speed decreases

(v)

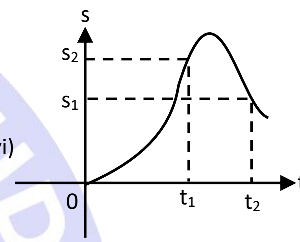


Graph not possible

As t → increases

s→decreases

(vi)

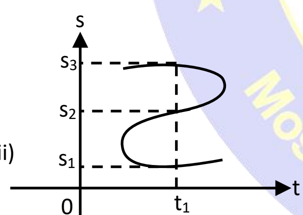


Graph not possible

As t → increases

s→decreases

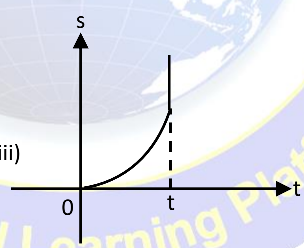
(vii)



Graph not possible

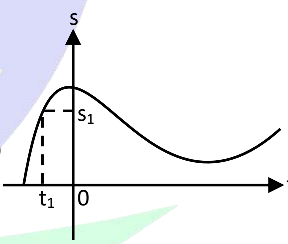
As particle is at three different place at one time

(viii)

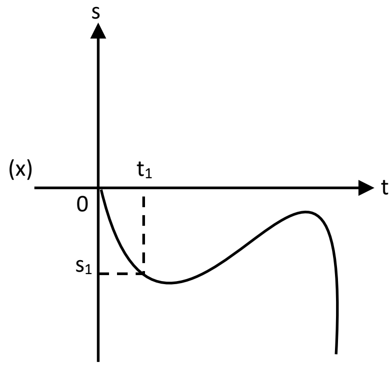


Graph not possible as time t is constant but distance is increasing.

(ix)



Graph not possible as time t is negative



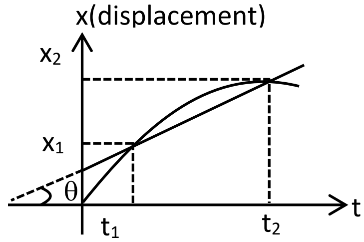
Graph not possible as distance
s is negative

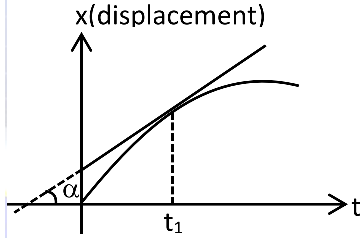
SPOT LIGHT

- By observing distance time graph we cannot predict about the path followed by the particles OR we cannot tell about the path that it is straight path or curve path

6.3 Displacement-Time Graph :

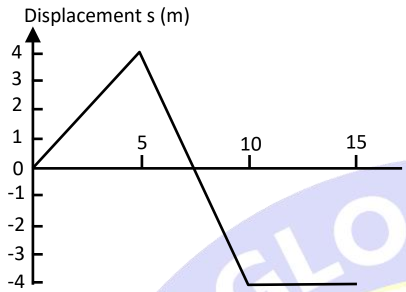
- Slope of straight line joining the displacement - time graph at $t = t_1$ and $t = t_2$ gives the average rate of change displacement with respect to time i.e. velocity for time interval $t = t_1$ to $t = t_2$ and slope of tangent at any point of x – t curve gives the instantaneous velocity at any time $t = t_1$

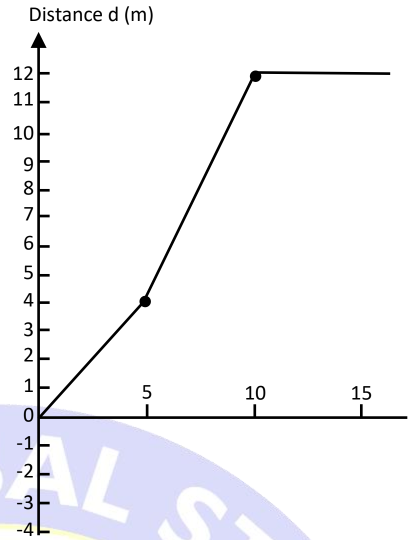


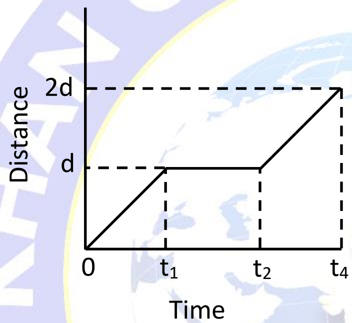


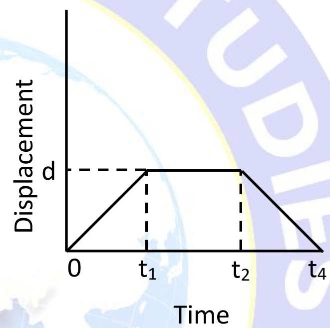
Average velocity $= \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \tan\theta$ instantaneous velocity at time $t = t_1 = \frac{dx}{dt} = \tan\alpha$

- Magnitude of slope of tangent at any point of x – t curve gives the instantaneous speed at that point
- Area of x – t curve has no physical significance
- Rate of change of slope of x – t curve $\left[\frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}\right]$ represents acceleration at any instant.
- To convert displacement–time curve into distance-time curve, we take the mirror image of section of negative slope about an imaginary line parallel to x-axis.

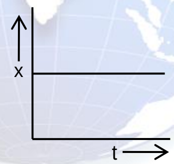








(i) ➢ the slope of x-t graph is zero

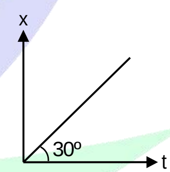


➢ v = 0

➢ so body is in rest condition

(ii) ➢ The slope of curve is constant so

➢ = constant = tan 30° = $\frac{1}{\sqrt{3}}$ unit, $\vec{a}$ = 0.



➢ So body moves with constant velocity & zero acceleration.

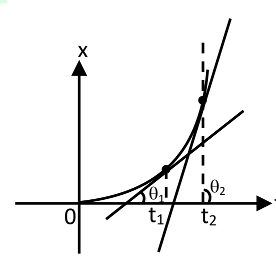
(iii) ➢ For the curve

➢ As t $\rightarrow$ increases, x$\rightarrow$increases

➢ initial slope is zero i.e. u = 0

➢ Slope is positive throughout i.e. velocity = +ve

➢ Magnitude of slope)$_{\text{at } t_1}$ < Magnitude of slope)$_{\text{at } t_2}$



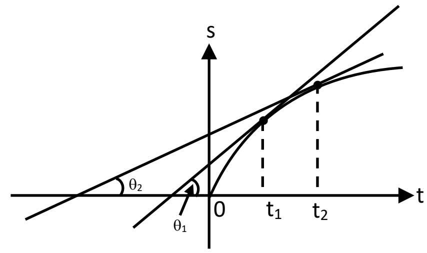
$$\left|\frac{dx}{dt}\right|_{\text{at } t_1} < \left|\frac{dx}{dt}\right|_{\text{at } t_2} \Rightarrow \text{speed})_{\text{at } t_1} < \text{speed})_{\text{at } t_2}$$

➢ Since, speed is increasing with time it means it is an accelerated motion

➢ $\frac{d^2x}{dt^2}$ = acceleration = +ve i.e. a = +ve

- So body starts from rest & moves with constant acceleration

(iv)
- For the curve
- Initial slope is +ve i.e. u = +ve
- Slope is +ve throughout i.e. v = +ve always
- Magnitude of slope)$_{\text{at } t_1}$ > Magnitude of slope)$_{\text{at } t_2}$



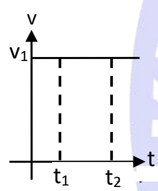
$$\left|\frac{dx}{dt}\right|_{\text{at } t_1} > \left|\frac{dx}{dt}\right|_{\text{at } t_2} \Rightarrow \text{speed})_{\text{at } t_1} > \text{speed})_{\text{at } t_2}$$

- Since, speed is decreasing with time it means it is an retarded motion.
- $\frac{d^2x}{dt^2} = -\text{ve}$ i.e. a = –ve
- So body starts moving with a certain velocity is being retarded

6.4 Speed-Time Graph :

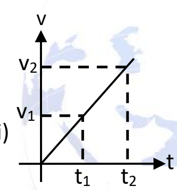
- Area of speed-time graph gives total distance covered.
- Average speed from v-t curve can be calculated as $v_{av} = \frac{\text{Area enclosed with time axis}}{\text{Time-interval}}$

(i)



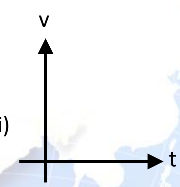
As t → increases
v → constant
At t = t_1, v = v_1
And at t = t_2, v = v_1

(ii)



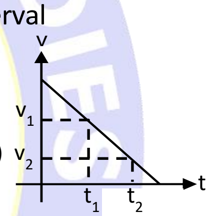
As t → increases
v → increases
At t = t_1, v = v_1
And at t = t_2, v = v_2

(iii)



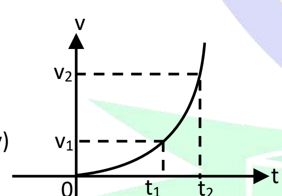
As t → increases
v → zero

(iv)



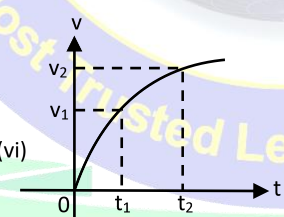
As t → increases
v → decreases
At t = t_1, v = v_1
And at t = t_2, v = v_1

(v)



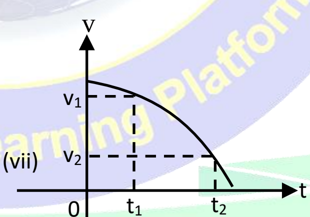
As t → increases
v → increases
At t = 0, v = 0

(vi)



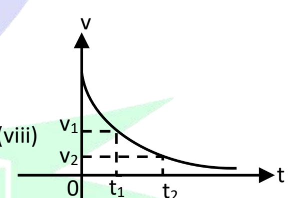
As t → increases
v → increases
At t = 0, v = 0

(vii)



As t → increases
v→ decreases
At t = 0, v ≠ 0

(viii)



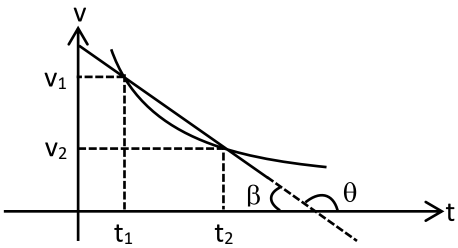
As t → increases
v → decreases
At t = 0, v ≠ 0

SPOT LIGHT

- By observing speed-time graph we cannot predict about the path followed by the particles OR we cannot tell about the path that it is straight path or curve path

6.5 Velocity-Time Graph:

- Slope of straight line joining the velocity - time graph at t = t_1 and t = t_2 gives the average rate of change velocity with respect to time i.e. acceleration for time interval t = t_1 to t = t_2 and slope of tangent of v-t graph gives instantaneous acceleration at any time t = t_1 that point

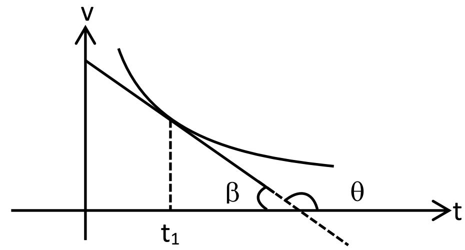


Average acceleration $= \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$ = tan θ

Here average acceleration is negative as

θ is obtuse angle so magnitude of average

acceleration $= \left|\frac{\Delta v}{\Delta t}\right| = \left|\frac{v_2 - v_1}{t_2 - t_1}\right|$ = tan β



Instantaneous acceleration $= \frac{dv}{dt}$ = tan θ

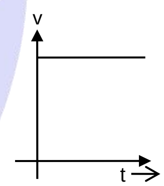
Here instantaneous acceleration is negative as

θ is obtuse angle so magnitude of instantaneous

acceleration at t = t_1 $= \left|\frac{dv}{dt}\right|$ = tan β

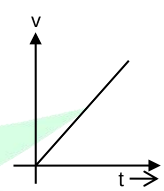
- Area of v-t graph provides change in displacement.
- Area of magnitude of v-t graph gives total distance covered.
- Average velocity from v-t curve can be calculated as $\vec{v}_{av} = \frac{\text{Area enclosed with time axis}}{\text{Time} - \text{interval}}$
- To convert v-t curve into speed-time curve, negative section of curve is made positive as speed can not be negative.

Following Standard v-t curve represents :–



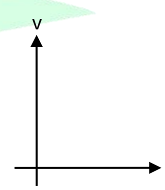
(i) The slope of curve is zero so
a = 0 & v = constant

- So body moves with constant velocity.



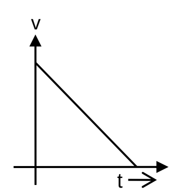
(ii) The slope of curve is positive i.e. = +ve also slope is constant which denotes
a = constant

- So body starts from rest & moves with constant acceleration.



(iii) v = 0, a = 0

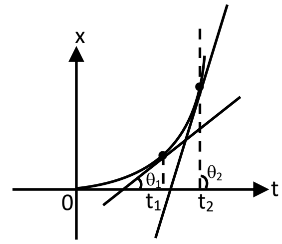
- So the body is at rest



(iv) The slope of curve is negative i.e. = –ve

- So the body moving with certain velocity is retarding with constant retardation.

(v) Slope is changing continuously & is always positive i.e. $\vec{a}$ = variable,

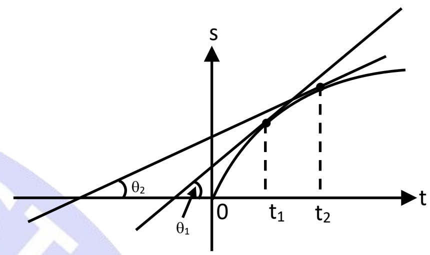


- Magnitude of slope)$_{at\ t_1}$ < (Magnitude of slope)$_{at\ t_2}$

  Also $\left|\frac{dv}{dt}\right|_{at\ t_1} < \left|\frac{dv}{dt}\right|_{at\ t_2} \Rightarrow (\text{acceleration})_{at\ t_1} < (\text{acceleration})_{at\ t_2}$

  $\Rightarrow$ Acceleration is increasing
- So the body is moving with variable & increasing acceleration.

