Chapter 02 Sequence And Series

RANKER'S STUFF

SINGLE CORRECT QUESTIONS

Q.1 The sum of n terms of the series

$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots$ is-

(1) $n \log \left(\frac{a}{b}\right)$

(2) n log (ab)

(3) $\frac{n^2}{2} \log \frac{a}{b} + \frac{n}{2} \log (ab)$

(4) $\frac{n^2}{2} \log \frac{a}{b} - \frac{n}{2} \log (ab)$

Q.2 If S denotes the sum to infinity and S_n the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is-

(1) 11 (2) 9

(3) 10 (4) 8

Q.3 The sum of 10 terms of the series

$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \ldots$ is -

(1) $\left(\frac{x^{20} - 1}{x^2 - 1}\right)\left(\frac{x^{22} + 1}{x^{20}}\right) + 20$

(2) $\left(\frac{x^{18} - 1}{x^2 - 1}\right)\left(\frac{x^{11} + 1}{x^9}\right) + 20$

(3) $\left(\frac{x^{18} - 1}{x^2 - 1}\right)\left(\frac{x^{11} - 1}{x^9}\right) + 20$

(4) None of these0

Q.4 If 0 < x, y, a, b < 1, then the sum of the infinite terms of the series

$\sqrt{x}(\sqrt{a} + \sqrt{x}) + \sqrt{x}(\sqrt{ab} + \sqrt{xy}) + \sqrt{x}(b\sqrt{a} + y\sqrt{x}) + \ldots$ is-

(1) $\frac{\sqrt{ax}}{1+\sqrt{b}} + \frac{x}{1+\sqrt{y}}$ (2) $\frac{\sqrt{x}}{1+\sqrt{b}} + \frac{x}{1+\sqrt{y}}$

(3) $\frac{\sqrt{x}}{1-\sqrt{b}} + \frac{\sqrt{x}}{1-\sqrt{y}}$ (4) $\frac{\sqrt{ax}}{1-\sqrt{b}} + \frac{x}{1-\sqrt{y}}$

Q.5 If sum of 3 terms of a G.P. is S. product is P, and sum of reciprocal of its terms is R, then $P^2 R^3$ equals to -

(1) S (2) S^3

(3) $2S^2$ (4) S^2/R

Q.6 If A and G are respectively A.M. and G.M. of roots of a quadratic equation, then it is-

(1) $x^2 + 2Ax + G^2 = 0$

(2) $x^2 - 2Ax + G^2 = 0$

(3) $x^2 - Ax + G = 0$

(4) None of these

Q.7 If t_n be the n^{th} term of an A.P. and if $t_7 = 9$, then the value of the common difference that would make $t_1 t_2 t_7$ least is-

(1) 33/40 (2) 33/20

(3) 33/10 (4) None of these

Q.8 If m^{th} terms of the series 63 + 65 + 67 + 69 + ... and 3 + 10 + 17 + 24 + ... be equal, then m =

(1) 11 (2) 12 (3) 13 (4) 15

Q.9 A ball falls from a height of 100 mts. on a floor. If in each rebound it describes 4/5 height of the previous falling height, then the total distance travelled by the ball before coming to rest is-

(1) ∞ (2) 500 mts

(3) 1000 mts (4) 900 mts

Q.10 If A, G and H are respectively A.M., G.M., and H.M. of three positive numbers a, b and c, then the equation whose roots are a, b and c is given by-

(1) $x^3 - 3Ax^2 + 3G^3 x + G^3 = 0$

(2) $x^3 - 3Ax^2 + 3(G^3/H) x - G^3 = 0$

(3) $x^3 + 3Ax^2 + 3(G^3/H) x - G^3 = 0$

(4) $x^3 - 3Ax^2 - 3(G^3/H) x + G^3 = 0$

Q.11 The G.M. of roots of the equation $x^2 - 2ax + b^2 = 0$ is equal to which type of mean of roots of $x^2 - 2bx + a^2 = 0$?

(1) A.M. (2) G.M.

(3) H.M. (4) None of these

Q.12 If a and ℓ be the first and last term of an A.P. and S be the sum of its all terms; then its common difference is-

(1) $\frac{\ell^2 + a^2}{2S - \ell - a}$ (2) $\frac{\ell^2 - a^2}{2S - \ell - a}$

(3) $\frac{\ell^2 - a^2}{2S + \ell + a}$ (4) None of these

Q.13 If x,y,z are in A.P. , then magnitude of its common difference is-

(1) $\sqrt{(x^2 - yz)}$ (2) $\sqrt{(y^2 - zx)}$

(3) $\sqrt{(z^2 - xy)}$ (4) None of these

Q.14 If sum of infinite G.P. is x and sum of square of its terms is y, then common ratio is-

(1) $\frac{x^2 + y}{x^2 - y}$ (2) $\frac{x^2 - y}{x^2 + y}$

(3) $\frac{x^2 + y^2}{x^2 - y^2}$ (4) $\frac{x^2 - y^2}{x^2 + y^2}$

Q.15 The sum of 10 terms of the series.