(vi) Slope is changing continuously & is always positive i.e. $\vec{a}$ = variable,



- Magnitude of slope)$_{at\ t_1}$ > Magnitude of slope)$_{at\ t_2}$

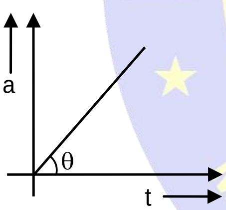
  Also $\left|\frac{dv}{dt}\right|_{at\ t_1} > \left|\frac{dv}{dt}\right|_{at\ t_2} \Rightarrow (\text{acceleration})_{at\ t_1} > (\text{acceleration})_{at\ t_2}$

  $\Rightarrow$ Acceleration is decreasing
- So the body is moving with variable & decreasing acceleration.

6.5 Acceleration–Time Graph :

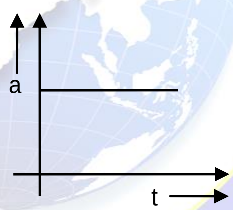
- Slope of curve provides an idea about the rate by which acceleration changes but it does not represent any physical quantity.
- Area under the curve denotes change in velocity of any particle in motion
- Average acceleration from the curve can be calculated as $a_{Avg.} = \frac{\text{Area enclosed with time axis}}{\text{Time} - \text{interval}}$

(i)



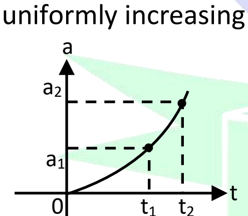
$a \propto t$, $\tan\theta$ = constant,
i.e. uniformly increasing acceleration

(ii)



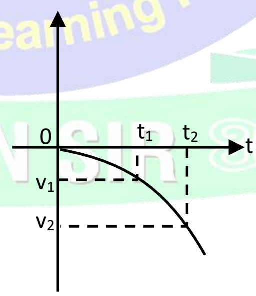
$a \propto t^0$ i.e. uniform or constant acceleration

(iii)



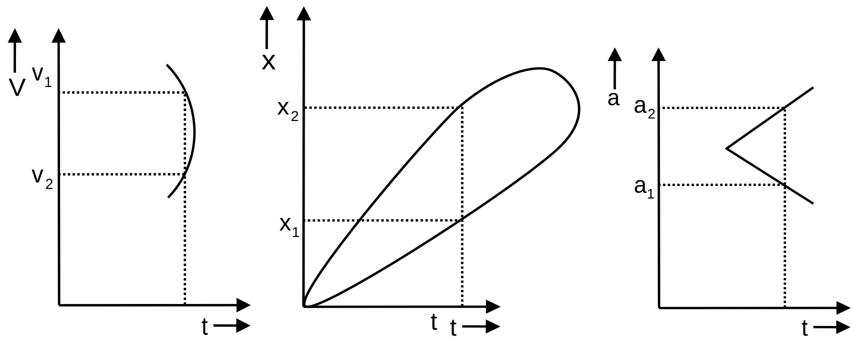
$t_1 \rightarrow$ increases
$a \rightarrow$ increases

(iv)



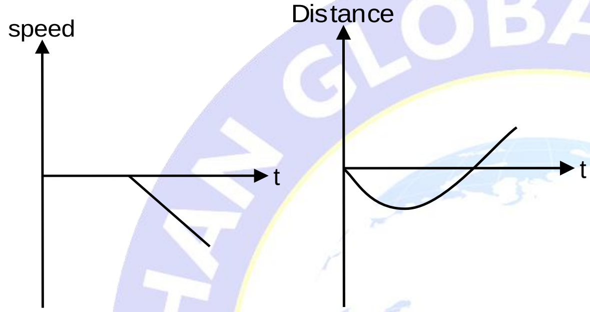
As t $\rightarrow$ increases
Magnitude of acceleration $\rightarrow$ increases
At t = 0, a = 0 and t = t_1 and t_2 a is negative

6.6 Following Graphs Do Not Exist In Practice:
Case-I



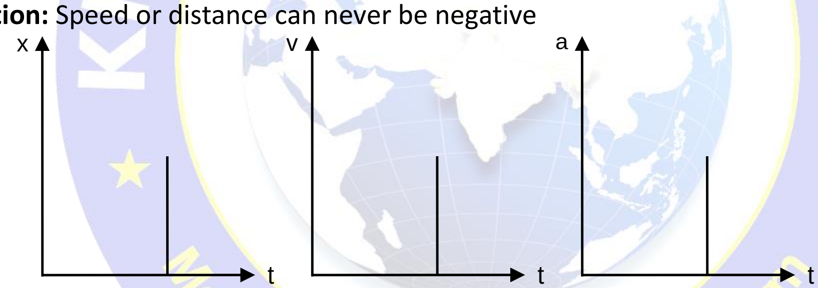
Explanation: In practice, at any instant body can not have two velocities or displacements or accelerations simultaneously.

Case-II



Explanation: Speed or distance can never be negative

Case III



Explanation: It is not possible to change any quantity without consuming time.

SOLVED EXAMPLES

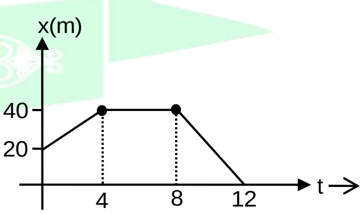
Example 33: For the following x – t curve calculate

(i) Displacement

(ii) Distance

(iii) At what time average velocity is zero.

(iv) Also Draw distance-time curve (in sec.)



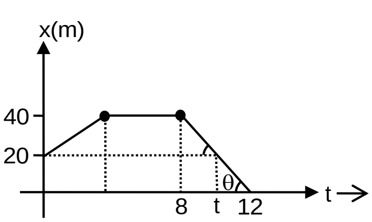
Solution: (i) Displacement

$= x_f - x_i = 0 - 20 = -20$ m

(ii) Distance

$= |40 - 20| + |40 - 40| + |0 - 40| = 60$ m

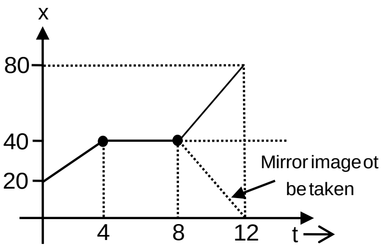
(iii) Average velocity is zero where particle attains its initial position



$= \frac{40-20}{t-8} = \frac{20-0}{12-t} \Rightarrow t = 10$ sec.

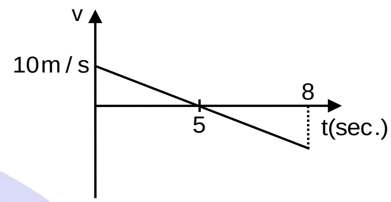
(iv) Since the slope of curve is negative from $t = 8$ to $t = 12$ s.

So distance-time curve will be drawn as



Example 34: For the given v – t curve calculate

(i) Displacement
(ii) Average velocity
(iii) Distance
(iv) Acceleration
(v) Speed-time graph



Solution: (i) Displacement = Area, Since $\frac{10}{5-0} = \frac{v}{8-5} \Rightarrow v = -6$

So Displacement

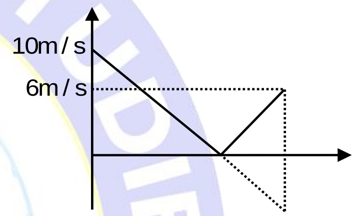
$= \left(\frac{1}{2}\times 10\times 5\right) - \frac{1}{2}(3 \times 6) = 16$ m

(ii) Average velocity $= \frac{16}{8} = 2$ m/s]

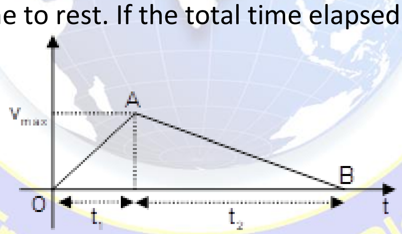
(iii) Distance $= \left|\frac{1}{2}\times 10\times 15\right| + \left|\frac{1}{2}\times 3\times -6\right| = 34$ m

(iv) acceleration = slope = v = -2 m/s^2

(v) Speed–time curve will be drawn as



Example 35: A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t, evaluate



(i) The maximum velocity attained and
(ii) The total distance travelled.

(1) $\frac{\alpha+\beta}{\alpha\beta t}, \frac{1}{2}\left(\frac{\alpha+\beta}{\alpha\beta t^2}\right)$ (2) $\frac{\alpha\beta t}{\alpha+\beta}, \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$

(3) $\frac{\alpha\beta}{(\alpha+\beta)t}, \frac{1}{2}\left(\frac{\alpha+\beta^2}{\alpha\beta t}\right)$ (4) $\frac{\alpha\beta}{(\alpha+\beta)t^2}, \frac{1}{4}\left(\frac{\alpha+\beta^2}{\alpha\beta t}\right)$

Solution: (i) Let the car accelerates for time t_1 and decelerates for time t_2 then $t = t_1 + t_2$ ......(i)

and corresponding velocity-time graph will be as shown in. fig.From the graph α = slope of line OA $= \frac{v_{max}}{t_1}$

or $t_1 = \frac{v_{max}}{\alpha}$ .....(ii)

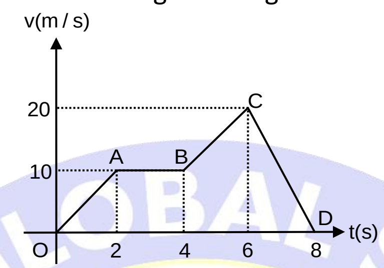
and $\beta = -$ slope of line OB $= \frac{v_{max}}{t_2}$ or $t_2 = \frac{v_{max}}{\beta}$ ...(iii)

From Eqs. (i),(ii) and (iii) $\frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t$ or $v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$ or $v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$

(ii) Total distance = area under v-t graph = $\frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta}$, Distance = $\frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$

Note: This problem can also be solved by using equations of motion (v = u + at, etc.).

Example 36: Velocity-time graph of a particle moving in a straight line is shown. Find out the displacement of particle.



(1) 80 m
(2) 120 m
(3) 60 m
(4) 90 m

Solution: Displacement = area under velocity-time graph.

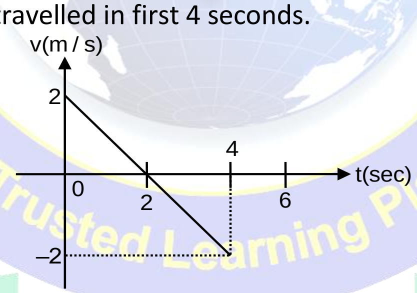
Hence, $s_{OA} = \frac{1}{2} \times 2 \times 10 = 10$ m $\Rightarrow s_{AB} = 2 \times 10 = 20$ m; v = +ve, a = 0

or s_{OAB} = 10+20 = 30 m, v = +ve, a = +ve $\Rightarrow s_{BC} = 2 \times \left(\frac{10+20}{2}\right) = 30$ m

or s_{OABC} = 30+30 = 60 m and $s_{CD} = 2 \times \left(\frac{20+0}{2}\right)$ = 20m, v = +ve, a = –ve

or s_{OABCD}= 60 + 20 = 80 m

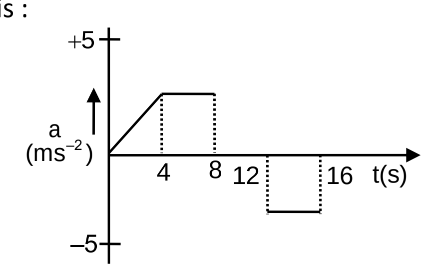
Example 37: Velocity time graph for a particle moving in a line is given. Calculate the displacement of the particle and distance travelled in first 4 seconds.



(1) 0, 2 m (2) 2 m, 4 m (3) 0, 4 m (4) 4 m, 4 m

Solution: Take the area above time axis as positive and area below time axis negative then displacement = (2-2) m = 0 while for distance take all areas as positive the distance covered s = (2 + 2)m = 4m

Example 38: The acceleration of a particle starts traveling along a straight line is shown in figure. The maximum speed of the particle is :



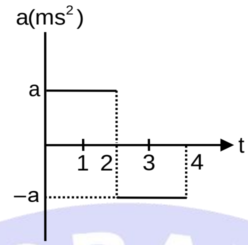
(1) 60 m/s (2) 40 m/s (3) 30 m/s (4) 50 m/s

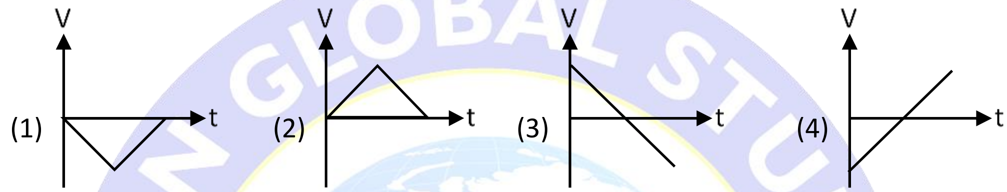
Solution: Particle will accelerate upto t = 8 sec as acceleration is positive till t = 8 sec

$$\Delta v = \int_0^8 a\,dt = \text{area under } a - t \text{ curve} = \frac{1}{2} \times 4 \times 5 + 4 \times 5 = 30$$

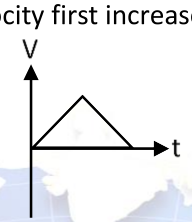
$v_f - v_i = 30 \Rightarrow v_f - 0 = 30 \Rightarrow \{v_{max} = v_f = 30 \text{ m/s}\}$

Example 39: A particle starts from rest and undergoes an acceleration as shown in figure. The velocity-time graph from figure will have a shape.





Solution: Till t = 2 s, there is constant acceleration a m/s^2 and after that there is constant deceleration –a m/s^2 (same magnitude). So velocity first increase from 0 to maximum and finally becomes zero.



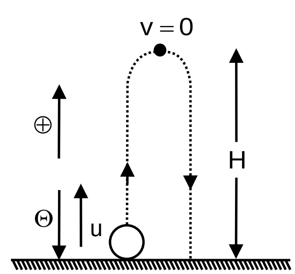
7. MOTION UNDER GRAVITY (FREE FALL) :

Acceleration produced in a body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

Value of g =9.8 m/s^2 = 980 cm/s^2 = 32 ft/s^2

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h<<earth's radius) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

7.1 If A Body Is Projected Vertically Upward:



Positive / Negative directions are a matter of choice. You may take another choice

- Taking initial positions as origin and initial direction of motion (i.e. vertically up) as a positive, a = -g

  So, if a body is projected with velocity *u* and after time *t* it reaches a height *h* then
- For maximum height(H), v = 0

  So from above equation $u = g\,t_1$

  It is called time of ascent $(t_1) = u/g$

  In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance.

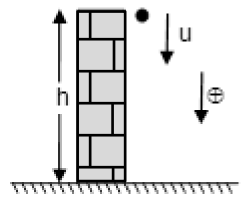
  Time of ascent (t_1) = Time of descent (t_2) = u/g

$\therefore$ Total time of flight $T = t_1 + t_2 = \frac{2u}{g}$ and $u^2 = 2gH \Rightarrow H = \frac{u^2}{2g}$

from $s = vt - at^2$ $\Rightarrow$ t(Time of ascent) = $\sqrt{\frac{2H_{max}}{g}}$

7.2 If A Body Is Projected Vertically Downward With Some Initial Velocity From Some Height:

Taking initial position as origin and direction of motion (i.e., downward direction)as a positive, we have

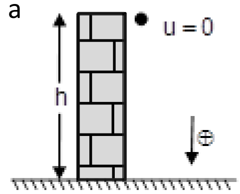


$v = u + gt$, $h = ut + \frac{1}{2}$

$v^2 = u^2 + 2gh$, $h_{n^{th}} = u + \frac{g}{2}(2n-1)$

7.3 If A Body Is Dropped From Some Height (Initial Velocity Zero):

Taking initial position as origin and direction of motion (i.e. downward direction) as a positive, here we have



$u = 0$ [As body starts from rest]

$a = +g$ so $v = g\,t$, $h = \frac{1}{2}gt^2$, $v^2 = 2gh$

7.4 if a body is projected vertically upward with some initial velocity from a certain height:

$v = -u + gt$

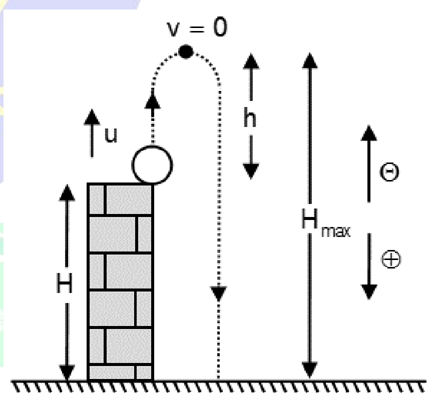
$H = -ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gH$

$h_{nth} = -u + \frac{g}{2}(2n-1)$

7.5 application of equation of motion in motion under gravity :

Taking initial position as origin and upward direction as negative.



$v = -u + gt$; $H = -ut + \frac{1}{2}gt^2 \Rightarrow v^2 = u^2 + 2gh$; $n^{th} = -u + \frac{g}{2}(2n-1)$

- Maximum height attained by the body $H_{max} = H + h = H + \frac{u^2}{2g}$
- Distance travelled by the body $H + 2h = H + \frac{u^2}{2g}$

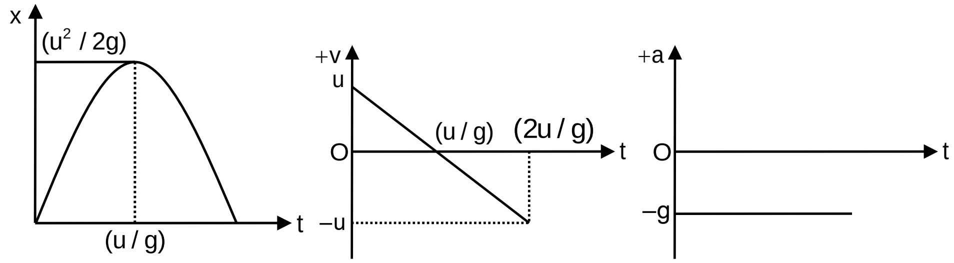
Time taken by the body to reach the ground $H = -ut + \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 -$

$ut - H = 0 \Rightarrow gt^2 - 2ut - 2H = 0$

After solving this equation we get the result.

- In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
- The magnitude of velocity at any point on the path is same whether the body is moving in upward or downward direction.
- Graph of displacement, velocity and acceleration with respect to time.

(For a body projected vertically upward)

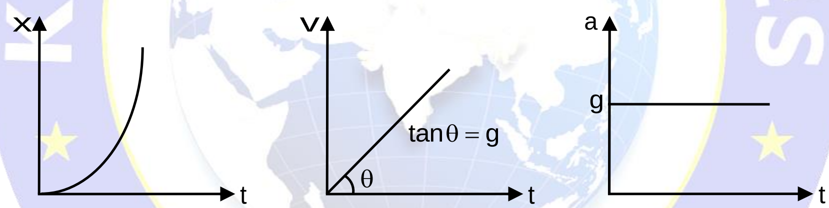


- As h = (1/2) gt^2, i.e. $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e. square of consecutive integers. (in case of free fall, from rest)
- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity

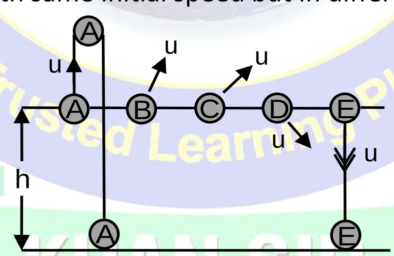
  i.e., $t = \sqrt{\left(\frac{2h}{g}\right)}$ and $v = \sqrt{2gh}$
- The distance covered in the n^{th} second, (For a body dropped from some height)

  $h_n = \frac{1}{2} g(2n-1)$
- Graph of distance, velocity and acceleration with respect to time :

  (For a body dropped from some height)

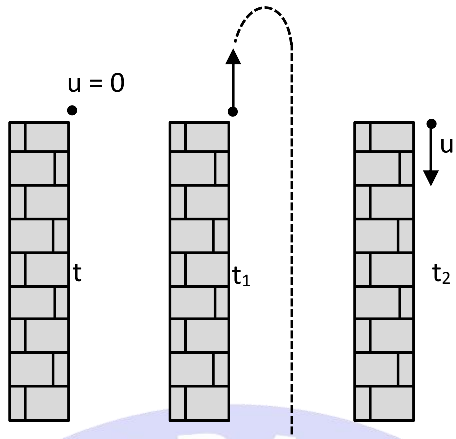


- If various particles are thrown with same initial speed but in different directions then.

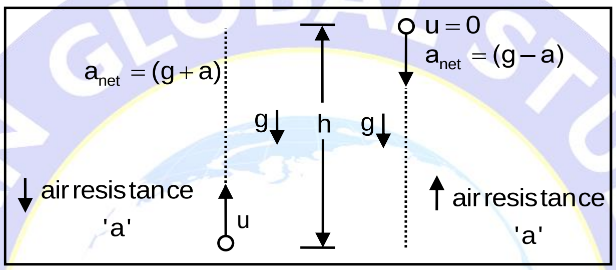


- They strike the ground with same speed at different times irrespective of their initial direction of velocities.
- Time would be least for particle E which was thrown vertically downward.
- Time would be maximum for particle A which was thrown vertically upward.
- A ball is dropped from a building of height h and it reaches ground after time t.

  From the same building if two balls are thrown (one upwards and other downwards) with the same speed u and they reach the ground after t_1 and t_2 seconds respectively then t = $\sqrt{t_1 t_2}$



7.6 Air Drag:



➢ A body is thrown vertically upwards, if constant Air resistance is to be taken into account :

For upward motion :-

Net acceleration $a_{net} = g + a$ (downwards)

If maximum height attained by the particle is 'H' then

$t_{ascent} = \sqrt{\frac{2H}{a_{Net}}}$ $\Rightarrow$ $t_{ascent} = \sqrt{\frac{2H}{g+a}}$

For downward motion : -

Net acceleration $a_{Net} = g - a$ (downwards) So $t_{descent} = \sqrt{\frac{2H}{g-a}}$ Also $t_{descent} > t_{ascent}$

➢ For downwards motion a and g will work in opposite directions because a always acts in direction opposite to motion and g always acts vertically downwards.

SOLVED EXAMPLES

Example 40: A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is 10 m/s^2. (i) How long does it take to reach its highest point ? (ii) How high does it rise above the point of projection ? (iii) How long will it take for the ball to reach a point 10 m above the point of projection ?

Solution: As here motion is vertically upwards, $a = g$ and $v = 0$

(i) From 1st equation of motion, i.e., $v = u + at$, $0 = 20 - 10t$, i.e., $t = 2s$

(ii) From 3rd equation of motion, i.e., $v^2 = u^2 + 2as$, $0 = (20)^2 - 2 \times 10 \times h$, i.e., $h = 20$ m

(iii) From 2nd equation of motion, i.e., $s = ut + \frac{1}{2}at^2$, $10 = 20t - \left(\frac{1}{2}\right) \times 10 \times t^2$

i.e., $t^2 - 4t + 2 = 0$ or $t = 2 \pm \sqrt{2}$, i.e., $t = 0.59$ s or 3.41 s

i.e., there are two such times, as the ball passes twice through h = 10 m once when going up and once when coming down.

Example 41: A juggler throws balls into air. He throws one wherever the previous one is at its highest point. How high do the balls rise if he throws n balls each sec ? Acceleration due to gravity is g.

Solution: As the juggler is throwing n balls each second and 2nd when the first is at its highest point, so time taken by one ball to reach of highest point,

$t = (1/n)$ s and as at highest point $v = 0$, from 1st equation of motion,

$0 = u - (g)(1/n)$, i.e., $u = (g/n)$ .... (i)

Now from 3rd equation of motion, i.e.,

$\upsilon^2 = u^2 + 2as$, $\Rightarrow$ $0 = u^2 - 2gh$, i.e., $h = (u^2/2g)$ or $h = \frac{g}{2n^2}$ $\left[\text{as from Eqn.(1)}, u = \frac{g}{n}\right]$

Example 42: A pebble is released from rest at a certain height and falls freely, reaching an impact speed of 4 m/s at the floor. Next, the pebble is thrown down with an initial speed of 3 m/s from the same height. What is its speed at the floor ?

(1) 3 m/s (2) 5 m/s (3) 8 m/s (4) 12 m/s

Solution: 1st case $v^2 = u^2 + 2gh$ $\Rightarrow$ $16 = 0 + 2 \times 10 \times h$ $\Rightarrow$ $h = \frac{16}{20}$ m

2nd case $v^2 = u^2 + 2gh$ $\Rightarrow$ $v^2 = 9 + 2 \times 10 \times \frac{16}{20}$ $\Rightarrow$ $v^2 = 9 + 16 \Rightarrow v^2 = 25$ [v = 5 m/s]

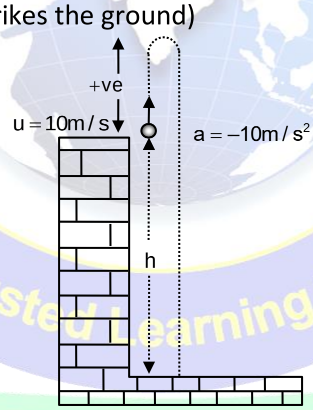
Example 43: A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10$ m/s^2)

(1) 1 s (2) 2 s (3) 3 s (4) 4 s

Solution: In the problem $u = +10$ m/s,

$a = -10$ m/s^2 and $s = -40$ m

(at the point where ball strikes the ground)



Substituting in $s = ut + \frac{1}{2}at^2$

$-40 = 10t - 5t^2$ or $5t^2 - 10t - 40 = 0$

or $t^2 - 2t - 8 = 0$

Solving this we have t = 4 s and – 2 s. Taking the positive value t = 4 s.

Example 44: A body is dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1st second, in 2nd second, in 3rd second etc.

(1) 1 : 3 : 5 ; 1 : 1 : 1 (2) 1 : 4 : 9 ; 1 : 3 : 5 (3) 1 : 3 : 5 ; 1 : 4 : 9 (4) 1 : 1 : 1 ; 1 : 3 : 5

Solution: From second equation of motion, i.e. $h = \frac{1}{2} gt^2$

$h_1 : h_2 : h_3 .... = \frac{1}{2} g(1)^2 : \frac{1}{2} g(2)^2 : \frac{1}{2} g(3)^2 = 1^2 : 2^2 : 3^2 .... = 1 : 4 : 9 : ....$

Now from the expression of distance travelled in n^{th} second $s_n = u + \frac{1}{2} a (2n-1)$

Here $u = 0$, $a = g$ So, $S_n = \frac{1}{2} g(2n-1)$

Therefore $S_1 : S_2 : S_3 ....$ $= \frac{1}{2} g(2 \times 1 - 1) : \frac{1}{2} g(2 \times 2 - 1) : \frac{1}{2} g(2 \times 3 - 1) = 1 : 3 : 5 ....$

Example 45: A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1 minute and it continues to move up.

(i) What is the maximum height reached ?

(1) 18 km (2) 36 km (3) 72 km (4) 108 km

(ii) After the fuel is finished, calculate the time for which it continues its upwards motion. (Take g = 10 m/s^2)

(1) 60 s (2) 120 s (3) 240 s (4) 180 s

Solution: (i) The distance travelled by the rocket during burning interval (1minute = 60s) in which resultant acceleration is vertically upwards and 10 m/s^2 will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 1800$ m = 18 km and velocity acquired by it will be $v = 0 + 10 \times 60 = 600$ m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero,

such that $0 = (600)^2 - 2gh_2$ or $h_2 = 18000$ m = 18 km [$g = 10$ ms^{-2}]

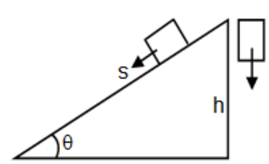
max height = 36 km

(ii) As after burning of fuel the initial velocity 600 m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till its velocity v = 0

$v = u + at \Rightarrow 0 = 600 - 10t \Rightarrow t = 60$ sec

Example 46: A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground: (i) earlier (ii) with greater speed?

Solution: In case of sliding motion on an inclined plane.



$\frac{h}{s} = \sin\theta \Rightarrow$ $s = \frac{h}{\sin\theta}$, $a = g \sin\theta$

$$t_s = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2}{g\sin\theta} \times \frac{h}{\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

$$v_s = \sqrt{2as} = \sqrt{2g\sin\theta \times \frac{h}{\sin\theta}} = \sqrt{2gh} = v_F$$

In case of free fall $t_F = \sqrt{\frac{2h}{g}}$ and $v_F = \sqrt{2gh} = v_s = v_p$

(i) $\because \sin\theta < 1$, $t_F < t_s$, i.e.,falling body reaches the ground first.