0.7 + .77 + .777 + ... is-

(1) $\frac{7}{9}\left(89 + \frac{1}{10^{10}}\right)$ (2) $\frac{7}{81}\left(89 + \frac{1}{10^{10}}\right)$

(3) $\frac{7}{81}\left(89 + \frac{1}{10^{9}}\right)$ (4) None of these

Q.16 The value of $a^{\log_b x}$ where a = 0.2, $b = \sqrt{5}$,

$x = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\infty\right)$, is-

(1) 1 (2) 2 (3) 1/2 (4) 4

Q.17 If the sum of four numbers in A.P. be 48 and that the product of the extremes is to the product of the means is 27 to 35 then the numbers are-

(1) 3, 9, 15, 21 (2) 9, 5, 7, 3

(3) 6, 10, 14, 18 (4) None of these

Q.18 If a, b, c, d are in G.P., then the value of $(a - c)^2 + (b - c)^2 + (b - d)^2 - (a - d)^2$ is-

(1) 0 (2) 1 (3) a + d (4) a – d

Q.19 The third term of an A.P. is 9 and the difference of the seventh and the second term is 20. If the number 2001 is the nth term of the sequence then n is-

(1) equal to 499 (2) is equal to 500

(3) equal to 501 (4) can have no value

Q.20 Given the geometric progression 3, 6,12, 24,..... the term 12288 would occur as the-

(1) 11th term (2) 12th term

(3) 13th term (4) 14th term

Q.21 The maximum sum of the series

$20 + 19\frac{1}{3} + 18\frac{2}{3} + ...$ is -

(1) 310 (2) 300

(3) 320 (4) None of these

Q.22 Let a, b be the roots of $x^2 - 3x + p = 0$ and let c, d be the roots of $x^2 - 12x + q = 0$, where a, b, c, d form an increasing G.P. Then the ratio of q + p : q – p is equal to -

(1) 8 : 7 (2) 11 : 10

(3) 17 : 15 (4) None of these

Q.23 If $\frac{a + bx}{a - bx} = \frac{b + cx}{b - cx} = \frac{c + dx}{c - dx}$ $(x \neq 0)$, then a, b, c, d are in -

(1) A.P. (2) G.P.

(3) H.P. (4) None of these

Q.24 The sum of the first n terms of the series $\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + ...$ is -

(1) $\frac{6n}{n+1}$ (2) $\frac{9n}{n+1}$ (3) $\frac{12n}{n+1}$ (4) $\frac{15n}{n+1}$

Q.25 If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then $\frac{S_3(1+8S_1)}{S_2^2}$ is equal to -

(1) 1 (2) 3 (3) 9 (4) 10

Q.26 The sum of three consecutive terms in a geometric progression is 14. If 1 is added to the first and the second terms and 1 is subtracted from the third, the resulting new terms are in arithmetic progression. Then the lowest of the original terms is -

(1) 1 (2) 2 (3) 4 (4) 8

Q.27 If S_n denotes the sum of n terms of an A.P., then $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to -

(1) 0 (2) 1 (3) 1/2 (4) 2

Q.28 A G.P. consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms at the even places is S_2, then S_2/S_1 is -

(1) Dependent on a

(2) Independent of r

(3) Independent of a and r

(4) Dependent on r

Q.29 If $x^{18} = y^{21} = z^{28}$, then 3, 3 $\log_y x$, 3 $\log_z y$, 7 $\log_x z$ are in -

(1) A.P. (2) G.P. (3) H.P. (4) None

Q.30 The sum of integers from 1 to 100 that are divisible by 2 or 3, is -

(1) 3300 (2) 3330

(3) 3000 (4) None of these

Q.31 The sum of an infinitely decreasing G.P. is equal to 4 and the sum of the cubes of its terms is equal to $\frac{64}{7}$. Then 5th term of the progression is-

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{16}$ (4) $\frac{1}{32}$

Q.32 If the sum of the first 2n terms of the A.P. 2, 5, 8, ... is equal to the sum of first n terms of the A.P. 57, 59, 61, ..., then n equals -

(1) 10 (2) 11 (3) 12 (4) 13

Q.33 Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + ...+ \frac{1+2+.......+n}{1^3+2^3+.....n^3}$; n = 1, 2, 3, ... Then S_n is not greater than-

(1) 1/2 (2) 1 (3) 2 (4) 4

Q.34 The number of common terms to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ... 466 is -

(1) 21 (2) 19 (3) 20 (4) 91

NUMERICAL VALUE TYPE QUESTIONS

Q.35 Let fourth term of an arithmetic progression be 6 and m^{th} term be 18. If A.P. has integral term only then the numbers of such A.P. s is______ .