(ii) $v_F = v_s$, i.e., both reach the ground with same speed.

Example 47: A particle is thrown vertically upward with the velocity of 15 m/s against gravity and air resistance of 5 m/s^2. Find the time of flight and maximum height attained.

(1) $\sqrt{3}$ s, 15 m (2) $(\sqrt{2}+1)$ s, 15 m (3) $(\sqrt{3}+1)$ s, 30 m (4) $(\sqrt{3}+1)$ s, $\frac{15}{2}$ m

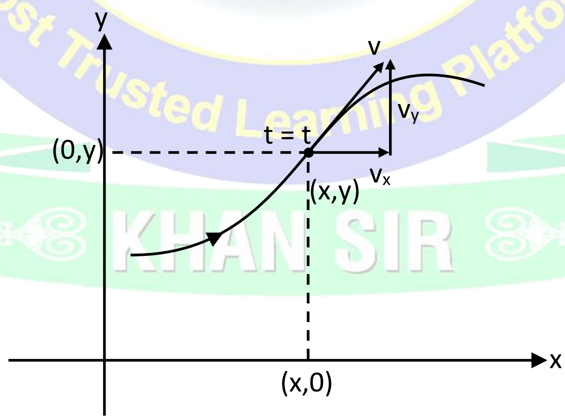
Solution: $v^2 = u^2 + 2ah \Rightarrow 0 = 15^2 - 2 \times (g + a)h \Rightarrow h = \frac{15 \times 15}{2(15)} \Rightarrow \left\{h_{max} = \frac{15}{2} m\right\}$

$t_{ascent} = \sqrt{\frac{2h}{g+a}} = \sqrt{\frac{2 \times 15}{2 \times 15}} = 1$ sec $\Rightarrow t_{descent} = \sqrt{\frac{2h}{g-a}} = \sqrt{\frac{2 \times 15}{2 \times 5}} = \sqrt{3}$ sec

Time of flight = T = $t_{ascent} + t_{descent} = (1 + \sqrt{3})$sec

8. GENERAL TWO DIMENTION (2-D) MOTION:

General two-dimensional motion is the motion where two of the three co-ordinates' changes with time to deal with such motion we consider the motion to be the superposition of two 1-D motion occurring simultaneously along two axes (say x axis and y axis) respectively. Here time is the connecting thread of two 1-D motions. Also all the concepts 1-D motion are applicable as they are, while motion along either axis is dealt but we use component of velocity in place of velocity and component of acceleration in place of acceleration.



Example: $\vec{r} = at^2\hat{i} + 2bt\hat{j}$, a and b are positive constant.

(a) Find equation of trajectory (relation between x and y) i.e. y (x)

(b) Time dependence of angle between $\vec{v}$ and $\vec{a}$ (acceleration)

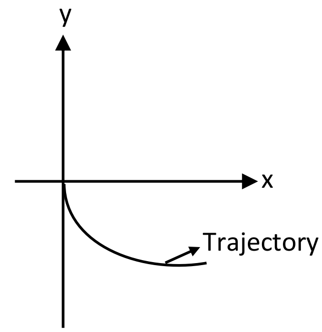
Solution: (a) To find the equation of trajectory

(i) Write equation of displacement along y axis as function at time,t.

(ii) Write equation of displacement along x axis as function of time,t.

(iii) Find the value of time,t from one equation and put the value of time in another equation.

$\vec{r} = at^2\hat{i} + 2bt\hat{j} \Rightarrow x\hat{i} + y\hat{j} = at^2\hat{i} + 2bt\hat{j}$



Motion along y axis

y = -2bt $\Rightarrow$ t = $\frac{y}{-2b}$ .... (1)

Motion along x axis

x = at^2 $\Rightarrow$ x = a$\left(\frac{y}{-2b}\right)^2 \Rightarrow$ x = $\left(\frac{a}{4b^2}\right)y^2$

(b) $\vec{v} = 2at\hat{i} - 2b\hat{j}$ and $\vec{a} = \frac{d\vec{v}}{dt} = 2a\hat{i} + 0\hat{j}$

Motion along x axis a_x = $\frac{dv_x}{dt}$ = 2a and motion along y axis a_y = $\frac{dv_y}{dt}$ = 0

$\vec{a} = 2a\hat{i}$

Say angle between $\vec{a}$ & $\vec{v}$ is θ

$$\cos\theta = \frac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|} = \frac{(2at\hat{i} - 2b\hat{j})\cdot(2a\hat{i} + 0\hat{j})}{\left[\sqrt{(2at)^2 + (2b)^2}\right]\left[\sqrt{(2a)^2 + (0)^2}\right]} = \frac{at}{\sqrt{a^2t^2 + b^2}} \Rightarrow \theta = \cos^{-1}\left(\frac{at}{\sqrt{a^2t^2 + b^2}}\right)$$

Example: A particle moves in xy plane with velocity $\vec{v} = a\hat{i} + bx\hat{j}$ where a and b are constants. At intial moment of time (at t = 0) the particle was located at x = y = 0 find equation of trajectory of the particle.

Solution: $\vec{v} = a\hat{i} + bx\hat{j} \Rightarrow v_x\hat{i} + v_y\hat{j} = a\hat{i} + bx\hat{j}$

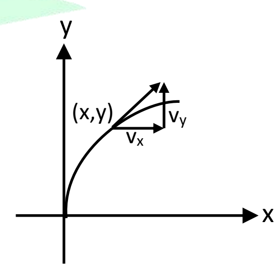
Motion along x axis $\rightarrow$ $v_x = a \Rightarrow \frac{dx}{dt} = a \Rightarrow \int_0^x dx = \int_0^t a dt \Rightarrow x = at \Rightarrow t = \frac{x}{a}$

Motion along y axis $\rightarrow$ $v_y = bat \Rightarrow \int_0^y dy = \int_0^t batdt \Rightarrow y = \frac{abt^2}{2}$

So equation of trajectory is $y = \frac{ab}{2}\left(\frac{x}{a}\right)^2 \Rightarrow y = \left(\frac{b}{2a}\right)x^2$

Example: A balloon start moving a constant velocity v_0 (ascending rate) d_2 wind the ballon acquires a horizontal velocity component v_x = ay when a is positive constant and y is hight of ascent find dependence of horizontal drift on hight of ascent

Solution: Motion along y axis$\rightarrow$ $v_y = v_0 = \frac{dy}{dt} \Rightarrow \int_0^y dy = \int_0^t v_0 dt \Rightarrow y = v_0 t \Rightarrow t = \frac{y}{v_0}$



Motion along x axis$\rightarrow$ $v_x = \frac{dx}{dt} = ay \Rightarrow \int_0^x dx = \int_0^t av_0 tdt \Rightarrow x = \frac{av_0t^2}{2}$, putting value of t

we get equation of trajectory$\rightarrow$ $x = \frac{av_0}{2} \times \frac{y^2}{v_0^2} \Rightarrow x = \left(\frac{a}{2v_0}\right)y^2$

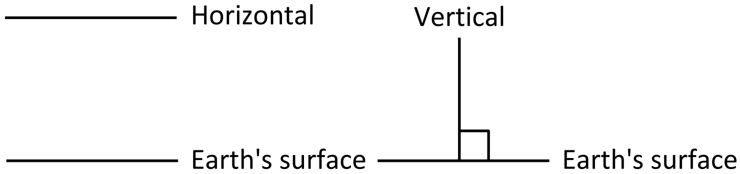
9. PROJECTILE MOTION :

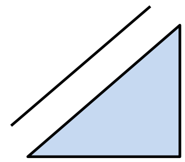
Assumption:

Horizontal and verticall is defined with respect to the earth surface

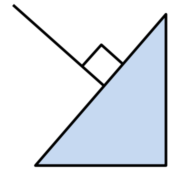
Horizontal: Parallel to earth surface

Vertical: Perpendicular to earth surface





[Parallel to inclined plane but not horizontal]



[Perpendicular to inclined plane but not verticall]

SPOT LIGHT

- Horizontal and vertical are not universal direction as horizontal direction for one observer standin at the pole is verticall for other observer standing at equator. Here we will talk about small range distance so for all observer horizontal and verticall will be same.

When a body is projected such that velocity of projection is not parallel to the force (θ other then 0° and 180°), then it moves along a curved path. If force on the body is constant then curved path of the body is parabolic This motion is studied under projectile motion.

(i) A particle thrown in the space which moves under the effect of gravity only is called a "projectile".

(ii) Any particle under the influence of the only force "gravity" $(m\vec{g})$ will have an acceleration equals "g" vertically downwards near the earth surface.

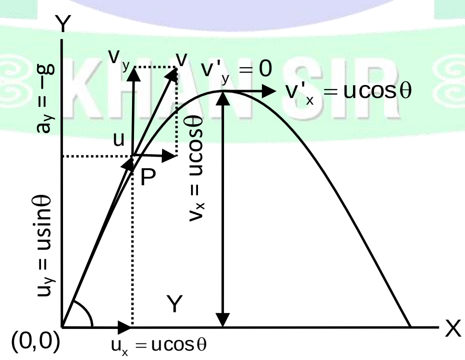
(iii) Value of "g" near earth's surface is $g = 10 m/s^2$ or $9.8 m/s^2$ or $980 cm/s^2$ or $32 feet/s^2$

(iii) It is an example of motion with constant (or uniform) acceleration. Thus, equations of motion can be used to analysis projectile motion.

(iv) If a particle posse a uniform acceleration in a directions oblique to its initial velocity, the resultant path will be parabolic. Let X-axis is along the ground and Y-axis is along the vertical, then path of projectile projected at an angle θ from the ground is as shown.

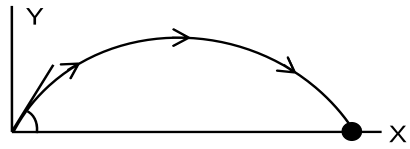
DETECTIVE MIND

Projectile motion may be 1-D motion (it is projected vertically upward) or 2-D motion (if it is projected at some angle with the horizontal except 90°)

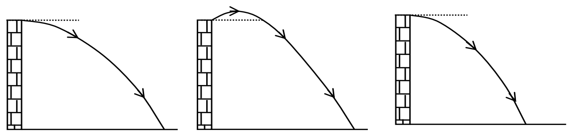


Types of Projectile

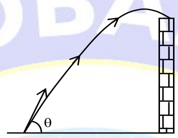
(i) Ground to Ground or Level to level Projectile



(ii) Hight to Ground Projectile



(iii) Ground to Hight Projectile



10. GROUND TO GROUND PROJECTION :

Projectile motion can be considered as two mutually perpendicular motions, which are independent of each other i.e. Projectile motion = Horizontal motion + Vertical motion

10.1 Horizontal Motion:

Initial velocity in horizontal direction = u_x = u cosθ

Acceleration along horizontal direction = a_x =0. (Neglect air resistance)

Therefore, horizontal velocity remains unchanged.

- At any instant horizontal velocity u_x = u cosθ
- At time t, x coordinate or displacement along X-direction is

$x = u_x t \quad \Rightarrow \quad x = (u\cos\theta)t$

10.2 Vertical Motion:

It is motion under the effect of gravity

Initial velocity in vertical direction u_y = u sinθ

Acceleration along vertical direction = a_y = – g

- At time t, vertical speed $v_y = u_y - gt = u\sin\theta - gt$
- In time t, displacement in vertical direction or "height" of the particle above the ground

$y = u_y t - \frac{1}{2} gt^2 = (u\sin\theta)\, t - \frac{1}{2} gt^2$

10.3 Net Motion:

Net initial velocity = = $u_x\,\hat{i} + u_y\,\hat{j} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Direction of u can be explained in terms of angle θ it makes with the horizontal

Net acceleration $= \vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$ (direction of g is downwards)

10.4 Coordinates of Particle At Time T:

Let (x, y) are coordinates of particle at general time t, then

$x = u_x t$ and $y = u_y t - \frac{1}{2} g t^2$

Net displacement in t time = $\sqrt{x^2 + y^2}$

10.5 Velocity Of Particle At Time T :

$= v_x\,\hat{i} + v_y\,\hat{j} = u_x\,\hat{i} + (u_y - gt)\,\hat{j} \qquad = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

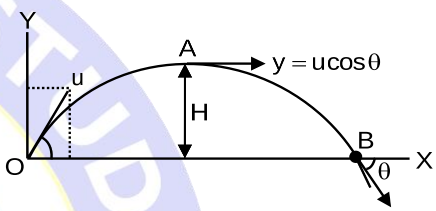
magnitude of velocity $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

If angle made by velocity with the ground is α, then

$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} \quad \Rightarrow \quad \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$

10.6 Change In Velocity And Momentum Of Projectile :

When particle returns to ground again at B point, its y coordinate is zero and the magnitude of its velocity is u at angle θ with ground. Total angular change = 2θ



Initial velocity $\vec{u}_1 = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Final velocity $\vec{u}_1 = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

Total change in velocity $|\Delta\vec{v}| = 2u\sin\theta$, Total change in momentum

$|\Delta\vec{p}| = m|\Delta\vec{v}| = 2u\sin\theta$

- change in speed = 0

10.7 Time Of Flight (T):

At time T particle will be at ground again, i.e. displacement along Y-axis becomes zero.

$\therefore y = u_y t - \frac{1}{2} g t^2 \qquad \therefore 0 = u_y T - \frac{1}{2} g T^2 \qquad$ or $\qquad T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Time of ascent = Time of descent $\quad \Rightarrow \quad \frac{T}{2} = \frac{u_y}{g} = \frac{u\sin\theta}{g}$

at time $\frac{T}{2}$ particle attains maximum height of its trajectory.

10.8 Maximum Height (H):

At maximum height, vertical component of velocity becomes zero. At this instant y coordinate is maximum

$\because v_y^2 = u_y^2 - 2gy \qquad \because 0 = u_y^2 - 2gy_{max} \Rightarrow H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

10.9 Horizontal Range Or Range (R):

It is the displacement of particle along X-direction during its complete flight.

$\because x = u_x t \;\therefore\; R = u_x T = u_x \frac{2u_y}{g} \Rightarrow R = \frac{2u_x u_y}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g} = \frac{u^2\sin 2\theta}{g}$

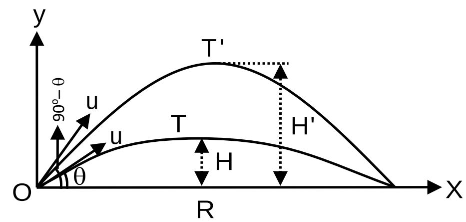
10.10 Maximum Horizontal Range (R_{max}):

If value of θ is increased from $\theta = 0°$ to 90°, then range increases from $\theta = 0°$ to 45° but it decreases beyond 45°. Thus range is maximum at θ = 45°

For maximum range, θ = 45° and $R_{max} = \frac{u^2\sin 2(45°)}{g} \Rightarrow R_{max} = \frac{u^2}{g}$

10.11 Comparison Of Two Projectiles Of Equal Range:

When two projectiles are thrown with equal speed at angles θ and $(90^o-\theta)$ with horizontal then their ranges are equal but maximum heights attained are different and time of flights are also different.



At angle θ, $R = \dfrac{u^2 \sin 2\theta}{g}$

At angle $(90^o - \theta)$, $R' = \dfrac{u^2 \sin 2(90^o - \theta)}{g} = \dfrac{u^2 \sin 2\theta}{g} \Rightarrow$ Thus, R' = R

Maximum heights of projectiles

$H = \dfrac{u^2 \sin^2 \theta}{2g}$ and $H' = \dfrac{u^2 \sin^2 (90^o - \theta)}{2g} = \dfrac{u^2 \cos^2 \theta}{2g}$

- $\dfrac{H}{H'} = \dfrac{\sin^2 \theta}{\cos^2 \theta} = \tan^2\theta$ • $HH' = \dfrac{u^4 \sin^2 \theta \cos^2 \theta}{4g^2} = \dfrac{R^2}{16} \Rightarrow R = 4\sqrt{HH'}$
- $H + H' = \dfrac{u^2 \sin^2 \theta}{2g} + \dfrac{u^2 \cos^2 \theta}{2g} \Rightarrow H + H' = \dfrac{u^2}{2g}$

Time of flight of projectiles

$T = \dfrac{2u \sin \theta}{g}$; $T' = \dfrac{2u \sin (90^o - \theta)}{2g} = \dfrac{2u \cos \theta}{g}$

- $\dfrac{T}{T'} = \dfrac{\sin \theta}{\cos \theta} = \tan\theta$ • $T\,T' = \dfrac{4u^2 \sin\theta \cos\theta}{g^2} = \dfrac{2R}{g} \Rightarrow T\,T' \propto R$

10.12 Equation Of Trajectory :

Along horizontal direction $x = u_x t$ or $x = (u\cos\theta)\, t$

Along vertical direction $y = u_y t - \dfrac{1}{2} g t^2$ or $y = (u \sin\theta)\, t - \dfrac{1}{2} g t^2$

On eliminating from these two equations

$$y = (u \sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2 \Rightarrow y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Again $$y = x \tan\theta \left[1 - \frac{gx}{2u^2 \sin\theta \cos\theta}\right] \Rightarrow y = x \tan\theta \left[1 - \frac{x}{R}\right]$$

10.13 Kinetic Energy of Projectile:

Kinetic energy = $\dfrac{1}{2}$ × Mass × (Speed)^2

Let a body is projected with velocity u at an angle θ.

Thus, initial kinetic energy of projectile, $K_0 = \dfrac{1}{2} mu^2$

Since velocity of projectile at maximum height is $u \cos \theta$. So $K_{hmax} = \dfrac{1}{2} mu^2 \cos^2\theta = K_0 \cos^2 \theta$

which is the minimum kinetic energy during whole motion.

- At maximum height, $v_y = 0$ and $v_x = u_x = u\cos\theta$ so at maximum height $v = \sqrt{v_x^2 + v_y^2}$

- At maximum height angle between velocity and acceleration is 90°.
- Magnitude of velocity at height 'h'.

  Vertical component of velocity at height h

  $v_y^2 = u_y^2 - 2gh \quad \Rightarrow \quad v_y^2 = (u\sin\theta)^2 - 2gh$

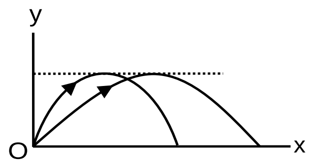
  Horizontal component of velocity at height h

  $v_x = u\cos\theta \quad \Rightarrow \quad |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u^2\sin^2\theta - 2gh)} \Rightarrow |\vec{v}| = \sqrt{u^2 - 2gh}$

- $T = \frac{2u_y}{g}$; $H = \frac{u_y^2}{2g}$; $R = \frac{2u_x u_y}{g}$

  T and H depend only upon initial vertical speed u_y
- If two projectiles thrown in different directions, have equal times of flight then their initial vertical speeds are same so that their maximum height are is also same.

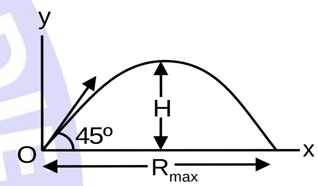
  If $H_A = H_B$ then $(U_y)_A = (u_y)_B$ and $T_A = T_B$



- For situation shown in figure for $\theta = 45°$

  Here $R_{max} = \frac{u^2}{g}$ and $H = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g}$

  $\therefore$ $R_{max} = 4H = 4$(maximum height attained)



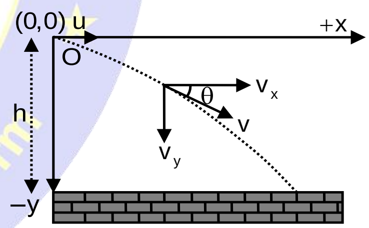
- When R = H $\Rightarrow$ $R = \frac{u^2(2\sin\theta\cos\theta)}{g}$ and $H = \frac{u^2\sin^2\theta}{2g}$

  $\Rightarrow \frac{R}{H} = 4\cot\theta = 1 \Rightarrow 4\cot\theta = 1 \Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4)$

11. HORIZONTAL PROJECTION FROM HEIGHT :

Consider a projectile thrown from point O at some height h from the ground with a velocity u in horizontal direction.

Now we shall deal the characteristics of projectile motion separately along horizontal and vertical directions i.e.



Horizontal direction:	Vertical direction:
(i) Initial velocity $u_x = u$	Initial velocity $u_y = 0$
(ii) Acceleration $a_x = 0$	Acceleration $a_y = -g$

11.1 Trajectory Equation :

The path traced by projectile is called its trajectory. After time t, x = ut and $y = -\frac{1}{2}gt^2$ negative sign indicates that the direction of vertical displacement is downward.

so $y = -\frac{1}{2}g\frac{x^2}{u^2} \quad \left(\because t = \frac{x}{u}\right)$

This is equation of a parabola. Above equation is called trajectory equation

11.2 Velocity at a general point P (x,y)

$v = \sqrt{v_x^2 + v_y^2}$

Horizontal velocity of the projectile after time t is $v_x = u$ (remains constant)

Velocity of projectile in vertical direction after time t is

$v_y = 0 - (g)t = -gt$ (downward) $\qquad \therefore v = \sqrt{v^2 + g^2t^2}$

and $\tan\theta = \frac{v_y}{v_x}$ or $\tan\theta = -\frac{gt}{u}$ (negative sign indicates clockwise direction)

11.3 Displacement:

The displacement of the particle is expressed by $\vec{s} = x\hat{i} + y\hat{j}$

Where $|\vec{s}| = \sqrt{x^2 + y^2} = \left|(ut)\hat{i} - \left(\frac{1}{2}gt^2\right)\hat{j}\right|$

11.4 Time of flight :

From equation of motion for vertical direction. $h = u_y t + \frac{1}{2}$

At highest point $u_y = 0 \Rightarrow h = gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$

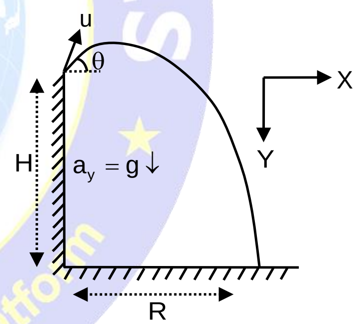
11.5 Velocity after falling a height h_1:

Along vertical direction = $0^2 + 2(h_1)(g) \Rightarrow v_y = \sqrt{2gh_1}$

Along horizontal direction $v_x = u_x = u$. So, velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$

12. OBLIQUE PROJECTION FROM A CERTAIN HEIGHT :

12.1 Projection from a height at an angle θ above horizontal :



$u_x = u\cos\theta$ $u_y = -u\sin\theta$

$x = (u\cos\theta)\, t$ $a_y = g$

$H = (-u\sin\theta)\, t + \frac{1}{2}gt^2$ $gt^2 - (2u\sin\theta)\, t - 2H = 0$

After solving the above equation we get the result

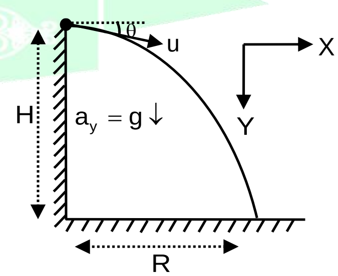
12.2 Velocity after falling height h :

Along vertical direction ; = $(-u\sin\theta)^2 + 2(h)(g)$

Along horizontal direction, $v_x = u_x = u\cos\theta$;

So, velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

12.3 Projection from a height at an angle θ below horizontal :



$u_x = u\cos\theta$ $u_y = u\sin\theta$

$c = (u\cos\theta)\, t$ $a_y = g$

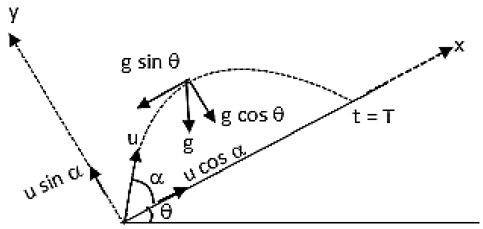
$H = (u\sin\theta)\, t + \frac{1}{2}gt^2$ $gt^2 + (2u\sin\theta)\, t - 2H = 0$

After solving the above equation we get the result.

12.4 Velocity after falling height h :

Along vertical direction, Along horizontal direction, $v_x = u_x = u\cos\theta$; So, velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

12.5 Projection on inclined surface



angle made by inclined plane with horizontal = θ

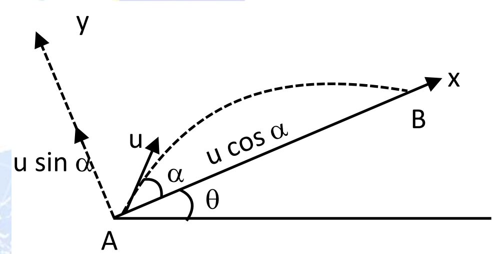
angle of projection with inclined plane = α

angle of projection with horizontal = $\alpha + \theta$

motion along x axis motion along y axis

motion along x axis	motion along y axis
$u_x = u\cos\alpha$	$u_y = u\sin\alpha$
$a_x = -g\sin\alpha$	$a_y = -g\cos\theta$
	$s_y = 0$
	for time flight apply $s_y = u_y t + 1/2\, a_y t^2$ along y axis
	$0 = u\sin\alpha\, t - 1/2\, g\cos\theta\, T^2$
	$T = 0,\ \dfrac{2u\sin\alpha}{g\cos\theta}$
	$\boxed{T = \dfrac{2u\sin\alpha}{g\cos\theta}}$

t = 0 is the time when particle is at the point of projection while projecting the projectile By to find AB (range along inclined plane) apply motion equation along x axis



$s_x = u_x T + 1/2\, a_x T^2$

$sx = u\cos\alpha\, T - \dfrac{1}{2}\, g\sin\theta\, T\, 2$

$= uT\left[\cos\alpha - \dfrac{g\sin\theta}{2}\dfrac{2u\sin\alpha}{ug\cos\theta}\right]$

$AB = uT\left[\dfrac{\cos\alpha\cos\theta - \sin\theta\sin\alpha}{\cos\theta}\right]$ we get AB (range) after putting the value T

AB (range) $\dfrac{2u^2\left[\cos(\theta+\alpha)\sin\alpha\right]}{g\cos^2\theta}$

SOLVED EXAMPLES

Example 48: Find the equation of trajectory of the body if the position vector of the particle is given as

(i) $r = at\hat{i} - bt^2\hat{j}$ (ii) $\vec{r} = a\sin\omega t\,\hat{i} - a\cos\omega t\,\hat{j}$

(iii) $\vec{r} = 3\sin\omega t\,\hat{i} + 4\cos\omega t\,\hat{j}$ (iv) $\vec{r} = 2\sin\omega t\,\hat{i} - 3\sin\omega t\,\hat{j}$

Solution: (i) As x = at and y = $-bt^2$ on solving the equation we get $y = -\dfrac{bx^2}{a^2}$: so, path is parabola

(ii) $x = a\sin\omega t$, $y = -a\cos\omega t \Rightarrow x^2 + y^2 = a^2$: so, path is circle

(iii) $x = 3\sin\omega t$, $y = 4\cos\omega t \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$: so, path is ellipse

(iv) $x = 2\sin\omega t$, $y = -3\sin\omega t \Rightarrow y = -\dfrac{3}{2}x$: so, path is straight line

Example 49: A projectile is thrown with speed u making angle θ with horizontal at t = 0. It just crosses two points of equal height at time t = 1s and t = 3s respectively. Calculate the maximum height attained by it? ($g = 10\ m/s^2$)

(1) 4 m (2) 12 m (3) 16 m (4) 20 m

Solution: Displacement in y direction

$$y = u_y \times 1 - \frac{1}{2} g \times (1)^2 = u_y \times 3 - \frac{1}{2} g(3)^2 \Rightarrow u_y = 2g = 20\ m/s$$

Maximum height attained $h_{max} = \frac{u_y^2}{2g} = 20m.$

Example 50: A stone is to be thrown so as to cover a horizontal distance of 3 m. If the velocity of the projectile is 7 m/s find:

(i) the angle at which it must be thrown

(1) 18.5° (2) 37° (3) 71.5° (4) Both (1) and (3)

(ii) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

(1) 3 m (2) 5 m (3) 8 m (4) 10 m

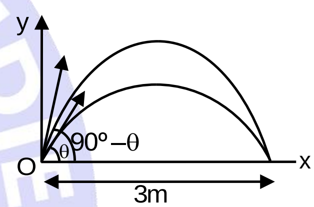
Solution: (i) Range $R = \frac{u^2}{g} \sin 2\theta \Rightarrow \sin 2\theta = \frac{gR}{u^2} = \frac{9.8 \times 3}{(7)^2} = 0.6 = \sin 37^o$

$\Rightarrow 2\theta = 37^o \Rightarrow \theta = 18.5^o$

angle of projection may also be $90^o - 18.5^o = 71.5^o$

(ii) For largest horizontal displacement $\theta = 45^o$ maximum range

$$R_{max} = \frac{u^2}{g} = \frac{(7)^2}{9.8} = \frac{49}{98} \times 10 = 5m.$$



Example 51: Two projectiles are projected at angles (θ) and $\left(\frac{\pi}{2} - \theta\right)$ to the horizontal respectively with same speed 20 m/s. One of them rises 10 m higher than the other. Find the angles of projection. (Take $g = 10\ m/s^2$)

(1) 37°, 53° (2) 30°, 60° (3) 25°, 55° (4) 18.5°, 71.5°

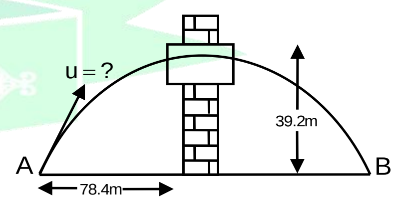
Solution: Maximum height $H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow h_1 = \frac{(20)^2 \sin^2\theta}{2g} = 20 \sin^2\theta$ & $h_2 = \frac{(20)^2 \sin^2(\pi/2 - \theta)}{2g} = 20\cos^2\theta$

$$h_2 - h_1 = 20[\cos^2\theta - \sin^2\theta] = 10 \Rightarrow 20\cos 2\theta = 10 \Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = 60^o \Rightarrow \theta = 30^o \text{ and } \theta' = 90^o - \theta = 60^o$$

Example 52: A boy stands 78.4 m away from a building and throws a ball which just enters a window at maximum height 39.2m above the ground. Calculate the velocity of projection of the ball.