Q.36 Let $a_1, a_2, a_3,......, a_{10}$ are in G.P. with $a_{101} = 25$ and $\sum_{i=1}^{201} a_i = 625$. Then the value of $\sum_{i=1}^{201} \frac{1}{a_i}$ equals ________ .

Q.37 Let a_n = 16, 4, 1,.... be a geometric sequence. Define P_n as the product of the first n terms. The value of $\sum_{n=1}^{\infty} \sqrt[n]{P_n}$ is ______.

Q.38 The terms a_1, a_2, a_3 form an arithmetic sequence whose sum is 18. The terms $a_1 + 1$, a_2, $a_3 + 2$, in that order, form a geometric sequence. Then the sum of all possible common difference of the A.P. is ______ .

Q.39 The sum of the infinite Arithmetic–Geometric progression 3, 4, 4, ... is _____.

Q.40 $\sum_{r=1}^{50}\frac{r^2}{r^2+(11-r)^2}$ is equal to ______.

Q.41 If $\sum_{r=0}^{100}\frac{1}{2^r+2^{50}}=\frac{n}{2^{50}}$, then the value of n is ____.

Q.42 Let S denote sum of the series

$\frac{3}{2^3}+\frac{4}{2^4.3}+\frac{5}{2^6.3}+\frac{6}{2^7.5}+....\infty$ Then the value of S^{-1} is ______.

STATEMENT TYPE QUESTIONS

Each question contains Statement-I and statement-II. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

(A) Statement-I is True, Statement-II is True, statement-II is a correct explanation for statement-I

(B) Statement-I is True, statement-II is True, statement-II is NOT a correct explanation for statement-I

(C) Statement-I is True statement-II is False

(D) Statement-I is False statement-II is True

Q.43 **Statement-I :** The sum of reciprocals of first n terms of series $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+...$ is n^2. and

Statement-II : A sequence is said to be H.P. if the reciprocals of its terms are in A.P.

(1) A (2) B (3) C (4) D

Q.44 **Statement-I:** Let m, n, a, b and c are non-zero real numbers such that a, b, c are in H.P., then $\frac{a}{m+na}$, $\frac{b}{m+nb}$, $\frac{c}{m+nc}$ are also in H.P.

Statement-II: If a, b, c are in G.P., then $a-\frac{b}{2}$, $\frac{b}{2}$, $c-\frac{b}{2}$ are in H.P.

(1) A (2) B (3) C (4) D

Q.45 **Statement-I** : If a, b, c are three positive numbers in G.P. then $\left(\frac{a+b+c}{3}\right)\left(\frac{3abc}{ab+bc+ac}\right)=\left(\sqrt[3]{abc}\right)^2$

Statement-II : (A.M.) (H.M) = $(G.M.)^2$ is true for positive numbers.

(1) A (2) B (3) C (4) D

Q.46 **Statement-I** : If x > 1, the sum to infinite number of $1+3\left(1-\frac{1}{x}\right)+5\left(1-\frac{1}{x}\right)^2+7\left(1-\frac{1}{x}\right)^3+$ .... is $2x^2-x$.

Statement-II : If 0 < y < 1, the sum of the series $1+3y+5y^2+7y^3+....$ is $\frac{1+y}{(1-y)^2}$

(1) A (2) B (3) C (4) D

Q.47 **Statement-I**: The infinite A.GP. 1, $\sqrt{3}$, 2, x ... has a finite sum, then x =2 and

Statement-II : The infinite A.GP. a, (a + d)r, (a + 2d)r^2 ..... has a finite sum only if |r|< 1.

(1) A (2) B (3) C (4) D

MORE THAN ONE CORRECT TYPE QUESTIONS

Q.48 For an increasing A.P. $a_1, a_2, ..., a_n$ if $a_1+a_3+a_5=-12$ and $a_1a_3a_5=80$, then which of the following is/are true?