(1) 19.8 m/s (2) 39.2 m/s

(3) 54.6 m/s (4) 24.4 m/s



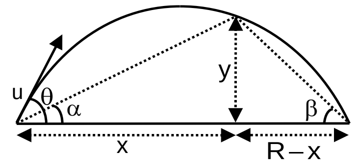
Solution: Maximum height $= \frac{u^2 \sin^2\theta}{2g}$ m....(i)

Range $= \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g} = 2 \times 78.4$....(ii)

from equation (i) divided by equation (ii) $\tan\theta = 1 \Rightarrow \theta = 45^o$

from equation (ii) range $= \frac{u^2 \sin 90^o}{g} = 2 \times 78.4 \Rightarrow u = \sqrt{2 \times 78.4 \times 9.8} = 39.2\ m/s$

Example 53: A particle thrown over a triangle from one end of a horizontal base falls on the other end of the base after grazing the vertex. If α and β are the base angles of triangle and angle of projection is θ, then prove that $\tan\theta = \tan\alpha + \tan\beta$.



Solution: From triangle $y = x\tan\alpha$ and $y = (R-x)\tan\beta$

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{yR}{x(R-x)}$$

$$\because y = x\tan\theta\left[1-\frac{x}{R}\right] \Rightarrow \tan\theta = \frac{yR}{x(R-x)}$$

$$\therefore \quad \tan\theta = \tan\alpha + \tan\beta$$

Example 54: A particle is projected from the ground at an angle such that it just clears the top of a pole after t_1 time in its path. It takes further t_2 time to reach the ground. What is the height of the pole ?

(1) $\frac{1}{2}g(t_1+t_2)$ (2) $\frac{1}{4}g\sqrt{t_1+t_2}$ (3) $\frac{g}{2}(t_1+t_2)^{1/2}$ (4) $\frac{1}{2}gt_1t_2$

Solution: Height of the pole is equal to the vertical displacement of the particle at time t_1

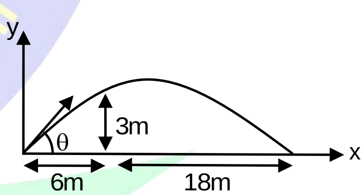
Vertical displacement $y = u_yt_1 + \frac{1}{2}gt_1^2$ .....(i) and total flight time $t_1 + t_2 = \frac{2u_y}{g} \Rightarrow u_y = \frac{g}{2}(t_1+t_2)$

put value u_y in equation (i) $\Rightarrow y = \frac{g}{2}(t_1+t_2)t_1\ \frac{1}{2} - g(t_1)^2 = \frac{1}{2}gt_1t_2$, so height of the pole $= \frac{1}{2}gt_1t_2$

Example 55: A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is :-

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\tan^{-1}\left(\frac{2}{3}\right)$ (3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $\tan^{-1}\left(\frac{3}{4}\right)$

Soluiton: From equation of trajectory, $y = x\tan\theta\left[1-\frac{x}{R}\right]$



$$\Rightarrow 3 = 6\tan\theta\left(1-\frac{6}{24}\right) \Rightarrow \tan\theta = \frac{2}{3}$$

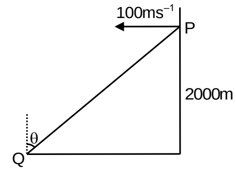
Example 56: A particle is projected with the velocity $v = a\hat{i}+b\hat{j}$ then calculate range, height and time of flight of particle.

Solution: Horizontal velocity $= v_x = a$ Vertical velocity $= v_y = b$

Then time of flight $= \frac{2b}{g}$ Range of projectile $= \frac{2b}{g} \times a = \frac{2ab}{g}$

Maximum height of projectile $= b^2 = 2gh \Rightarrow h = \frac{b^2}{2g}$

Example 57: An aeroplane is traveling horizontally at a height of 2000 m from the ground. The aeroplane when at a point P, drops a bomb to hit a stationary target Q on the ground. In order that the bomb hits the target, what angle θ must the line PQ make with the vertical ?



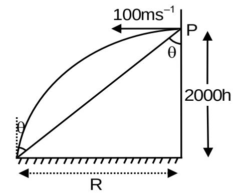
[g = 10 ms^{-2}]

(1) 37° (2) 30° (3) 45° (4) 53°

Solution: Let t be the time taken by bomb to hit the target.

$h = 2000 = \frac{1}{2}gt^2 \Rightarrow t = 20s$ $R = ut = (100)(20) = 2000$ m

$\tan\theta = \frac{R}{h} = \frac{2000}{2000} = 1 \Rightarrow \theta = 45°$



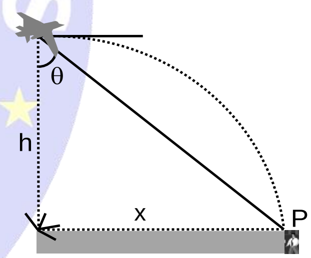
Example 58: A relief aeroplane is flying at a constant height of 1960 m with 600 km/hr speed above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water ? (g = 9.8 m/s^2)

(1) 60° (2) 30° (3) 45° (4) 90°

Solution: Plane is flying at a speed = $600 \times \frac{5}{18} = \frac{500}{3}$ m/s horizontally (at a height 1960m)

time taken by the kit to reach the ground

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times1960}{9.8}} = 20$ s



in this time the kit will move horizontally by

$x = ut = \frac{500}{3} \times 20 = \frac{10,000}{3}$ m

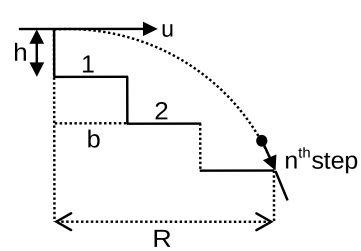
So $\tan\theta = \frac{x}{h} = \frac{10,000}{3\times1960} = \frac{10}{5.88} = 1.7 = \sqrt{3}$

or $\theta = 60°$

Example 59: A ball rolls off the top of a stairway with a horizontal velocity u. If each step has height h and width b the ball will just hit the edge of n^{th} step. Find the value of n.

(1) $\frac{2hu^2}{gb^2}$ (2) $\frac{hu^2}{gb^2}$ (3) $\frac{gb^2}{2hu^2}$ (4) $\frac{gb^2}{hu^2}$

Solution: If the ball hits the nth step, the horizontal and vertical distances traversed are nb and nh respectively. Let be the time taken by the ball for these horizontal and vertical displacements. Velocity along horizontal direction = u (remains constant) and initial vertical velocity = zero.



$nb = ut$ and $nh = 0 + \frac{1}{2}gt^2$

Eliminating t from the equation

$$nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2 \Rightarrow n = \frac{2hu^2}{gb^2}$$

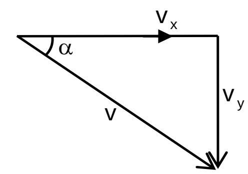
Example 60: A particle is projected horizontally with a speed 20 m/s from the top of a tower. After what time will the velocity of particle be at 45° angle from the initial direction of projection? [Let g = 10 ms^{-2}].

(1) 1 s (2) 2 s (3) 4 s (4) 6 s

Solution: Let x and y axes be adopted along horizontal and vertically downward directions respectively.

After time t, $v_x = u_x = 20\ ms^{-1}$, velocity in y direction

$v_y = u_y + a_y t = 0 + gt = gt$



$$\tan\alpha = \frac{v_y}{v_x} = \frac{gt}{20} = \text{ if } \alpha = 45^o$$

$$\text{than } 1 = \frac{10t}{20} \Rightarrow t = 2s$$

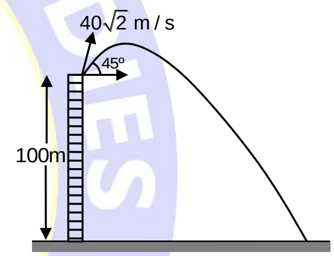
Example 61: For shown projection from a tower, find the :

(i) time of flight,

(ii) horizontal range,

(iii) maximum height attained from ground,

(iv) striking speed. ($g = 10\ m/s^2$)



Solution: (i) By using $s = ut + \frac{1}{2}at^2$ in vertical direction

$$-100 = 40(T) + \frac{1}{2}(-10)T^2 \qquad \Rightarrow T^2 - 8T - 20 = 0 \Rightarrow T = 10\ s$$

(ii) Horizontal range $R = u_x T = (40)(10) = 400$ m

(iii) Maximum height $= 100 + \frac{u_y^2}{2g} = 100 + \frac{(40)^2}{2(10)} = 180$ m

(iv) Striking speed $v = \sqrt{u^2 + 2gh} = \sqrt{(40\sqrt{2})^2 + 2(10)(100)} = 20\sqrt{13}$ m/s

RELATIVE MOTION:

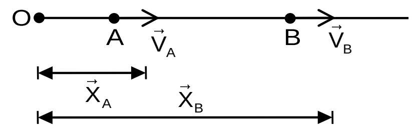
13. FRAME OF REFRENCE :

It is the place from where observation are taken or the place from where any change in parameter of motion (distance, displacement, velocity, acceleration, energy, force etc.) is noticed.

- The observer must be as rest with respect to that place.
- The term of "frame of refrence" stands for observer only so it has nothing to do with the object.
- A man sitting inside a moving train may called him self his seat overt the train to be his "frame of reference" as another man moving inside the same train can not call the train to be his "fram of reference" as he is not rest with respect to the train.

13.1 RELATIVE VELOCITY IN ONE DIMENSION :

When we consider the motion of a particle, we assume a fixed point relative to which the given particle is in motion. For example, if we say that water is flowing or wind is blowing or a person is running with a speed u, we mean that these all are relative to the earth (which we have assumed to be fixed).



Displacement of B with respect to A = Displacement of B as measured from A

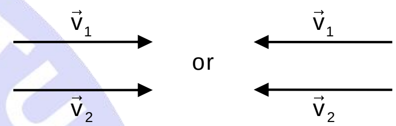
$\Rightarrow \quad \vec{x}_{BA} = \vec{x}_B - \vec{x}_A \qquad \Rightarrow \quad \dfrac{d\vec{x}_{BA}}{dt} = \dfrac{d\vec{x}_B}{dt} - \dfrac{d\vec{x}_A}{dt}$

$\Rightarrow \quad \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ Relative = Actual - Reference

13.2 While moving in same frame of reference :

- **For same direction :**

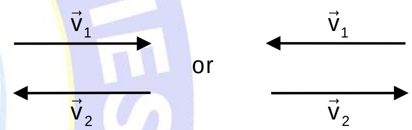
When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.



$|\vec{v}_{12}|$ or $|\vec{v}_{21}|$ = $v_1 - v_2$

- **For opposite directions :**

When two particles are moving in the opposite directions, then magnitude of their relative velocity is always equal sum of their individual speeds.



$|\vec{v}_{12}|$ or $|\vec{v}_{21}|$ = $v_1 + v_2$

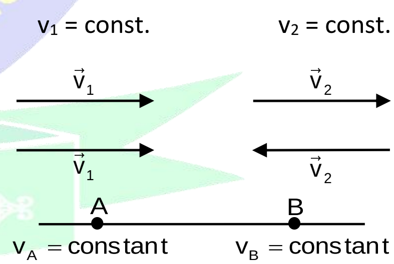
Note :-

When two particles move simultaneously then the concept of relative motion becomes applicable conveniently.

13.3 Numerical Applications

- When two particles are moving along a straight line with constant speeds then their relative acceleration must be zero and, in this condition, relative velocity is the ratio of relative displacement to time.

when $a_{rel.} = 0$



- When two particles move in such a way that their relative acceleration is non zero but constant then we apply equation of motion in the relative form.

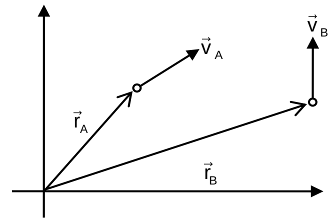
$a_A = 0 \qquad a_B = a$

$a_{AB} = a_A - a_B \qquad a_{AB} = 0 - a = -a$ = constant

Equation of Motion (Relative)

- $v_{rel.} = u_{rel.} + a_{rel.}t$
- $s_{rel.} = u_{rel.}t + \dfrac{1}{2} a_{rel.}t^2$
- $v_{rel.}^2 = u_{rel.}^2 + 2a_{rel.}\, s_{rel.}$
- $s_{rel.} = \dfrac{1}{2}(u_{rel.} + v_{rel.})t$

13.4 RELATIVE VELOCITY IN A PLANE



- **For 2D motion:**

Relative velocity of A with respect to B can be calculated as

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\Rightarrow |\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$

13.5 For two particles to collide :

- their combined relative displacement becomes zero.
- their combined vertical velocities will be same: if they are projected from same level (in case of projectiles)
- their combined motion can be converted into 1-D motions.

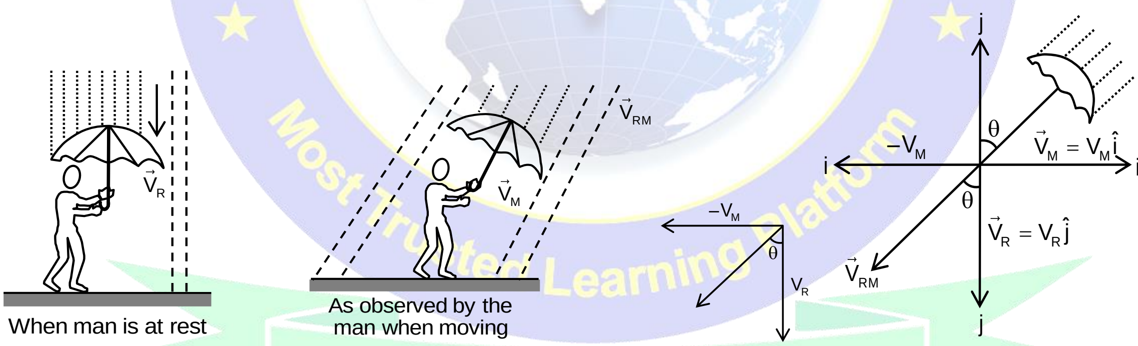
13.6 Relative path of a projectiles w.r.t. another projectile :

Two projectiles are thrown from ground with different velocities at different angles. Since both projectiles have equal accelerations so their relative acceleration is zero. Thus path of one projectile w.r.t. other is a straight line and motion of one projectile w.r.t. other is uniform.

- If $u_1 \cos\theta_1 = u_2 \cos\theta_2$ then relative path is a vertical line.
- If $u_1 \sin\theta_1 = u_2 \sin\theta_2$ then relative path is a horizontal line.

14. RAIN-MAN PROBLEM :

(i) If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ the velocity of rain relative to observer will be



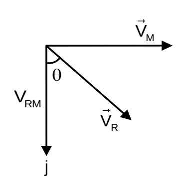
$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M \Rightarrow \vec{v}_{RM} = -\vec{v}_R\hat{j} - \vec{v}_M\hat{i}$

which by law of vector addition has magnitude

$v_{RM} = \sqrt{v_R^2 + v_M^2}$

The direction of $\vec{v}_{RM}$ is such that it makes an angle θ with the vertical given by $\theta = \tan^{-1}(v_m/v_R)$ as shown in figure.

(ii) If rain is already falling at an angle θ with the vertical with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ find that the rain drops are hitting on his head vertically downwards



Here $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M \Rightarrow \vec{v}_{RM} = (v_R \sin\theta - v_M)\hat{i} - v_R\cos\theta\hat{j}$

Now for rain to appear falling vertically, the horizontal component of $\vec{v}_{RM}$

should be zero, i.e.

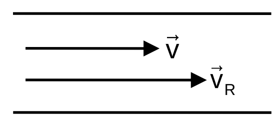
$$v_R \sin\theta - v_M = 0 \Rightarrow \sin\theta = \frac{v_M}{v_R} \text{ and } \quad |\vec{v}_{RM}| = v_R\cos\theta = v_R\sqrt{1-\sin^2\theta} = v_R\sqrt{1-\frac{v_M^2}{v_R^2}} \Rightarrow v_{RM} = \sqrt{v_R^2 - v_M^2}$$

15. RIVER-BOAT (OR MAN) PROBLEM :

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water.

If water is also flowing with the velocity $\vec{v}_R$ then velocity of man relative to ground

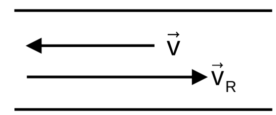
$\vec{v}_m = \vec{v} + \vec{v}_R$



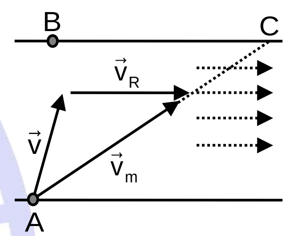
(i) If the swimming is in the direction of flow of water or along the downstream then

$v_m = v + v_R$

(ii) If the swimming is in the direction opposite to the flow of water or along the upstream then $v_m = v - v_R$



(iii) If the man is crossing the river i.e. $\vec{v}$ and $\vec{v}_R$ are non colinear then use vector algebra. $\vec{v}_m = \vec{v} + \vec{v}_R$



15.1 CROSSING THE RIVER :

To Cross a River

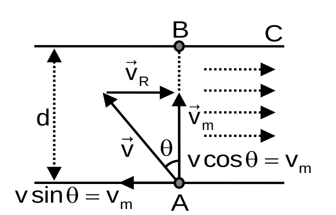
- Minimum distance of approach
  OR
  Cross the river along shortest possible path
  OR
  Cross the river and reach a point just opposite to the starting path
- Minimum time of approach
  OR
  Cross the river in shortest possible time

➢ **For shortest path:**

If a man wants to cross the river such that his "displacement should be minimum", it means he intends to reach just opposite point across the river. He should start swimming at an angle θ with the perpendicular to the flow of river towards upstream.

Such that its resultant velocity $\vec{v}_M = (\vec{v} + \vec{v}_R)$.

It is in the direction of displacement AB.

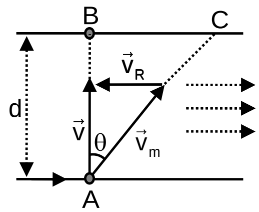


To reach at B $\quad v \sin\theta = v_R \Rightarrow \sin\theta = \frac{v_R}{v}$

component of velocity of man along AB is $v\cos\theta$

so time taken $t = \frac{d}{v\cos\theta} = \frac{d}{\sqrt{v^2 - v_R^2}}$

➢ **For minimum time:**



To cross the river in minimum time, the velocity along AB ($v \cos \theta$) should be maximum.

It is possible if $\theta = 0$, i.e. swimming should start perpendicular to water current.

Due to effect of river velocity, man will reach at point C along resultant velocity, i.e. his displacement will not be minimum but time taken to cross the river will be minimum.

$$t_{min} = \frac{d}{v}$$

In time t_{min} swimmer travels distance BC along the river with speed of river v_R distance travelled along river flow (drift of man)

SOLVED EXAMPLES

Example 62: Buses A and B are moving in the same direction with speeds 20 m/s and 15 m/s respectively. Find the relative velocity of A w.r.t. B and relative velocity of B w.r.t. A.

Solution: Let their direction of motion be along + x-axis then $\vec{v}_A = (20 m/s)\hat{i}$ and $\vec{v}_B = (15 m/s)\hat{i}$

(i) Relative velocity of A w.r.t. B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ = (actual velocity of A) – (velocity of B)

$= (20 m/s)\,\hat{i} - (15 m/s)\,\hat{i} = 5 m/s\,\hat{i}$

i.e. A is moving with speed 5m/s w.r.t. B in the same direction.

(ii) Relative velocity of B w.r.t. A is $\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$ (actual velocity of B) – (velocity of A)

$= (15 m/s)\,\hat{i} - (20 m/s)\,\hat{i} = (-5 m/s) = (5 m/s)(-\hat{i})$

i.e. B is moving in opposite direction w.r.t. A, at a speed 5 m/s.

Example 63: A police van moving on a highway with a speed of 30 km/hr. fires a bullet at a thief's car which is speeding away in the same direction with a speed of 190 km/hr. If the muzzle speed of the bullet is 150 m/s, find the speed of the bullet with respect to the thief's car.

(1) 760 km/hr (2) 540 km/hr (3) 380 km/hr (4) 190 km/hr

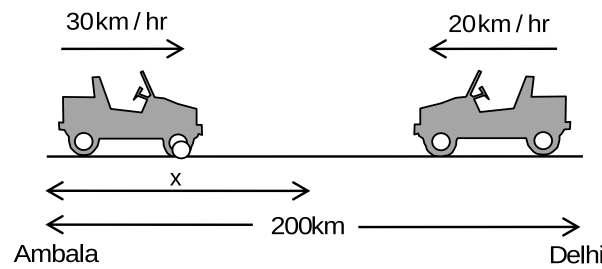
Solution: $v_b \rightarrow$ velocity of bullet $\quad v_p \rightarrow$ velocity of police van

$v_t \rightarrow$ velocity of thief's car $\quad v_{bp} = v_b - v_p \Rightarrow \quad v_{bp} = 150 \times \frac{18}{5} = 540$ km/hr

$v_b = v_{bp} + v_p = 540 + 30 = 570$ km/hr $\quad v_{bt} = v_b - v_t = 570 - 190 = 380$ km/hr.

Example 64: Delhi is at a distance of 200 km from Ambala. Car A sets out from Ambala at a speed of 30 km/hr. and car B set out at the same time from Delhi at a speed of 20 km/hr. When will they cross each other? What is the distance of that crossing point from Ambala ?

(1) 1 hr, 60 km
(2) 2 hr, 120 km
(3) 6 hr, 240 km
(4) 4 hr, 120 km



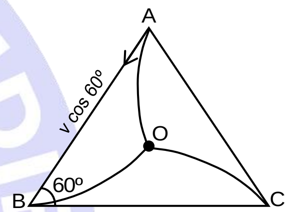
Solution: $\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$ = 30 – (– 20) = 50 km/hr

They will meet after time t

Given by t = $\frac{s}{v_{AB}} = \frac{200}{50}$ = 4hr

Distance from Ambala where they will meet is x = 30 × 4 = 120 km

Example 65: Three boys A, B and C are situated at the vertices of an equilateral triangle of side d at t =0. Each of the boys move with constant speed v. A always moves towards B, B towards C and C towards A. When and where will they meet each other.



(1) $\frac{d}{3v}$ (2) $\frac{2d}{v}$

(3) $\frac{2d}{3v}$ (4) $\frac{3d}{4v}$

Solution: By symmetry they will meet at the centroid of the triangle. Approaching velocity of A and B towards each other is v + v cos 60º and they cover distance d when they meet.

So that time taken, is given by

$$\therefore t = \frac{d}{v + v\cos 60^\circ} = \frac{d}{v + \frac{v}{2}} = \frac{2d}{3v}$$

Example 66: Two cars approach each other on a straight road with velocities 10 m/s and 12 m/s respectively. When they are 150 meters apart, both drivers apply their brakes and each car decelerates at 2 m/s^2 until they stops. How far apart will they be when both come to a halt ?

(1) 61 m (2) 150 m (3) 89 m (4) 36 m

Solution: Let x_1 and x_2 be the distance travelled by the cars before they stop under deceleration.

From IIIrd equation of motion $v^2 = u^2 + 2as$,

$\Rightarrow 0 = (10)^2 - 2 \times 2\, x_1 \Rightarrow x_1 = 25$m and $0 = (12)^2 - 2 \times 2x_2 \Rightarrow x_2 = 36$ m

Total distance covered by the two cars = $x_1 + x_2$ = 25 + 36 = 61m

Distance between the two cars when they stop = 150 – 61 = 89 m.

Example 67: Two trains A & B 100 km apart are travelling towards each other on different tracks with starting speed of 50 km/h for both. The train A accelerates at 20 km/h^2 and the train B retards at the rate 20 km/h^2 . The distance covered by the train A when they cross each other is :

(1) 45 km (2) 55 km (3) 65 km (4) 60 km

Solution: (4) From relative motion equation

$s_r = u_r t + \frac{1}{2} a_r t^2 \Rightarrow u_r = 100$ km/hr $a_r = 0, s_r = 100$ $100 = 100 \times t + 0 \Rightarrow t = 1$ hr

Distance travelled by A $S = 50 \times 1 + \frac{1}{2} \times 20 \times (1)^2 = 60$ km

Example 68: Two trains A and B which are 100 m and 60 m long are moving in opposite directions on parallel tracks. Velocity of the shorter train is 3 times that of the longer one. If the trains take 4 seconds to cross each other find the velocities of the trains?

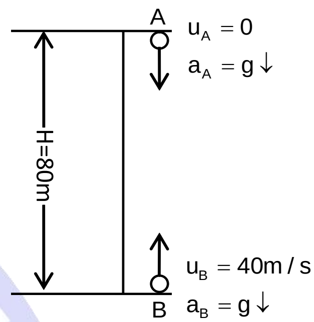
(1) $v_A = 20$ m/s, $v_B = 30$ m/s (2) $v_A = 5$ m/s, $v_B = 15$ m/s
(3) $v_A = 10$ m/s, $v_B = 30$ m/s (4) $v_A = 30$ m/s, $v_B = 10$ m/s

Solution: Given that $v_B = 3v_A$

Trains move in opposite directions then relative velocity $v_{rel} = \frac{d_{rel}}{time} \Rightarrow v_A + v_B = \frac{100+60}{4}$

$\Rightarrow 4v_A = 40 \Rightarrow v_A = 10$ m/s, $v_B = 3v_A = 30$ m/s

Example 69: A stone is dropped from a tower of height 80 m. At the same instant another stone is thrown from the foot of the tower with a speed of 40 m/s. When & where the stones will cross each other.
(1) At t = 2 s ; h = 30 m (2) At t = 4 s ; h = 60 m
(3) At t = 2 s ; h = 60 m (4) At t = 4 s ; h = 120 m



Solution: Here $S_{rel} = 80$ m,
$u_{rel} = 40$ m/s,
$a_{rel} = 0$ (Relative motion under gravity is a uniform motion)

So time $= \frac{S_{rel}}{U_{rel}} = 2$ s

Height from ground

$H = ut + \frac{1}{2}at^2 \Rightarrow H = 40(2) - \frac{1}{2}g(2)^2 = 60$ m

Example 70: A boy standing on a stationary lift (open from above) throws a ball upward with the maximum initial speed he can, equal to 49 m s^{-1}. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?
(1) 6 s (2) 8 s (3) 10 s (4) 16 s

Solution: When lift is stationary, time of flight $= \frac{2u}{g} = \frac{2\times 49}{9.8} = 10$ sec

When lift is moving constant speed $\Rightarrow S_{rel} = U_{rel}.t - \frac{1}{2}gt^2 \Rightarrow S_{rel} = 0$, $U_{rel} = 49$ m/sec

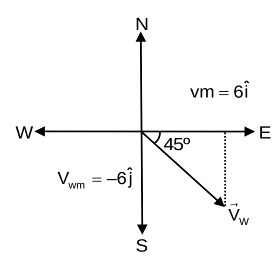
$0 = 49t - \frac{1}{2} \times 9.8 \times t^2 \Rightarrow t = 10$ sec

Example 71: A person moves due east at a speed 6m/s and feels the wind is blowing towards south at a speed 6m/s.
(i) Find actual velocity of wind blow.
(1) 12 m/s along S-E (2) 6 m/s along S-E
(3) $12\sqrt{2}$ m/s along S-E (4) $6\sqrt{2}$ m/s along S-E
(ii) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.
(1) 12 m/s along S-W (2) 6 m/s along S-W
(3) $12\sqrt{2}$ m/s along S-W (4) $6\sqrt{2}$ m/s along S-W

Solution: (i) $\vec{v}_{act} = \vec{v}_{rel} + \vec{v}_{ref}$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m = -6\hat{j} + 6\hat{i} \Rightarrow \vec{v}_w = 6\hat{i} - 6\hat{j}$

$|\vec{v}| = 6\sqrt{2}$ m/s and it is blowing along S-E



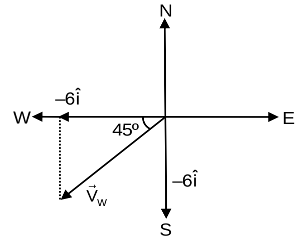
(ii) Person doubles its velocity then

$\vec{v}_m = 12\hat{i}$ but actual wind velocity remains unchanged.

$$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = \left(6\hat{i} - 6\hat{j}\right) - 12\hat{i}$$

$$\Rightarrow \vec{v}_{wm} = 6\hat{i} - 6\hat{j}$$

Now relative velocity of wind is $6\sqrt{2}$ m/s along S-W.