(1) $a_1=-10$ (2) $a_2=-1$

(3) $a_3=-4$ (4) $a_5=+2$

Q.49 If the sum of n terms of an A.P. is given by $S_n=a+bn+cn^2$, where a, b, c are independence of n, then

(1) a = 0

(2) common difference of A.P. must be 2b

(3) common difference of A.P. must be 2c

(4) first term of A.P. is b + c

Q.50 If sum of an infinite G.P. p, 1, $1/p^2$, … is 9/2, then value of p is

(1) 2 (2) 3/2 (3) 3 (4) 9/2

Q.51 The terms of an infinitely decreasing G.P. having common ratio r in which all the terms are positive, the first term is 4, and the difference between the third and fifth terms in 32/81, then

(1) r = 1/3

(2) r = $2\sqrt{2}/3$

(3) Sum of infinite terms is 6

(4) none of these

Q.52 Let $a_1, a_2, a_3, \ldots, a_n$ be in G.P. such that $3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$. Then common ratio of G.P. can be

(1) 2 (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $-\frac{1}{2}$

COMPREHENSION TYPE QUESTIONS

Q.53 In a G.P., the sum of the first and last terms is 66, the product of the second and the last second is 128, and the sum of the terms is 126.

(i) If an increasing G.P. is considered, then the number of terms in G.P. is

(1) 9 (2) 8 (3) 12 (4) 6

(ii) If the decreasing G.P. is considered, then the sum of infinite terms is

(1) 64 (2) 128

(3) 256 (4) 729

(iii) In any case, the difference of the least and greatest terms is

(1) 78 (2) 126

(3) 126 (4) None of these

Q.54 Consider the sequence in the form of groups (1), (2, 2), (3,3,3), (4,4,4,4), (5,5,5,5,5), …..

(i) The 2000th term of the sequence is not divisible by

(1) 3 (2) 9

(3) 7 (4) none of these

(ii) The sum of first 2000 terms is

(1) 84336 (2) 96324

(3) 78466 (4) None of these

(iii) The sum of the remaining terms in the group after 2000th term in which 2000th term lies is

(1) 1088 (2) 1008

(3) 1040 (4) none of these

MATCH THE COLUMN TYPE QUESTIONS

Q.55 If α and β are roots of the equation $x^2 - 8x + 4 = 0$, then match the following lists:

List I	List II
(A) If $\alpha, a_1, a_2, a_3, a_4, a_5, \beta$ are in A.P., then $\sum_{i=1}^{5} a_i$ is divisible by	**(P)** 3
(B) If $\alpha, a_1, a_2, a_3, a_4, a_5, \beta$ are in G.P., then $\sum_{i=1}^{5} \log_2 a_i$ is divisible by	**(Q)** 4
(C) If $\alpha^2, (a_1)^2, (a_2)^2, (a_3)^2, (a_4)^2, (a_5)^2, \beta^2$ are in A.P., then $\sum_{i=1}^{5} a_i^2$ is divisible by	**(R)** 5
(D) If $\alpha, a_1, a_2, a_3, a_4, a_5, \beta$ are in H.P., then $\left(\frac{1}{\sum_{i=1}^{5}\frac{1}{a_i}} + 1\right)$ is divisible by	**(S)** 7

Q.56 Match the following lists

List I	List II
(A) If x, y, z are real and $4x^2 + 9y^2 + 16z^2 - 6xy - 12y - 8zx = 0$, then x, y, z are	**(P)** in A.P.
(B) If $21(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$, then x, y, z are	**(Q)** in G.P.

(C)	If x, y, z > 0 and $216x^3 + 64y^3 + 27z^3 = 72xyz$, then x, y, z are	**(R)** in H.P.
(D)	If $ax^2 + 2px + b = 0$ has root '–1', $ax^2+2qx+b=0$ has equal roots and the line $\frac{x}{a}+\frac{y}{b}=\frac{2}{r}$ passes through the point (1,1) then p,q,r are	**(S)** not A.P.,G.P. or H.P.

ANSWER KEY

RANKER'S STUFF

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	1	1	4	2	1	2	3	4	2	1	2	2	2	2
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	4	3	1	3	2	1	3	2	1	3	2	1	4	1	4
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	2	2	3	3	9	1	32	-1	27	25	50	2	2	3	3
Que.	46	47	48	49	50	51	52	53(i)	53(ii)	53(iii)	54(i)	54(ii)	54(iii)		
Ans.	1	1	1,3,4	1,3,4	2,3	1,2,3	2,4	4	2	4	4	1	**2**		

Q.55 A $\rightarrow$ Q, R; B $\rightarrow$ R; C $\rightarrow$ Q, R, S; D $\rightarrow$ P.

Q.56 A $\rightarrow$ R; B $\rightarrow$ Q; C $\rightarrow$ P; D $\rightarrow$ Q.