Example 72: A man is going east in a car with velocity of 20 km/hr, a train appears to move towards north to him with a velocity of $20\sqrt{3}$ km/hr. What is the actual velocity and direction of motion of train ?

(1) 30 km/hr 60° north of east (2) 45 km/hr 45° north of east

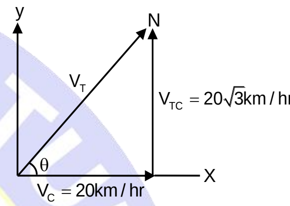
(3) 40 km/hr 60° north of east (4) 25 km/hr 30° south of east

Solution: $\vec{v}_{TC} = \vec{v}_T - \vec{v}_C \Rightarrow \vec{v}_{TC} + \vec{v}_C = 20\sqrt{3}\,\hat{j} + 20\hat{i}$

$|\vec{v}_T| = \sqrt{(20\sqrt{3})^2 + (20)^2} = \sqrt{1600}$ = 40 km/hr

$\tan\theta = \dfrac{20\sqrt{3}}{20} \Rightarrow \theta = 60°$

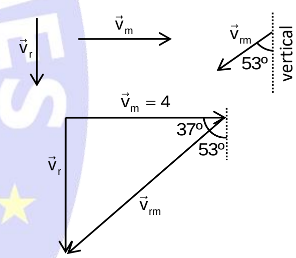
So direction of motion of train is 60° N of E or E - 60° - N



Example 73: A man at rest observes the rain falling vertically. When he walks at 4 km/h, he has to hold his umbrella at an angle of 53° from the vertical. Find the velocity of raindrops.

(1) $\dfrac{2}{3}$ km/hr (2) 3 km/hr

(3) $\dfrac{5}{3}$ km/hr (4) $\dfrac{4}{3}$ km/hr

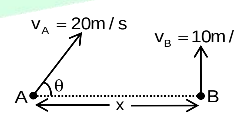


Solution: Assigning usual symbols and to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following equation $\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$

The above equation suggests that a still man observes velocity $\vec{v}_r$ of rain, relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

Therefore $v_r = v_m \tan 37° = 3$ km/h

Example 74: Two particles A and B are projected from the ground simultaneously in the directions shown in the figure with initial velocities v_A = 20 m/s and v_B = 10 m/s respectively. They collide after 0.5 s. Find out the angle θ and the distance x.



(1) θ = 30°, x = 5m (2) θ = 60°, x = $5\sqrt{2}$ m

(3) θ = 30°, x = $5\sqrt{3}$ m (4) θ = 60°, x = $10\sqrt{3}$ m

Solution: Both particle will collide if they are at same height in same time.

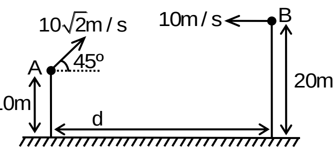
$y_A = y_B \Rightarrow (u_y)_A t - \dfrac{1}{2}gt^2 = (u_y)_B t - \dfrac{1}{2}gt^2 \Rightarrow (u_y)_A = (u_y)_B \Rightarrow (v_A \sin\theta) = v_B$

$\Rightarrow 20 \sin\theta = 10 \Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30°$

In 0.5s horizontal distance covered by A is

$x = (u_x)_A\, t = (20 \cos 30^o)0.5 = 10\times\frac{\sqrt{3}}{2} = 5\sqrt{3}m$

Example 75: Two particles are projected from the two towers simultaneously, as shown in the figure. What should be the value of 'd' for their collision?



(1) 10 m (2) 16 m (3) 20 m (4) 24 m

Solution: Their is no relative acceleration of between A and B.

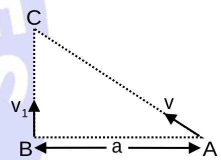
So time of collision $t = \frac{y_{BA}}{(v_y)_{BA}}$ where y_{BA} = vertical displacement of B w.r.t. A = 10 m.

$(v_y)_{BA}$ = vertical velocity of B w.r.t.

A = $0 - (-10\sqrt{2}\sin 45^o)$ = 10 m / s $\Rightarrow$ t = 10 / 10 = 1s

d = horizontal distance travelled by B w.r.t. A = $(v_x)_{BA}$. t = $(10+10\sqrt{2}\cos 45^o)$ × 1 = 20m.

Example 76: Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB, with velocity v_1.The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, then find t.



(1) $\sqrt{\frac{a}{v-v_1}}$ (2) $\sqrt{\frac{a^2}{v_1^2+v^2}}$

(3) $\sqrt{\frac{a^2}{v^2-v_1^2}}$ (4) $\frac{a^2}{v^2-v_1^2}$

Solution: Let the two boys meet at point C after time 't'

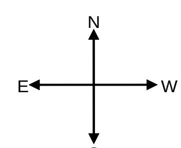
Then AC = vt, BC = v_1t

but $(AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow v^2t^2 = a^2 + v_1^2t^2 \Rightarrow t^2 = (v^2 - v_1^2) = a^2$

$\Rightarrow t = \sqrt{\frac{a^2}{v^2-v_1^2}}$

Example 77: Two bodies A and B are 10 km apart such that B is to the south of A. A and B start moving with the same speed 20 km/hr eastward and northward respectively then find.



(i) relative velocity of A w.r.t. B.

(ii) minimum separation attained during motion.

(iii) time elapsed from starting, to attain minimum separation.

Solution: (i) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 20\hat{i} - 20\hat{j} \Rightarrow |\vec{v}_{AB}| = 20\sqrt{2}$

(ii) Minimum distance = $\frac{v_2\times d}{\sqrt{v_1^2+v_2^2}} = \frac{20\times 10}{20\sqrt{2}} = 5\sqrt{2}$

(iii) Time to attain minimum separation = $\dfrac{5\sqrt{2}}{20\sqrt{2}} = \dfrac{1}{4} \times 60 = 15$ min

Example 78: A boat moves along the flow of river between two fixed points A and B. It takes t_1 time when going downstream and takes t_2 time when going upstream between these two points. What time it will take in still water to cover the distance equal to AB.

(1) $\left(\dfrac{2t_1t_2}{t_1+t_2}\right)$ (2) $\left(\dfrac{t_1t_2}{t_1+t_2}\right)$

(3) $\left(\dfrac{t_1+t_2}{2t_1t_2}\right)$ (4) $\left(\dfrac{t_1+t_2}{2}\right)$

Solution: $t_1 = \dfrac{AB}{v_b + v_R}$, $t_2 = \dfrac{AB}{v_b - v_R}$ or $v_b + v_R = \dfrac{AB}{t_1}$ and $v_b - v_R = \dfrac{AB}{t_2}$

$\Rightarrow 2v_b = \dfrac{AB}{t_1} + \dfrac{AB}{t_2} = AB\left(\dfrac{t_1+t_2}{t_1t_2}\right)$ or $\left(\dfrac{2t_1t_2}{t_1+t_2}\right) = \dfrac{AB}{v_b}$ time taken by the boat to cover AB

Example 79: A boat can be rowed at 5 m/s in still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(i) In which direction must it be steered to cross the river perpendicular to current?

(ii) How long will it take to cross the river in a direction perpendicular to the river flow ?

(iii) In which direction must the boat be steered to cross the river in minimum time ? How far will it drift?

Solution: (i) To cross the river perpendicular to current i.e. along shortest path, $\sin\theta = \dfrac{u}{v} = \dfrac{3}{5} \Rightarrow \theta = 37^\circ$

(ii) Time taken by boat, $t = \dfrac{d}{v\cos\theta} = \dfrac{200}{5\times\dfrac{4}{5}} = 50s$

(iii) To cross the river in minimum time, $\theta = 0^\circ$.

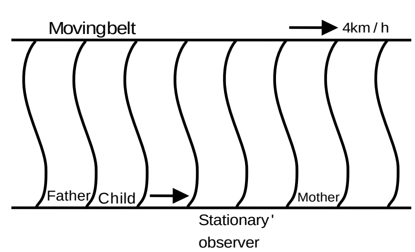
Therefore, $t_{min} = \dfrac{d}{v} = \dfrac{200}{5}s = 40s$ Drift = $u(t_{min})$ = 3(40)m = 120 m

Example 80: A child runs to and fro with a speed 9 km h^{-1} (with respects to the belt) on a long horizontally moving belt (fig.) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on stationary platform outside, what is the

(i) speed of the child while running in the direction of motion of the belt ?

(ii) speed of the child while running opposite to the direction of motion of the belt ?

(iii) time taken by the child in (i) and (ii) ?



Which of the answers will alter if motion is viewed by one of the parents ?

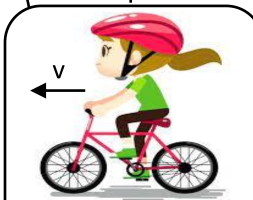
Solution: For an observer on stationary plate from out side

(i) speed of child w.r.t. observer = 9 + 4 = 13 km/h

(ii) speed of child w.r.t. observer = 9 – 4 = 5 km/h

(iii) time taken by child = $\frac{50}{9\times\frac{5}{18}}$ = 20 sec. a and b answer alter if motion viewed by one of parents.

Motion



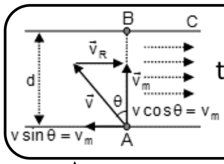
Relative motion

2-D motion

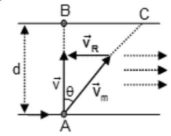
$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\Rightarrow |\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$

River-boat



$t_{\text{shortest path}} = \dfrac{d}{v\cos\theta} = \dfrac{d}{\sqrt{v^2 - v_R^2}}$



$t_{min} = \dfrac{d}{v}$

1-D motion

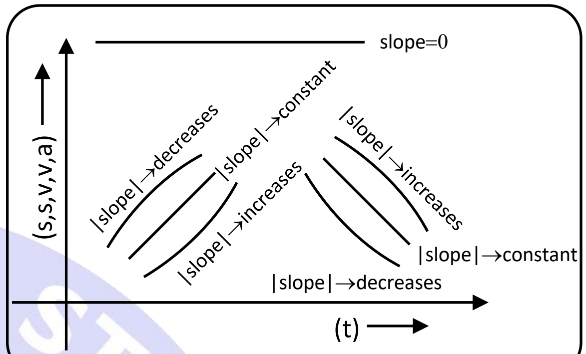
$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$ (same direction)

$\vec{v}_{12} = \vec{v}_1 + \vec{v}_2$ (opposite direction)

Equation of motion

- $v_{rel.} = u_{rel.} + a_{rel.}\, t$
- $s_{rel.} = u_{rel.} t + \dfrac{1}{2} a_{rel.} t^2$
- $v_{rel.}^2 = u_{rel.}^2 + 2a_{rel} s_{rel}$
- $s_{rel.} = \dfrac{1}{2}(u_{rel.} + v_{rel.})t$

Graph



1-D Motion parameter

Distance
- Length of actual path
- Scalar quantity
- Can never be -ve
- Always increases
- May be constant
- May not be zero
- SI unit m
- CGS unit cm

Displacement
- Shortest distance
- Can be +ve or -ve
- May increase or decrease
- May be zero
- Vector quantities
- SI unit m
- CGS unit cm
- |Displacement| ≤ distance

Speed
- Time rate of distance
- Scalar quantity
- Can never be -ve
- May increase or decrease
- May be zero
- SI unit m/s
- CGS unit cm/s

Velocity
- Time rate of displacement
- Vector quantity
- Can be +ve or -ve
- May increase or decrease
- May be zero
- SI unit m/s
- CGS unit cm/s
- |velocity| = speed

Acceleration
- Time rate of velocity
- Vector quantity
- Can be +ve or -ve
- May increase or decrease
- May be zero
- SI unit m/s^2
- CGS unit cm/s^2

Equations
- $v = u + at$
- $s = ut + \dfrac{1}{2}at^2$
- $v^2 - u^2 = 2as$
- $s = v_{av}\, t = \dfrac{(u+v)}{2} t$
- $s = vt - \dfrac{1}{2}at^2$
- $s_{nth} = u + \dfrac{1}{2}a(2n-1)$

If $v = f(t)$: $a = \dfrac{dv}{dt}$

If v = f(x): $a = \dfrac{v\,dv}{dx}$

a = f(t): $\int_u^v dv = \int_{t_1}^{t_2} a\,dt$

a = f(x): $\int_u^v v\,dv = \int_{x_1}^{x_2} a\,dx$

Rate of change of speed = $\vec{a} \cdot \hat{v}$

2-D Projectile motion

Ground to ground projection
- $x = (u\cos\theta)t$
- $y = u_y t - \dfrac{1}{2}gt^2 = (u\sin\theta)\,t - \dfrac{1}{2}gt^2$
- Time of flight $= \dfrac{2u_y}{g} = \dfrac{2u\sin\theta}{g}$
- Max. height $\dfrac{u_y^2}{2g} = \dfrac{u^2\sin^2\theta}{2g}$
- $R = \dfrac{2u_x u_y}{g} = \dfrac{2(u\cos\theta)(u\sin\theta)}{g} = \dfrac{u^2\sin 2\theta}{g}$
- $y = x\tan\theta - \dfrac{1}{2}g\dfrac{x^2}{u^2\cos^2\theta}$

or $y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$